Exotic bulk viscosity and its influence on neutron star r-modes

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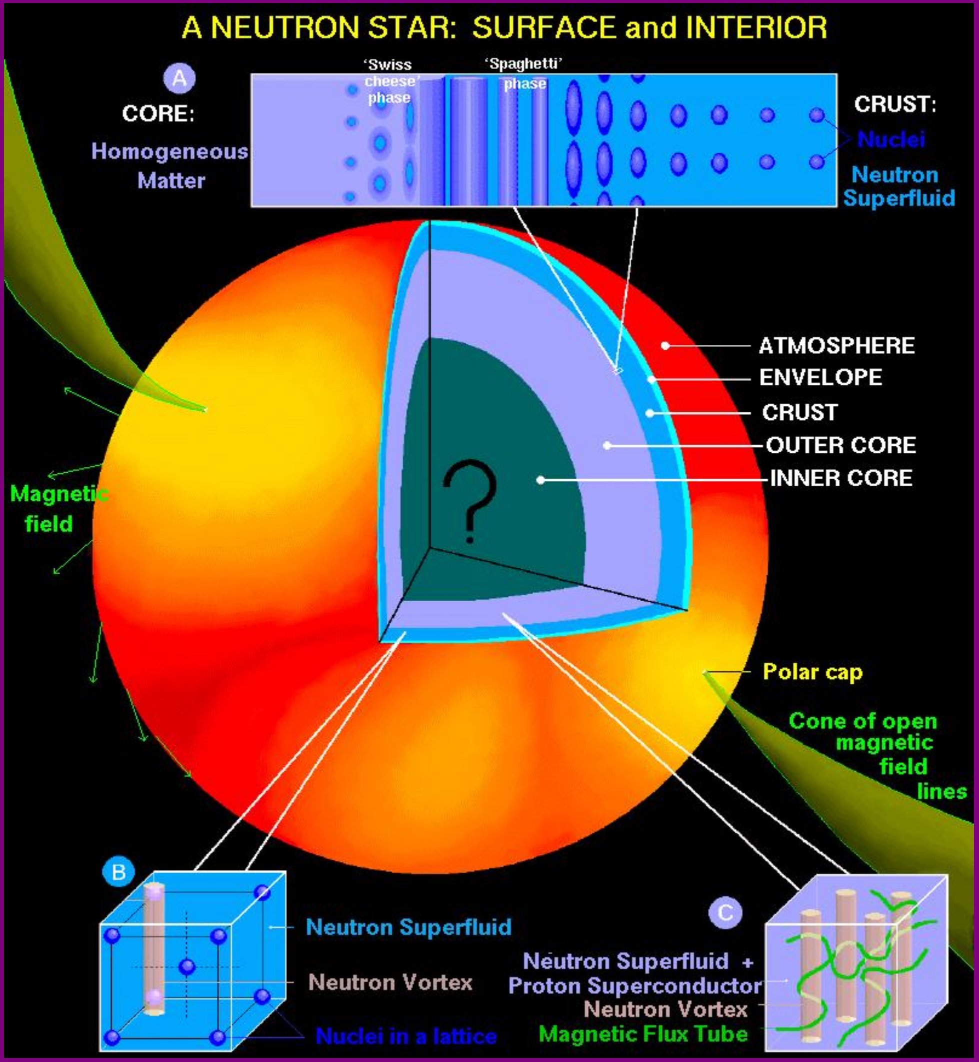
Outline of talk

- Exotic matter in neutron star interior
- Rapidly rotating neutron stars and r-mode instability
- Effect of hyperon-hyperon interaction on bulk viscosity and r-mode stability
- Outlook

Structure of a neutron star



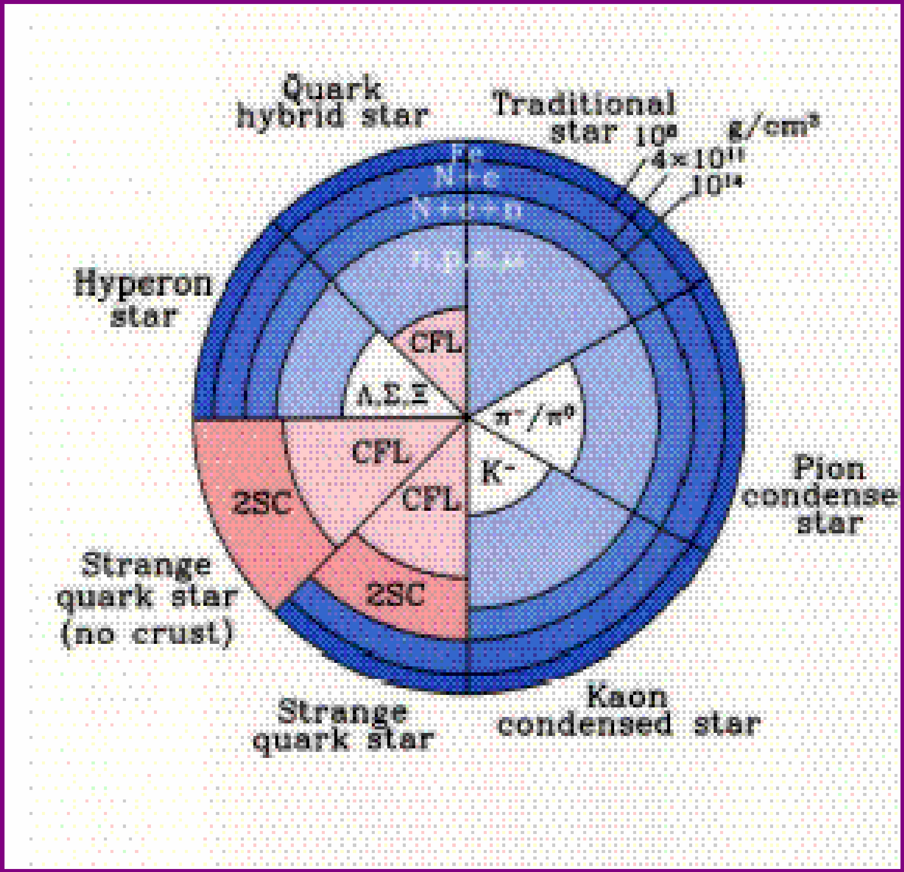
- *Atmosphere (atoms)*

  $n \leq 10^{4}\ g/cm^3$
- *Outer crust (free electrons, lattice of nuclei)*

  $10^{4} - 4 \times 10^{11}\ g/cm^3$
- *Inner crust (lattice of nuclei with free electrons and neutrons)*
- *Outer core (atomic particle fluid)*
- *Inner core (exotic subatomic particles?)*

