What do exotic equations of state have to offer?

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Isolated Neutron Stars

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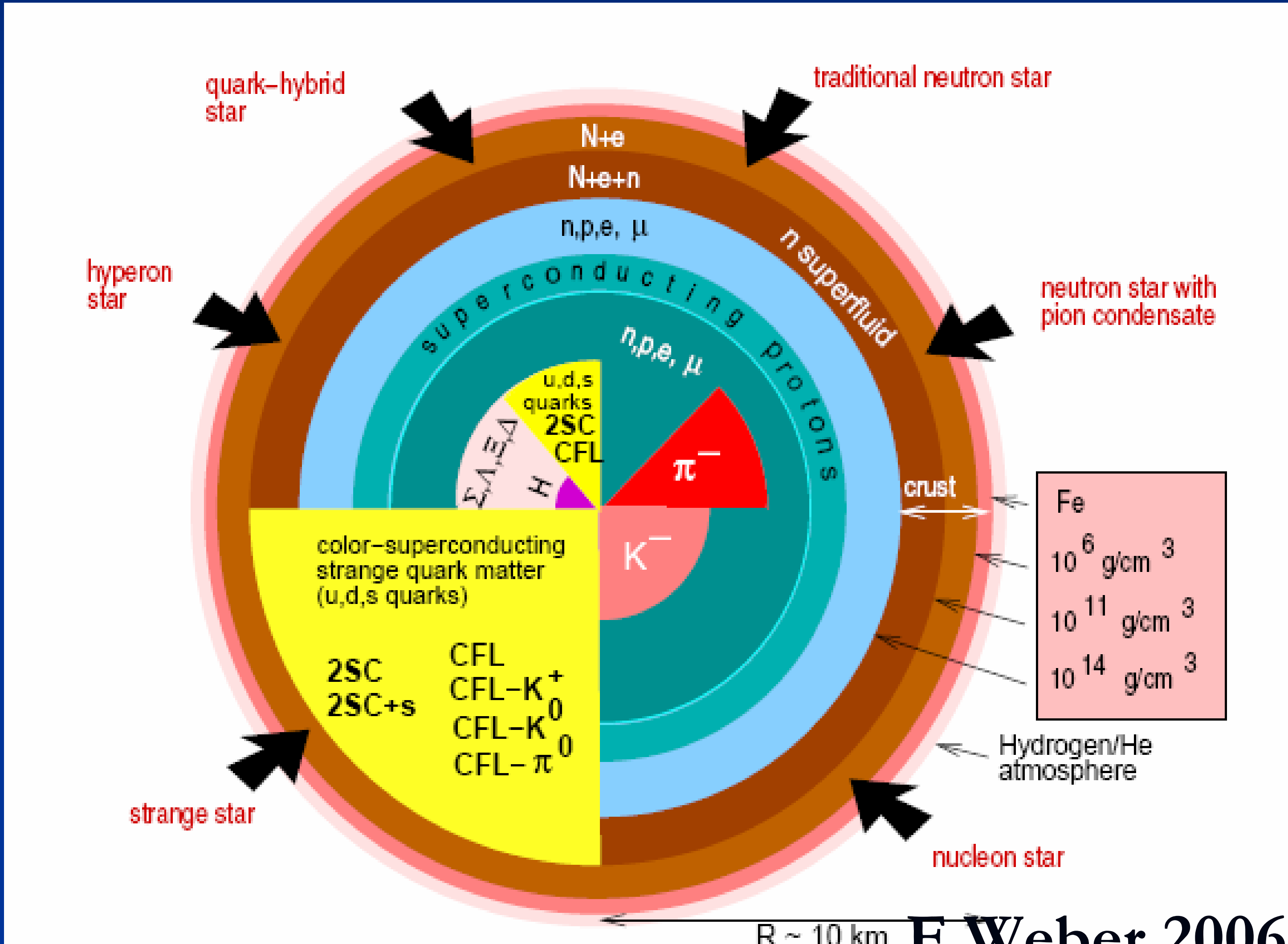
What is "exotic": many possibilities Condensates, quarks, self-bound quark matter, Q-matter… (Lattimer & Prakash 2001)