  $n \geq 10^{14}\ g/cm^3$

Possible interior neutron star phases



- Hyperons
- Bose-Einstein condensates of pions and kaons
- Quarks

Hyperons

- Hyperons produced at the cost of nucleons

$$n + p \rightarrow p + \Lambda + K^0 \;,\; n + n \rightarrow n + \Sigma^- + K^+$$

- Chemical equilibrium through weak processes

$$p + e^- \rightarrow \Lambda + \nu_e \,,\; \Lambda + e^- \rightarrow \Xi^- + \nu_e$$

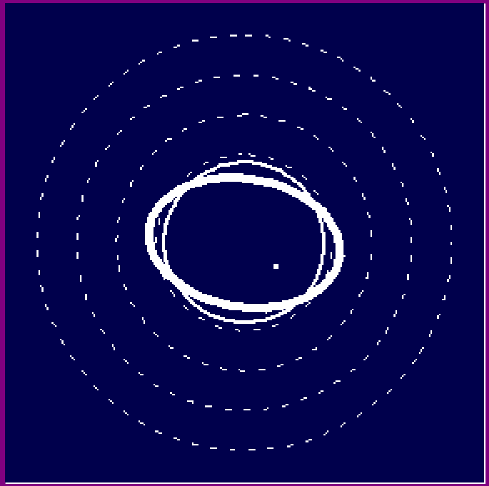
- General condition for β-equilibrium

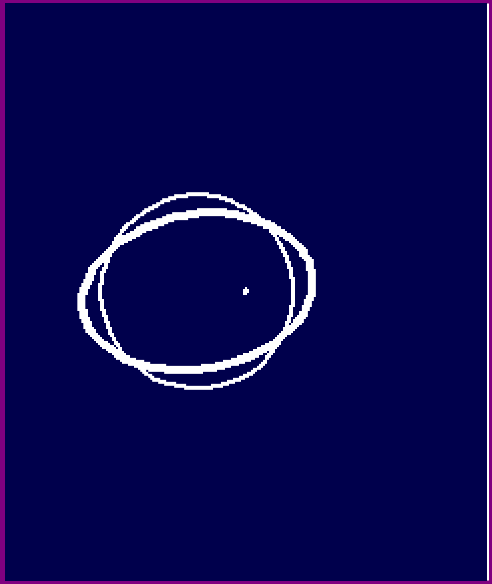
$$\mu_i = b_i \mu_n - q_i \mu_e$$

r-mode instability

- *A non-axisymmetric instability driven by gravitational radiation*
- *Rotation provides the restoring force (Coriolis)*
- *R-mode instability is responsible for slowing down a rapidly rotating, newly born neutron star*
- *r-modes are unstable to CFS (**C**handrasekhar 1970, **F**riedman and **S**chutz 1978) instability*

Gravitational Radiation Reaction driven instability





- *For slowly rotating stars,GR removes positive angular momentum from forward moving mode, and negative angular momentum from backward mode $\Rightarrow$ GR damps the perturbation*
- *For a star with large Ω, a mode that moves backward relative to the star appears as a forward moving mode relative to the inertial observer (CFS instability)*
- *GR removes positive angular momentum from a mode whose angular momentum is negative*

  (moves backward relative to the fluid)
- *By making angular momentum of the perturbation increasingly negative, GR drives the mode*

Growth vs Damping

- *Bulk viscosity: arises because the pressure and density variations associated with the mode oscillation drive the fluid away from chemical equilibrium. It estimates the energy dissipated from the fluid motion as weak interaction tries to re-establish equilibrium*
- *Viscosity tends to counteract the growth of the GW instability*
- *Viscosity would stabilize any mode whose growth time is longer than the viscous damping time*
- *There must exist a critical angular velocity Ω_c above which the perturbation will grow, and below which it will be damped by viscosity*
- *If $\Omega > \Omega_c$, the rate of radiation of angular momentum in gravity waves will rapidly slow the star, till it reaches Ω_c and can rotate stably*

Hadronic Phase

$$\begin{aligned}
\mathcal{L} &= \sum_B \bar{\psi}_B(i\gamma_\mu\partial^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu \\
&-\frac{1}{2}g_{\rho B}\gamma_\mu\vec{\tau_B}\cdot\vec{\rho^\mu})\psi_B \\
&+\frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - {m_\sigma}^2\sigma^2\right) - U(\sigma) \\
&-\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}{m_\omega}^2\omega_\mu\omega^\mu \\
&-\frac{1}{4}\rho_{\mu\nu}\cdot\rho^{\mu\nu} + \frac{1}{2}{m_\rho}^2\rho_\mu\cdot\rho^\mu + \mathcal{L}_{YY} \\
&+\sum_{e^-,\mu^-}\bar{\psi}_\lambda\left(i\gamma_\mu\partial^\mu - m\right)\psi_\lambda.
\end{aligned}$$

where, $U(\sigma) = \frac{1}{3}bm_N\,(g_{\sigma N}\sigma)^3 + \frac{1}{4}c\,(g_{\sigma N}\sigma)^4.$

Hyperon-Hyperon interaction:

$$\begin{aligned}
\mathcal{L}_{YY} &= \sum_B \bar{\psi}_B\left(g_{\sigma^* B}\sigma^* - g_{\phi B}\gamma_\mu\phi^\mu\right)\psi_B \\
&+\frac{1}{2}\left(\partial_\mu\sigma^*\partial^\mu\sigma^* - m_{\sigma^*}^2\sigma^{*2}\right) \\
&-\frac{1}{4}\phi_{\mu\nu}\phi^{\mu\nu} + \frac{1}{2}m_\phi^2\phi_\mu\phi^\mu\ .
\end{aligned}$$

Using mean field approximation,the meson fields are replaced by their expectation values:

$$\sigma \rightarrow <\sigma>, \omega_\mu \rightarrow <\omega_\mu>, \rho_{\mu\alpha} \rightarrow <\rho_{\mu\alpha}>$$

$$\sigma^* \rightarrow <\sigma^*>, \phi_\mu \rightarrow <\phi_\mu>$$

The equations of motion for meson fields are:

$$\begin{aligned}
m_\sigma^2 \sigma &= \sum_B g_{\sigma B} n_B^S - \frac{\partial U}{\partial \sigma}, && (1)\\
m_{\sigma^*}^2 \sigma^* &= \sum_B g_{\sigma^* B} n_B^S, && (2)\\
m_\omega^2 \omega_0 &= \sum_B g_{\omega B} n_B, && (3)\\
m_\phi^2 \phi_0 &= \sum_B g_{\phi B} n_B, && (4)\\
m_\rho^2 \rho_{03} &= \sum_B g_{\rho B} I_{3B} n_B. && (5)
\end{aligned}$$

The scalar and baryon number density

$$\begin{aligned}
n_B^S &= \frac{2J_B+1}{2\pi^2} \int_0^{k_{F_B}} \frac{m_B^*}{(k^2 + {m_B^*}^2)^{1/2}} k^2 dk, && (6)\\
n_B &= (2J_B+1) \frac{k_{F_B}^3}{6\pi^2}. && (7)
\end{aligned}$$

where the effective baryon mass

$$m_B^* = m_B - g_{\sigma B}\sigma - g_{\sigma^* B}\sigma^* \qquad (8)$$

The total energy density, $\varepsilon = \varepsilon_B + \varepsilon_l$

$$
\begin{aligned}
\varepsilon \;=\;& \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 + \frac{1}{2}m_{\sigma*}^2\sigma^{*2} \\
&+ \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 \\
&+ \sum_B \frac{2J_B+1}{2\pi^2}\int_0^{k_{F_B}} (k^2+m_B^{*2})^{1/2}k^2\;dk \\
&+ \sum_l \frac{1}{\pi^2}\int_0^{K_{F_l}} (k^2+m_l^2)^{1/2}k^2\;dk
\end{aligned}
$$

The pressure

$$
\begin{aligned}
P \;=\;& -\frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 \\
&- \frac{1}{2}m_{\sigma*}^2\sigma^{*2} + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 \\
&+ \frac{1}{3}\sum_B \frac{2J_B+1}{2\pi^2}\int_0^{k_{F_B}} \frac{k^4\;dk}{(k^2+m_B^{*2})^{1/2}} \\
&+ \frac{1}{3}\sum_l \frac{1}{\pi^2}\int_0^{K_{F_l}} \frac{k^4\;dk}{(k^2+m_l^2)^{1/2}}\,.
\end{aligned}
$$

Parameters of the theory

The nucleon-meson couplings are determined using the following properties of symmetric nuclear matter :

- *Binding energy* $B/A = -16.3$ MeV
- *Isospin asymmetry energy coefficient* , $a_{sym} = 32.5$ MeV.
- *Saturation density* $n_0 = 0.16\ fm^{-3}$
- *Compressibility* $K = 240$ MeV
- *Effective nucleon mass* $m^*/m = 0.78$

Compressibility: $K = 9 \frac{dp}{dn}$

Isospin asymmetry energy coefficient : $a_{sym} = \frac{1}{2} \left[\frac{\partial^2 (\varepsilon/n)}{\partial t^2} \right]_{t=0}$

where $t = (n_n - n_p) / n$

- Hyperon-meson coupling constants

$$\frac{1}{2}g_{\omega\Lambda} = \frac{1}{2}g_{\omega\Sigma} = g_{\omega\Xi} = \frac{1}{3}g_{\omega N},$$
$$\frac{1}{2}g_{\rho\Sigma} = g_{\rho\Xi} = g_{\rho N} \quad ; \quad g_{\rho\Lambda} = 0,$$
$$2g_{\phi\Lambda} = 2g_{\phi\Sigma} = g_{\phi\Xi} = -\frac{2\sqrt{2}}{3}g_{\omega N}.$$

The scalar meson (σ) coupling to hyperons

$$U_Y^N(n_0) = -g_{\sigma Y}\sigma + g_{\omega Y}\omega_0.$$

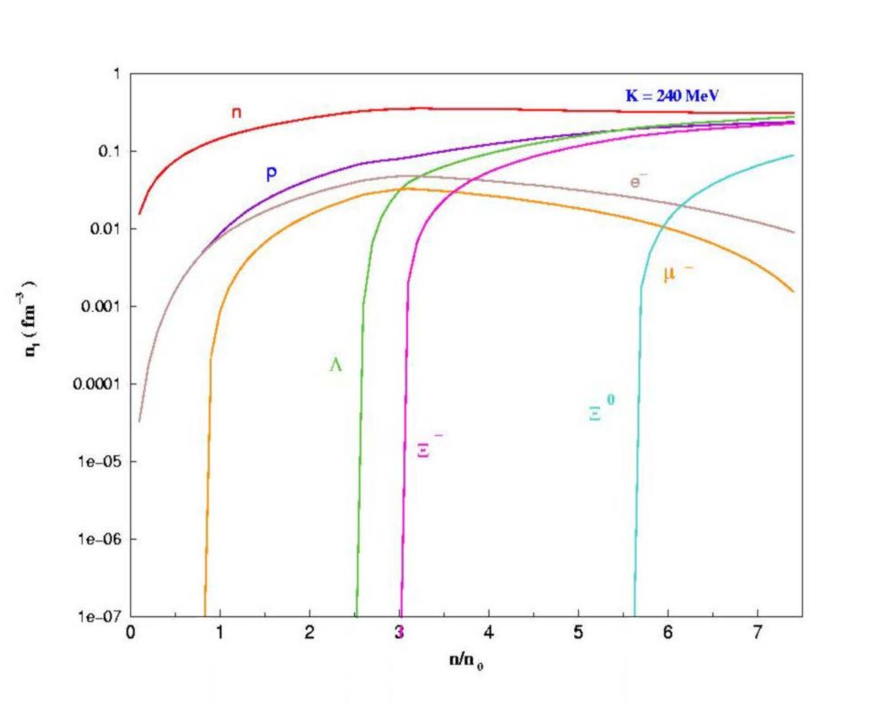
From Hypernuclei data

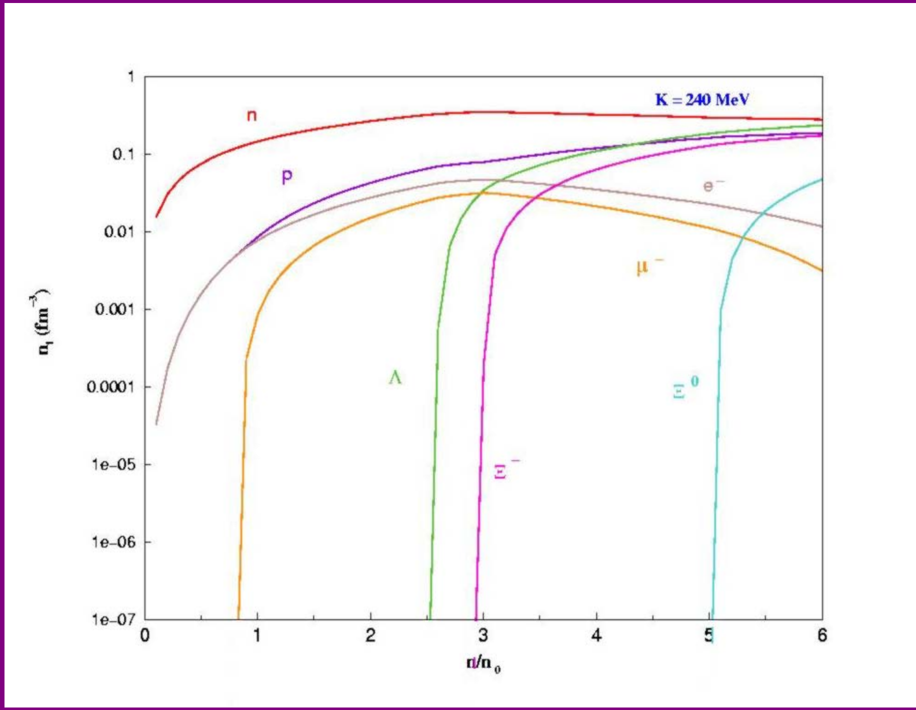
$$U_\Lambda^N = -30MeV, \;\; U_\Sigma^N = +30MeV \;\; \& \;\; U_\Xi^N = -18MeV$$