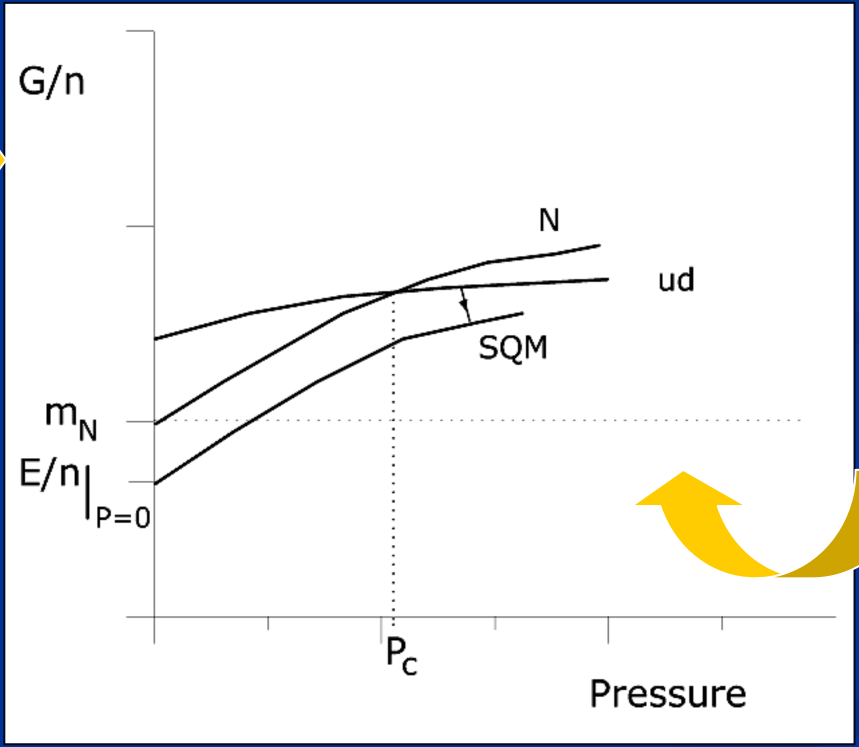


F.Weber 2006

Work on hypothetical QCD phases (Bodmer 1971, Terazawa 1979, Witten 1984) :

E/A < 939 MeV even at P=0 !!!

Gibbs free energy per particle



Non-equilibrium transition !!!

$$n \rightarrow uds + energy$$

Exothermic process, possibly important for supernova dynamics (PRL 63, 716, 1989)

Self-bound quark matter : Bodmer-Witten-Terazawa hypothesis

Fermi liquid in which strange quarks reduce the energy per baryon number unit below ~ 930 MeV

$$\boxed{P = \frac{1}{3}(\rho - 4B)}$$

Equation of state (EOS)

Ultra-relativistic quarks

Vacuum energy

Soft or stiff EOS? Effects on stellar models

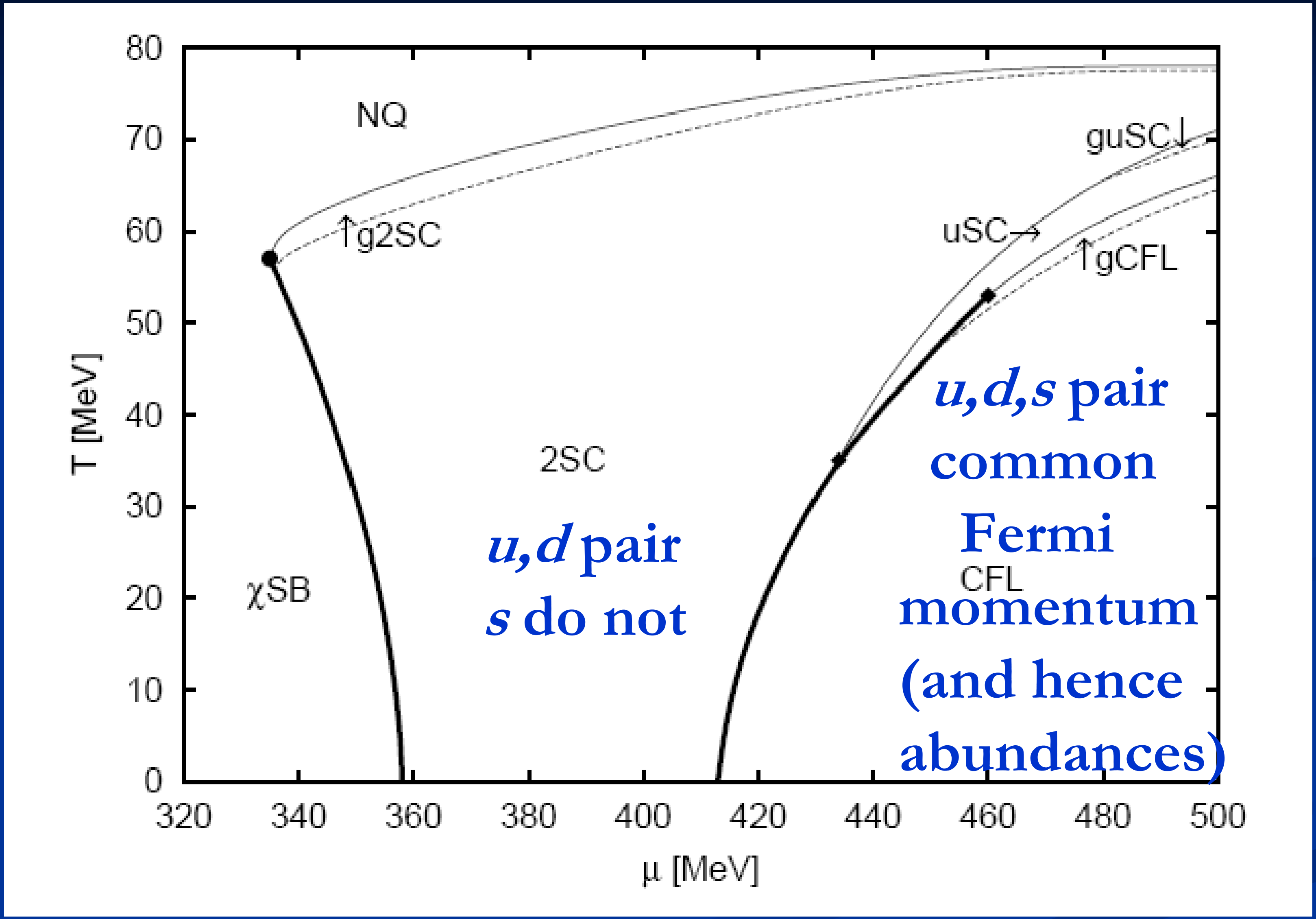
Beyond the simplest Fermi liquid picture
2SC and Color-Flavor Locked quark matter :