The σ^*-Y coupling constants

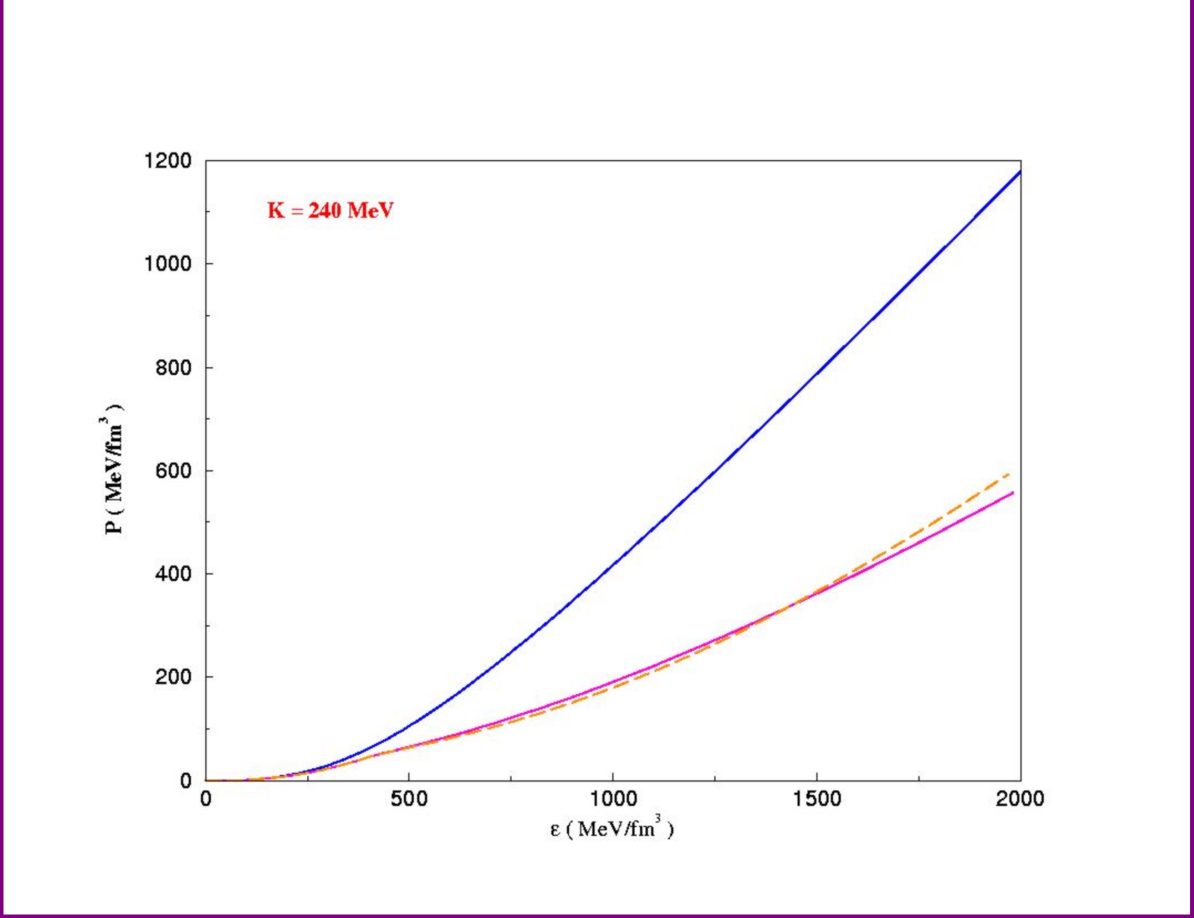
$$U_\Xi^{(\Xi)}(n_0) = U_\Lambda^{(\Xi)}(n_0) = 2U_\Xi^{(\Lambda)}(n_0) = 2U_\Lambda^{(\Lambda)}(n_0) = -40\ MeV.$$

Hyperon populations





EOS including hyperons



Non-rotating Neutron Stars

- The metric of a static, spherical space-time

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

- Energy-momentum tensor (EOS p *vs* ε in $T^{\mu\nu}$)

$$T^{\mu\nu} = (\varepsilon + p)u^{\mu} u^{\nu} + p\, g^{\mu\nu}$$

- Energy-momentum conservation + Einstein eqns

$$T^{\mu\nu}{}_{;\nu} = 0\,, \quad G^{\mu\nu} = R^{\mu\nu} - \tfrac{1}{2} g^{\mu\nu} R = 8\pi T^{\mu\nu}$$

- TOV equations + EOS $p=p(\varepsilon)$

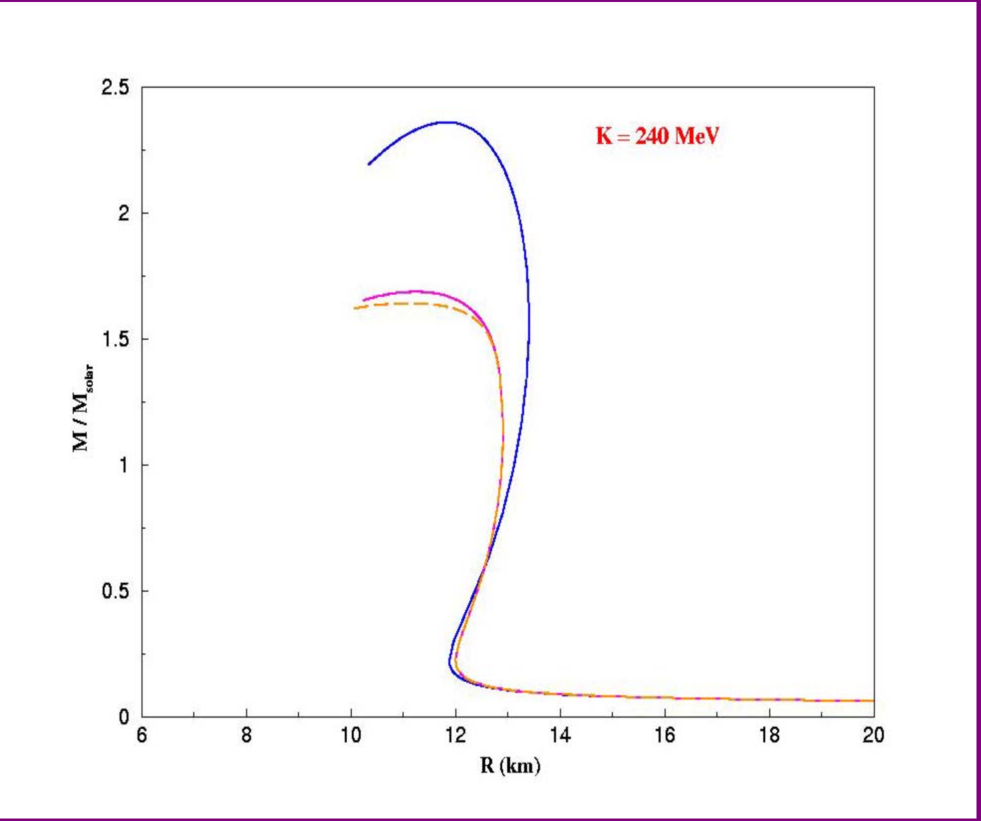
$$\frac{dm}{dr} = 4\pi\varepsilon r^2$$

$$\frac{d\nu}{dr} = 2e^{\lambda}(m + 4\pi p r^3)$$

$$\frac{dp}{dr} = -(\varepsilon + p)\frac{d\nu}{dr}$$

$$e^{-\lambda} = \left(1 - 2\frac{m}{r}\right)$$

Mass-Radius Relationship



- Allowed EOS : $M^{max}_{theo} > M^{highest}_{obs}$

Coefficient of Bulk Viscosity ζ

Ref : Landau and Lifshitz, Fluid Mehanics,2nd ed. (Oxford,1999)
Lindblom ,Owen and Morsink , Phys. Rev. D 65, 063006

$$\boxed{\zeta = \frac{n\,\tau}{(1-i\,\omega\,\tau)} \left(\frac{\partial p}{\partial x}\right)_n \frac{d\,\bar{x}}{d\,n}}$$

infinite frequency ("fast") adiabatic index

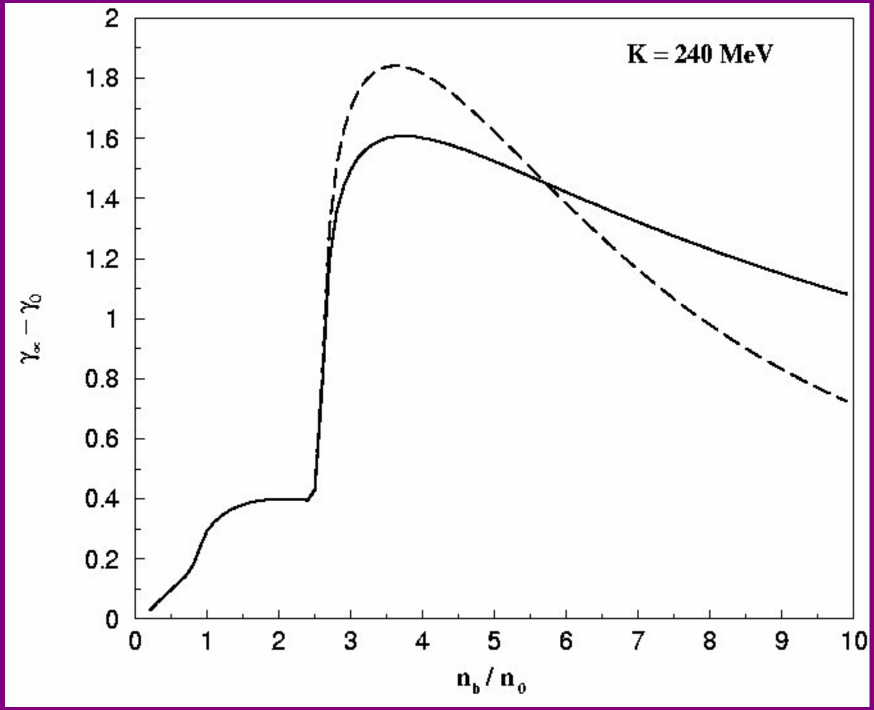
$$\gamma_\infty = \frac{n}{p} \left(\frac{\partial p}{\partial n} \right)_x$$

zero frequency ("slow")adiabatic index

$$\gamma_0 = \left[\left(\frac{\partial p}{\partial n} \right)_x + \left(\frac{\partial p}{\partial x} \right)_n \cdot \frac{d\bar{x}}{dn} \right]$$

$$\gamma_\infty - \gamma_0 = -\frac{n_B^2}{p} \frac{\partial p}{\partial n_n} \frac{d\bar{x}}{dn}$$

$$\boxed{\zeta = \frac{p(\gamma_\infty - \gamma_0)\,\tau}{(1 - i\,\omega\,\tau)}}$$



We consider the non-leptonic reaction, $n + p \leftrightarrow p + \Lambda$

$x_n = n_n / n_B$: *fraction of baryons comprised of neutrons*

$$(\partial_t + v \cdot \nabla) x_n = -(x_n - \bar{x}_n) / \tau = -\Gamma_n / n_B$$

where Γ_n *is the production rate of neutrons / volume, which is proportional to the chemical potential imbalance*

$$\delta\mu = \mu - \bar{\mu}$$

The relaxation time is given by

$$\frac{1}{\tau} = \frac{\Gamma_\Lambda}{\delta\mu} \frac{\delta\mu}{n_B \, \delta x_n}$$

where $\delta x_n = x_n - \bar{x}_n$

The reaction rate Γ *may be calculated using*

$$\Gamma = \frac{1}{4096\pi^8} \int \prod_{i=1}^{4} \frac{d^3 \mathbf{p}_i}{\varepsilon_i} |M|^2 \, \delta^{(3)}(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4) F(\varepsilon_i) \, \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4)$$

where

$$|\mathcal{M}_\Lambda|^2 = 4\, G_F^{\,2} \sin^2 2\theta_c \,[\; 2\, m_n\, m_p^{\,2}\, m_\Lambda (1 - g_{np}^{\,2})(1 - g_{p\Lambda}^{\,2})$$

$$- m_n\, m_p\, \mathbf{p_2} \cdot \mathbf{p_4} (1 - g_{np}^{\,2})(1 + g_{p\Lambda}^{\,2})$$

$$- m_p\, m_\Lambda\, \mathbf{p_1} \cdot \mathbf{p_3} (1 + g_{np}^{\,2})(1 - g_{p\Lambda}^{\,2})$$

$$+ \mathbf{p_1} \cdot \mathbf{p_2}\;\; \mathbf{p_3} \cdot \mathbf{p_4} \{(1 + g_{np}^{\,2})(1 + g_{p\Lambda}^{\,2}) + 4\, g_{np}\, g_{p\Lambda}\}$$