The strength of pairing $\boxed{\Delta}$

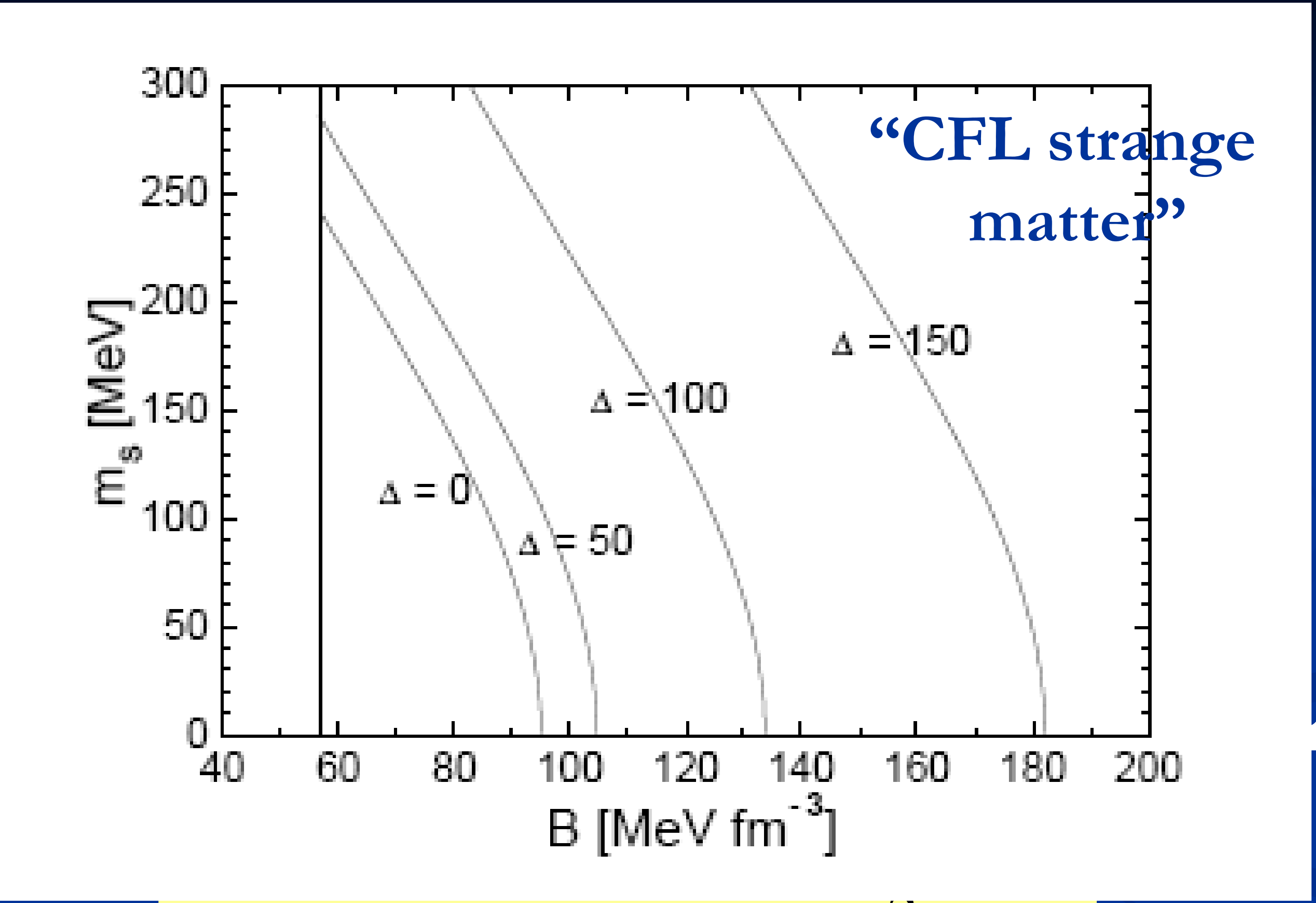
"Old" pairing, gaps < 1 MeV (Bailin & Love, 1984)
Perturbative QCD spirit

"New" pairing, gaps ~ 100 MeV (Alford, Wilczek, Rajagopal,…1998-99)
Better calculations, non-perturbative phenomenon

Which species pair ? (other possibilities as well….)