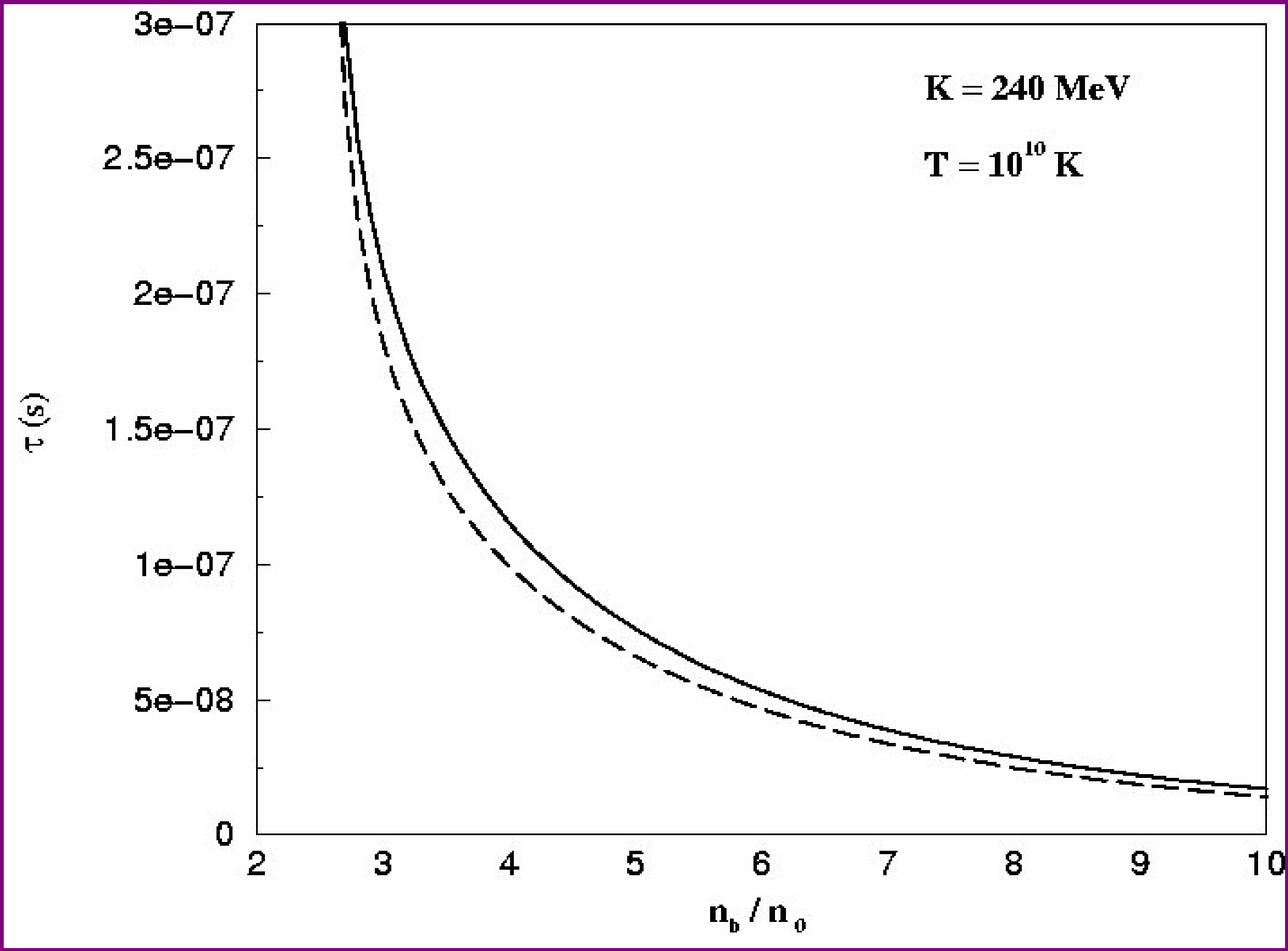
$$+ \mathbf{p_1} \cdot \mathbf{p_4}\;\; \mathbf{p_2} \cdot \mathbf{p_3} \{(1 + g_{np}^{\,2})(1 + g_{p\Lambda}^{\,2}) - 4\, g_{np}\, g_{p\Lambda}\}]$$

After performing the energy and angular integrals,

$$\Gamma = \frac{1}{192\pi^3} < |\mathcal{M}|^2 > \mathbf{p_4}\, (kT)^2\, \delta\mu$$

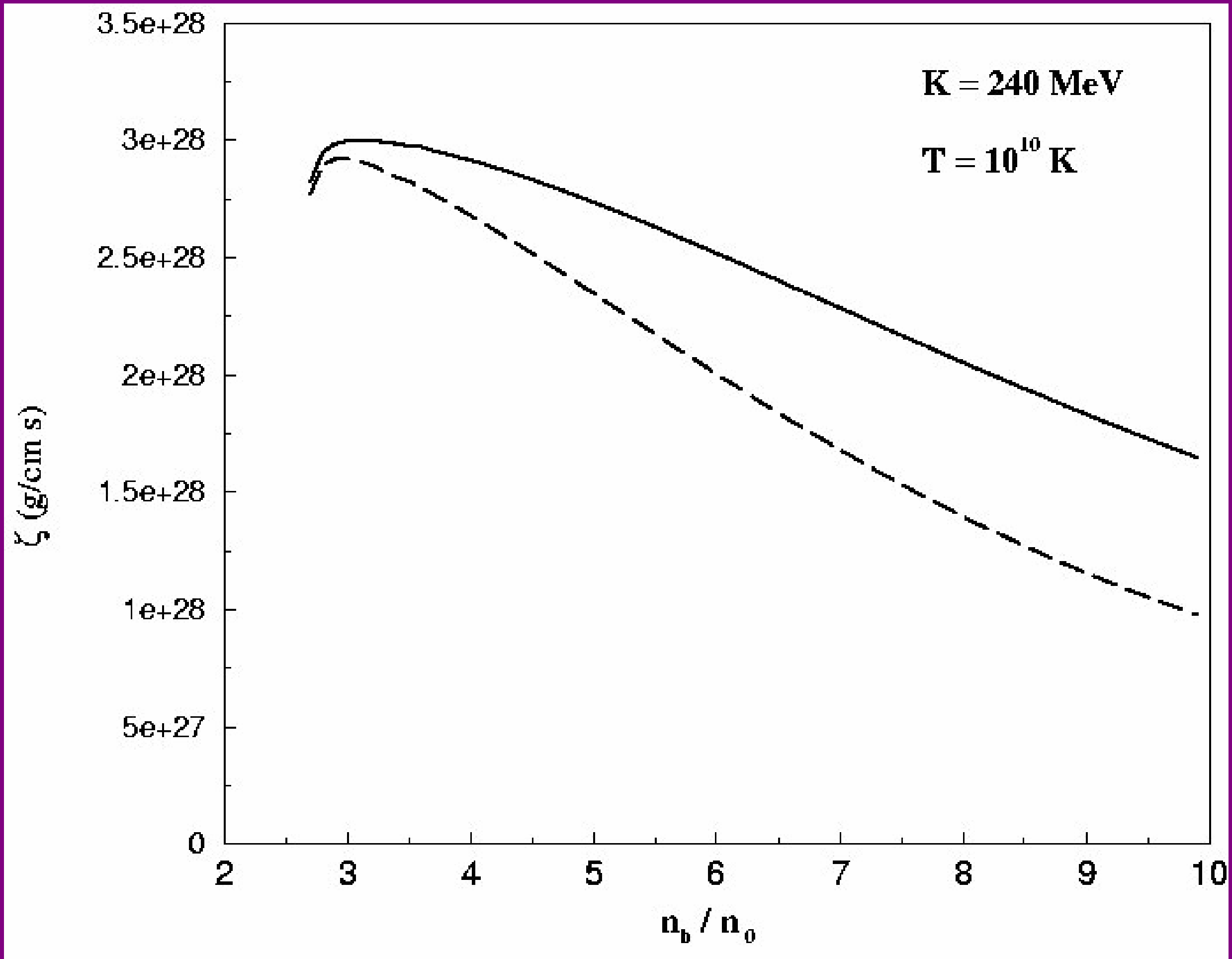
where $< |\mathcal{M}|^2 >$ *is the angle-averaged value of* $|\mathcal{M}|^2$

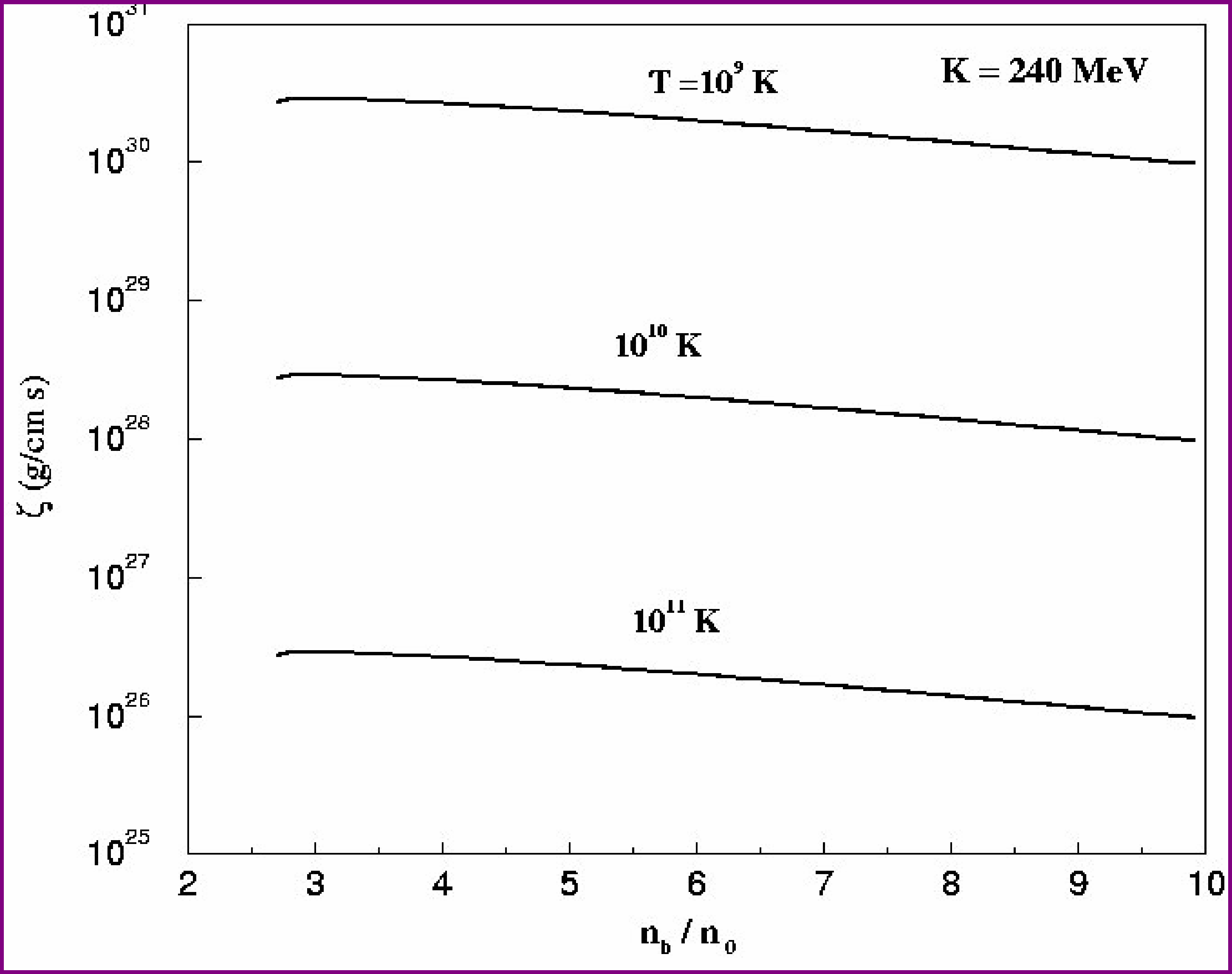
$$\boxed{\frac{1}{\tau} = \frac{(kT)^2}{192\pi^3}\; p_\Lambda < |\mathcal{M}_\Lambda|^2 > \frac{\delta\mu}{n_B\, \delta x_n}}$$