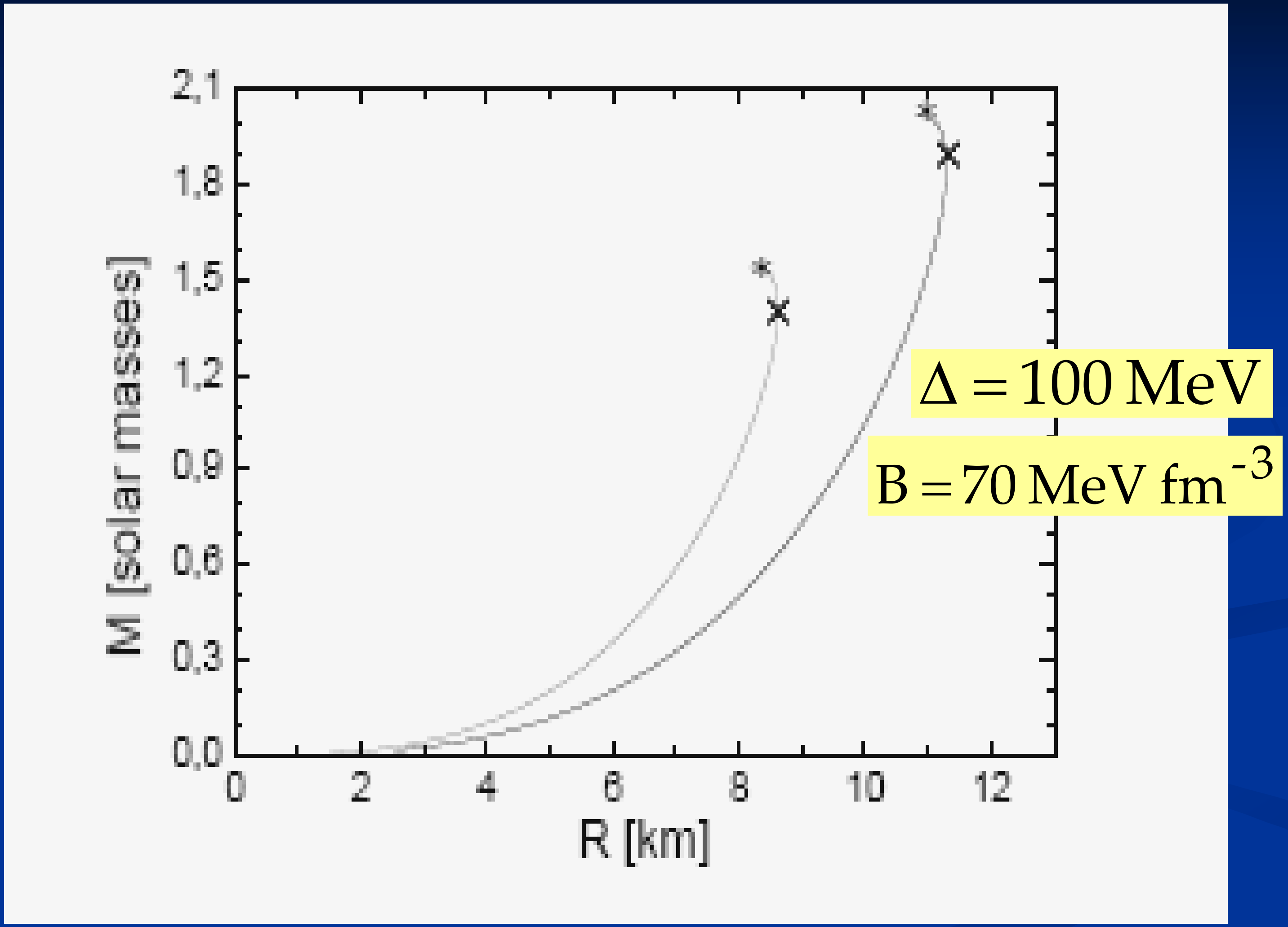


S. Rüster et al. hep-p/0503184 NJL description

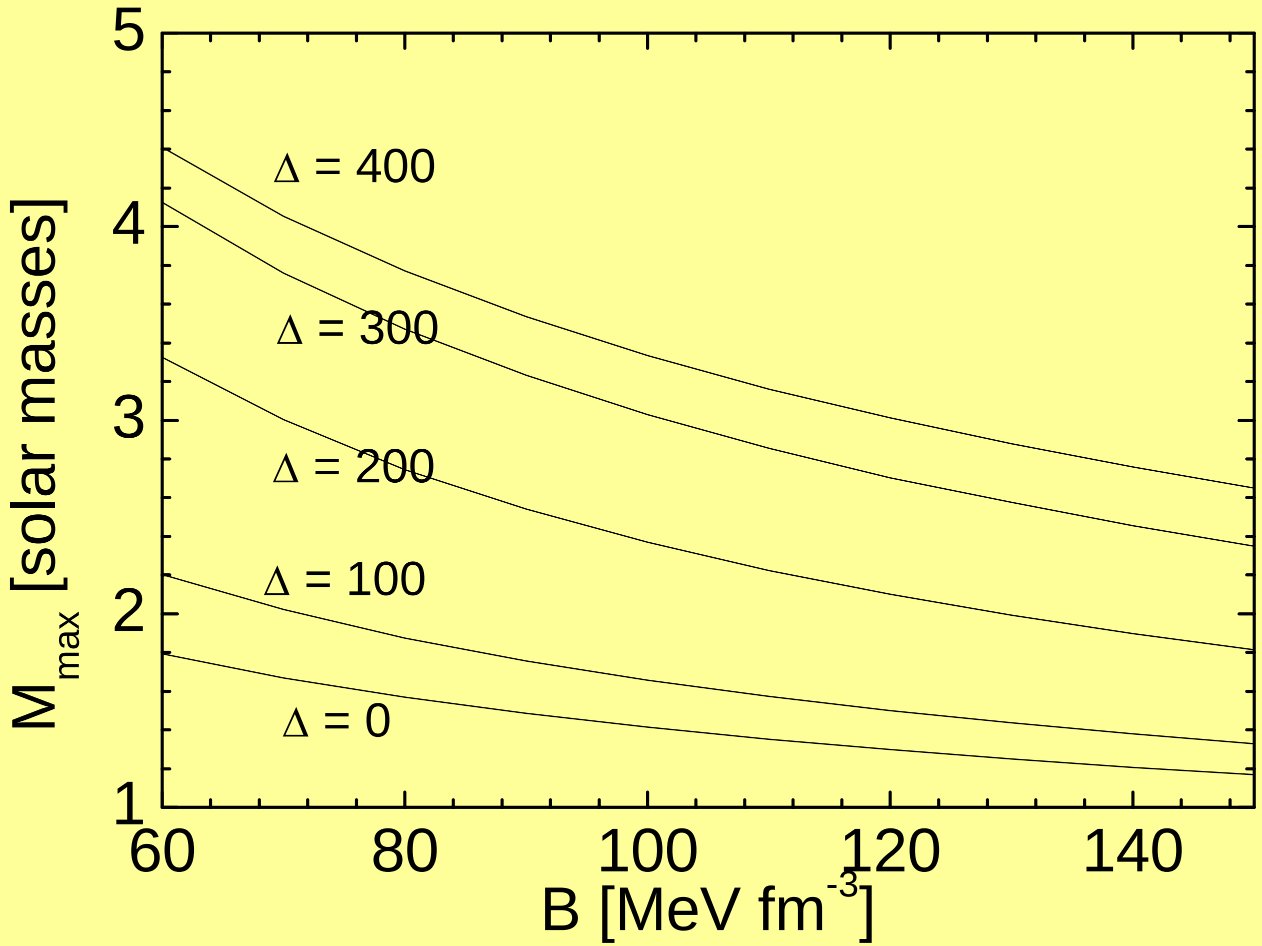


$$B = -\Omega_{free}(m_s, \mu_{fermi}) + \frac{3}{\pi^2}\Delta^2\mu^2$$

(G. Lugones & J.E. Horvath, hep-ph/0211070)



(G. Lugones & J.E. Horvath, A&A403, 173, 2003
Bagged self-bound quark matter)



Δ = 400
Δ = 300
Δ = 200
Δ = 100
Δ = 0
5
4
3
2
1
60
80
100
120
140
M_max [solar masses]
B [MeV fm^-3]

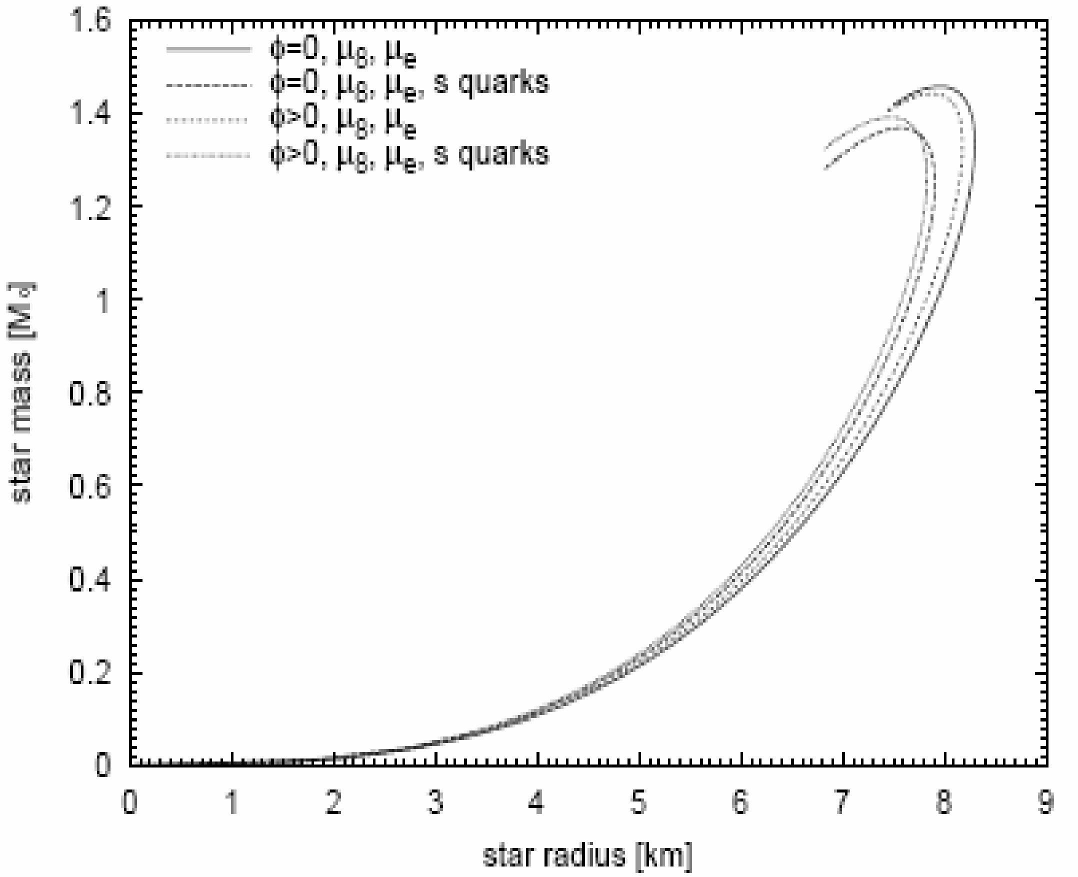
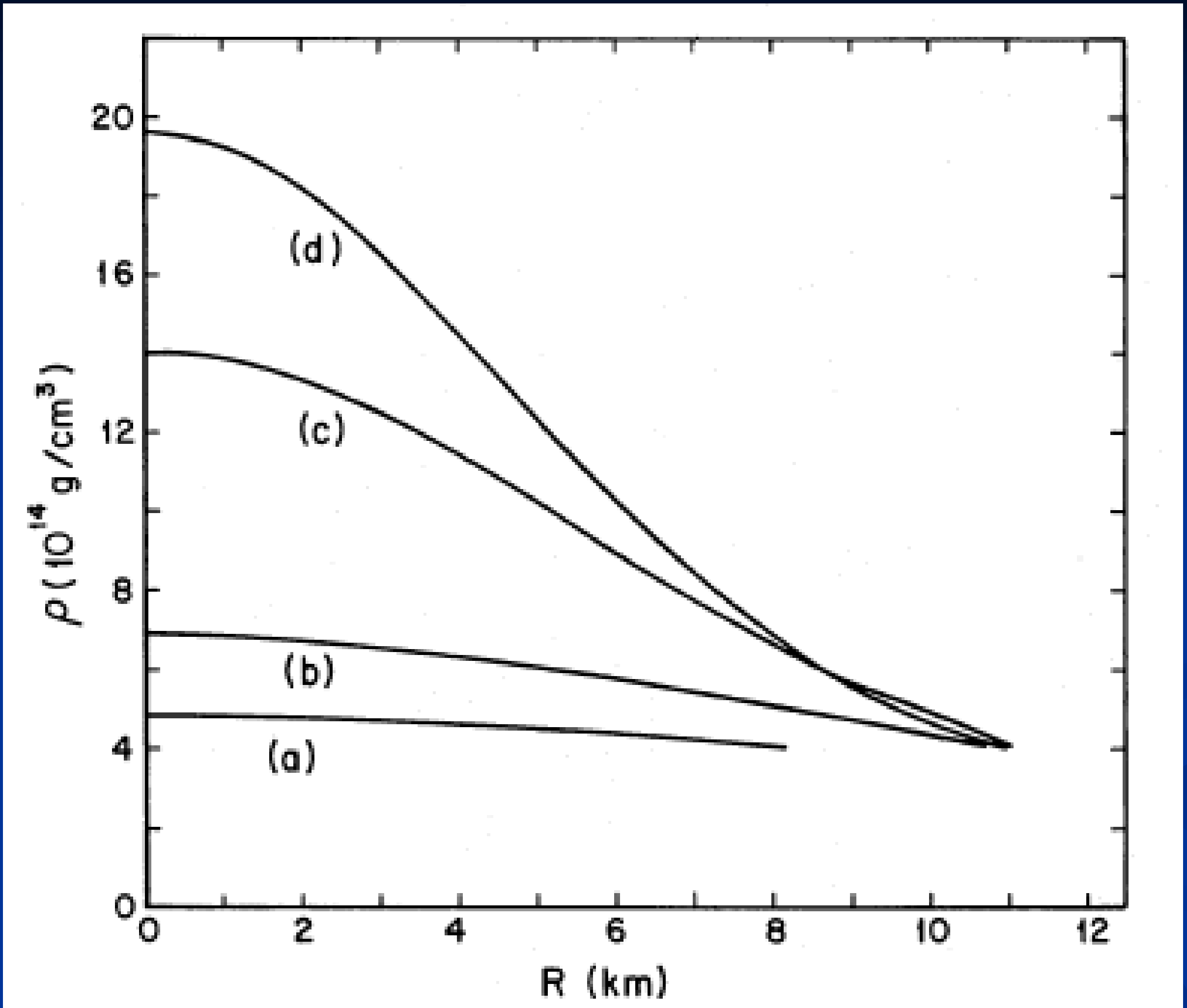


FIG. 7: The mass-radius relation for electric- and color-neutral quark stars. Full and dashed lines are for normal-conducting quark matter, short-dashed and dotted lines are for color-superconducting quark matter. Full and short-dashed lines are computed without, dashed and dotted lines are computed with strange quarks.

(S. Rüster & D. Rischke, nucl-th/0309022
NJL self-bound quark matter)



Alcock, Farhi & Olinto, 1986
self-bound strange quark matter

$$\mathrm{Mmax} \propto \mathrm{B}^{-1/2}$$

New analytic exact solutions of reltivistic structure with applications to self-bound stars (J.E.H. et al 2006)

$$\rho(r) = \rho_c \times e^{-(r/r_0)^2}$$

Gaussian *ansatz* motivated by numerical profiles

$$P = \alpha\rho + \beta \quad \Longleftrightarrow \quad P = \frac{1}{3}(\rho - 4B)$$

Linear EOS

Explicit metric elements

$$e^{\nu} = \frac{\text{Const.}}{\left(c^2\rho_c e^{-(r/r_0)^2} - B\right)^{\frac{1}{2}}}$$

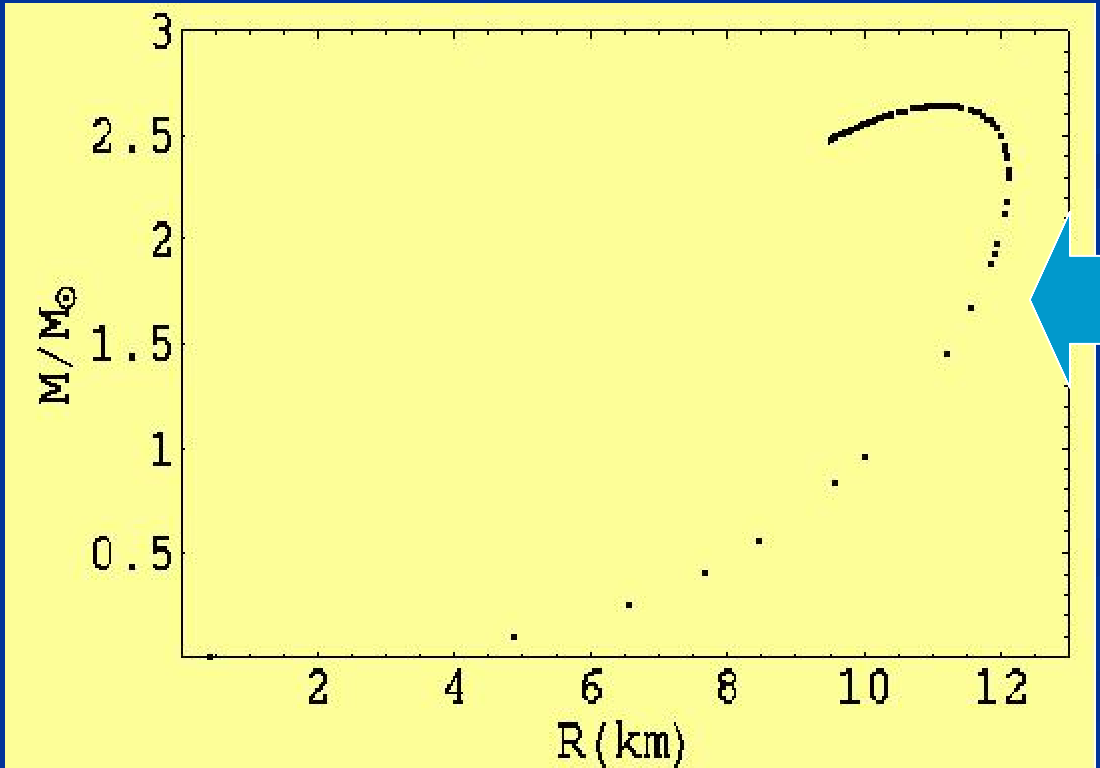
$$e^{\lambda} = \frac{3c^4}{r_0^2}\frac{\left(c^2\rho_c e^{-(r/r_0)^2}(r^2 + r_0^2) - r_0^2 B\right)}{\left(c^2\rho_c e^{-(r/r_0)^2} - B\right)\left(8\pi G c^2\rho_c r^2 e^{-(r/r_0)^2} - 32\pi G^2 B + 3c^4\right)}$$

Exploiting the boundary conditions

$$P(r = R) = 0 \Rightarrow R = r_0 \times \sqrt{\ln\left(\frac{\rho_c c^2}{4B}\right)} \equiv \kappa \times r_0$$

$$M = \frac{2}{3}\pi\rho_c R^3\left(-\frac{1}{\kappa^2}e^{-\kappa^2} + \frac{1}{4\kappa^3}\sqrt{\pi}\,\mathrm{erf}(\kappa)\right)$$

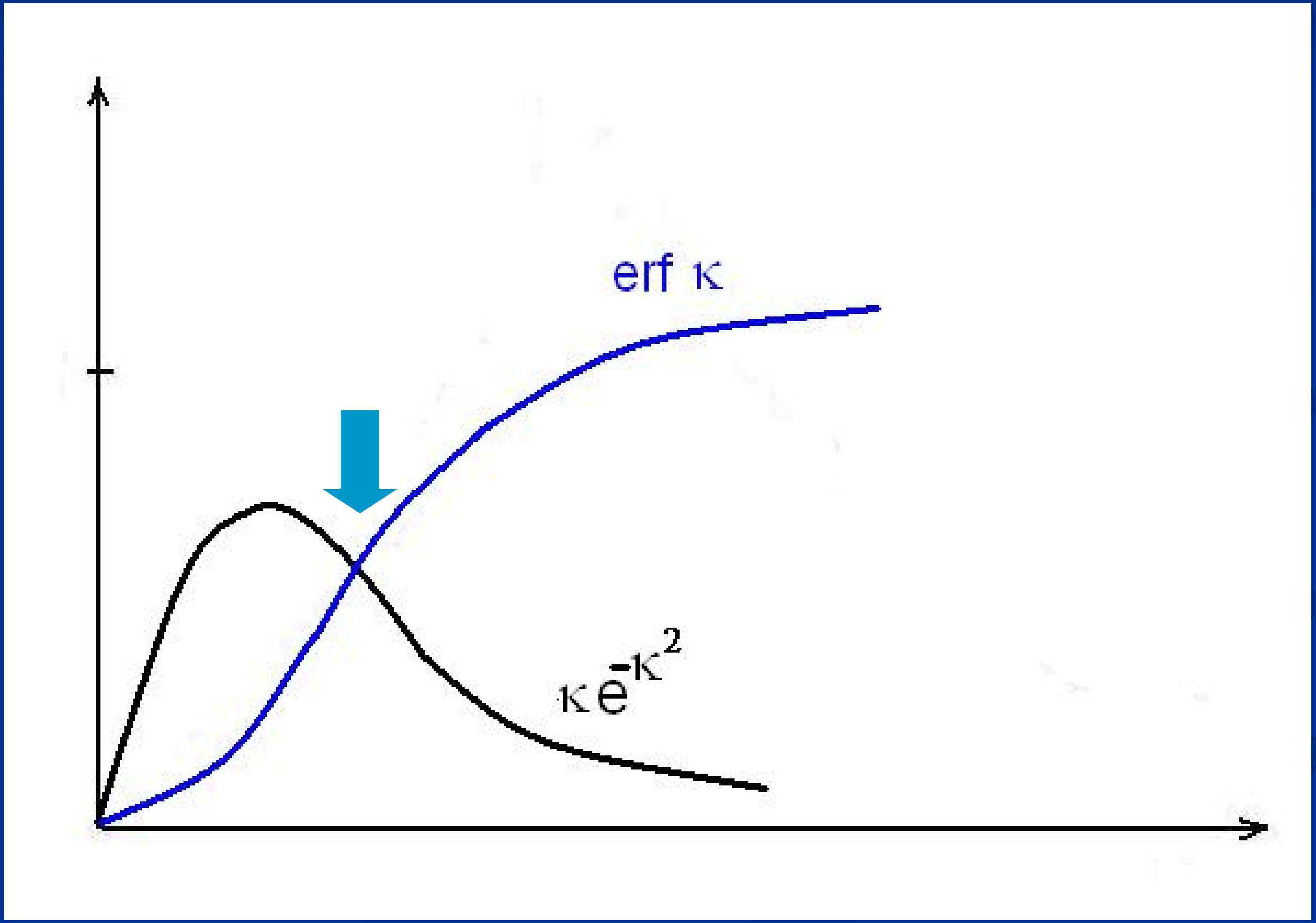
Mass depends cubically on R, but the coefficients are functions of the density quotient and vary with ρ_c hence the sequence "bends" at high mass