K = 240 MeV
T = 10^10 K
τ (s)
n_b / n_o
3e−07
2.5e−07
2e−07
1.5e−07
1e−07
5e−08
0
2
3
4
5
6
7
8
9
10

$$Re\,\zeta = \frac{p\,(\gamma_\infty - \gamma_0)\,\tau}{1 + (\omega\,\tau)^2}$$

where $\omega = 2/3\ \Omega_{max}$





K = 240 MeV
T =10^9 K
10^10 K
10^11 K
ζ (g/cm s)
n_b / n_0
10^31
10^30
10^29
10^28
10^27
10^26
10^25
2
3
4
5
6
7
8
9
10

r-mode damping time $\tau_{B(h)}$

- The rotating frame energy E for r-modes is

$$E = \tfrac{1}{2}\alpha^2\,\Omega^2 \frac{1}{R^2} \int_0^R \varepsilon\, r^2\, dr$$

Ref : Lindblom , Owen and Morsink , Phys Rev Lett. 80 (1998) 4843

- Time derivative of corotating frame energy due to BV is

$$\frac{dE}{dt} = -\,4\,\pi \int_0^R \mathrm{Re}\,\zeta < \left| \nabla . \delta v \right|^2 > r^2\, dr$$

The angle averaged expansion squared is determined numerically

$$< \left| \nabla . \delta v \right|^2 > = \frac{\alpha^2 \Omega^2}{690} \left(\frac{r}{R}\right)^6 \left[1 + 0.86 \left(\frac{r}{R}\right)^2 \right] \left(\frac{\Omega^2}{\pi G \varepsilon}\right)^2$$

Ref : Lindblom , Mendell and Owen, Phys Rev D 60 (1999) 064006

The time scale $\tau_{B(h)}$ on which hyperon bulk viscosity damps the mode is

$$\frac{1}{\tau_{B(h)}} = -\,\frac{1}{2E}\,\frac{dE}{dt}$$

Critical Angular Velocity

- *The imaginary part of the frequency of the r-mode*

$$\boxed{\frac{1}{\tau_r} = -\frac{1}{\tau_{GR}} + \frac{1}{\tau_{B(h)}} + \frac{1}{\tau_{B(u)}}}$$

$$\frac{1}{\tau_{GR}} = \frac{131072\,\pi\,\Omega^6}{164025} \int_0^R \varepsilon(r)\, r^6\, dr$$

where τ_{GR} *= timescale over which GR drives mode unstable*

$\tau_{B(u)}$ *= Urca bulk viscosity timescale*

- *The mode is stable when* $\tau_r < 0$ *, and unstable when* $\tau_r > 0$
- *For a star of given T and M , the critical angular velocity* Ω_c *is that for which the imaginary part of the r-mode frequency vanishes i.e.,* $\frac{1}{\tau_r} = 0$

Rotating Neutron Stars

- The metric of stationary axisymmetric rotating star

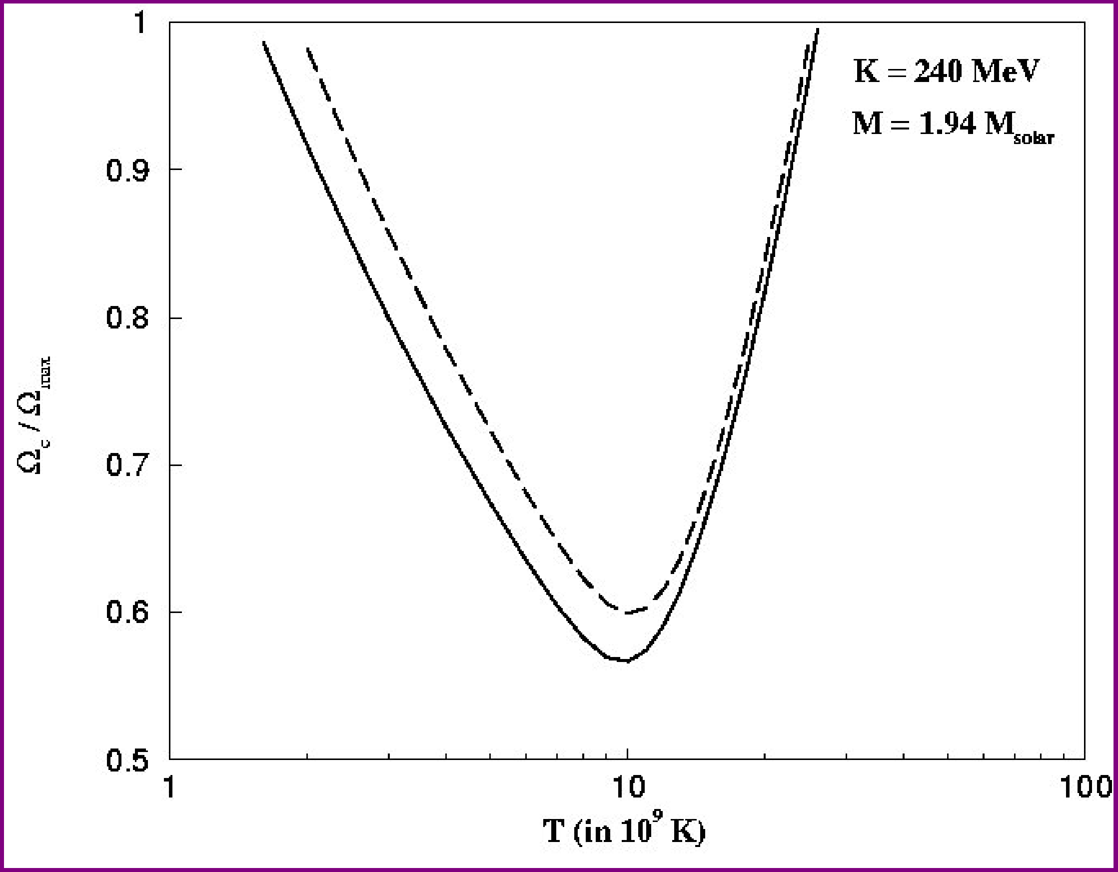
$$ds^2 = -e^{\nu(r,\theta)}dt^2 + e^{\mu(r,\theta)}(dr^2 + r^2 d\theta^2) + e^{\psi(r,\theta)} r^2 \sin^2\theta (d\varphi - \omega(r,\theta)dt)^2$$

in slow rotation limit

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) - 2\omega(r) r^2 \sin^2\theta dt d\varphi$$

- Dragging of local inertial frames
- Fast rotating models: numerical codes (RNS)

Friedman and Stergioulas, Ap.J. 444 (1995) 306



D.Chatterjee, D.B. astro-ph/0602538

Outlook

- Influence of Bose-Einstein condensation on bulk viscosity
- Non-leptonic process $n \to p + K^-$ *may be important*