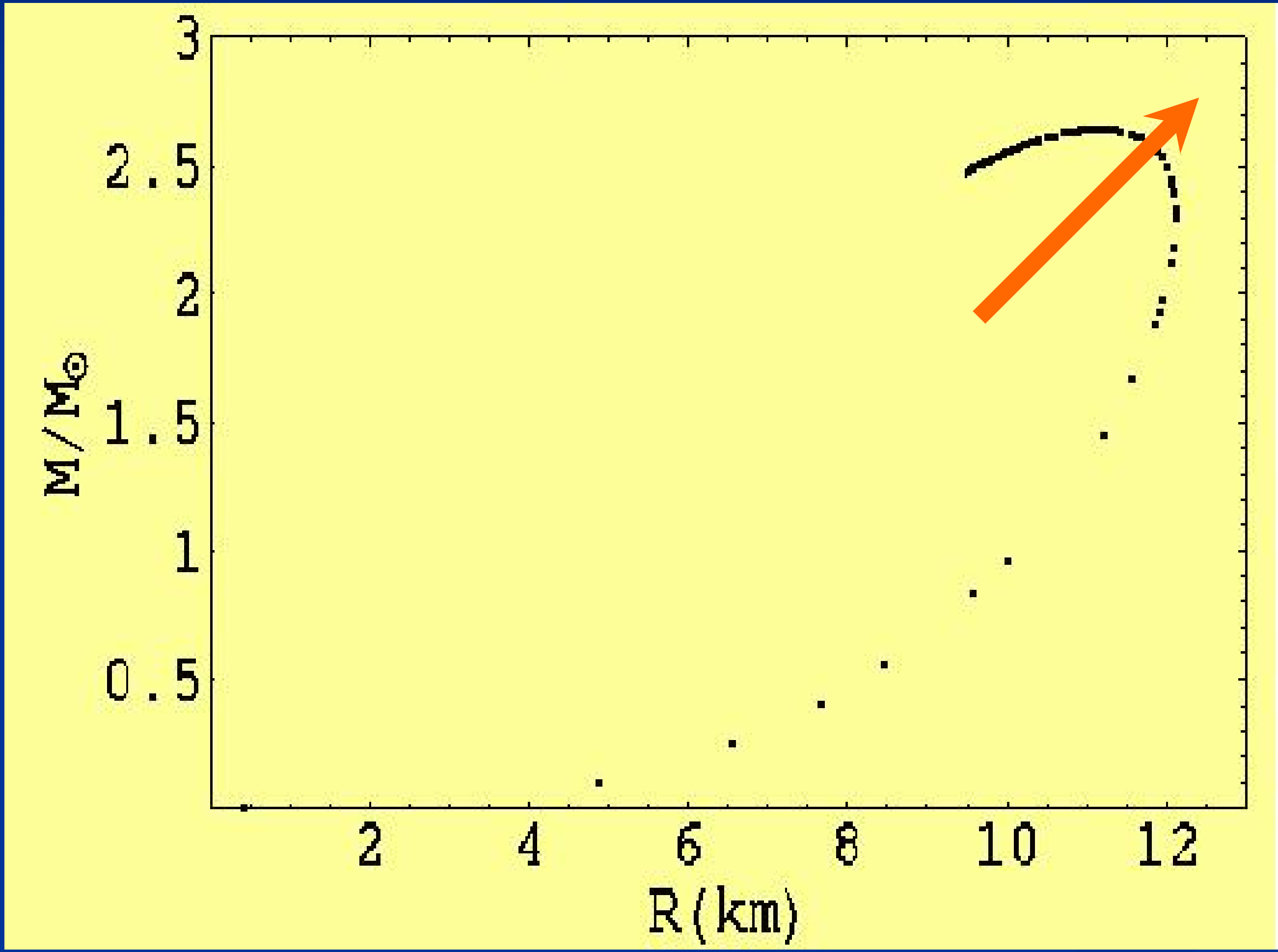
Analytical solutions with given B

$$\frac{\partial M}{\partial R} = 0$$

Analytical calculation of the *locus* of maxima as a function of R



No big surprises, just a neater description with the potentiality of easy applications



Large masses
→Large radii

Not possible to keep radius small while Mmax grows

Why care about self-bound models ?

Role of hyperons in hadronic matter : included in some NR form, they tend to soften the EOS. Threshold at 2-3 ρ_0

Interactions of hyperons with p,n still uncertain Generally H-n and H-p interactions are not included in the calculations

Existing EOS which behave quite stiffly either

a) Do not include hyperons
b) Include hyperons but use mean-field theories (e.g. Walecka-type) instead of a microscopic approach

(M.Baldo, F. Bugio & co-workers…)

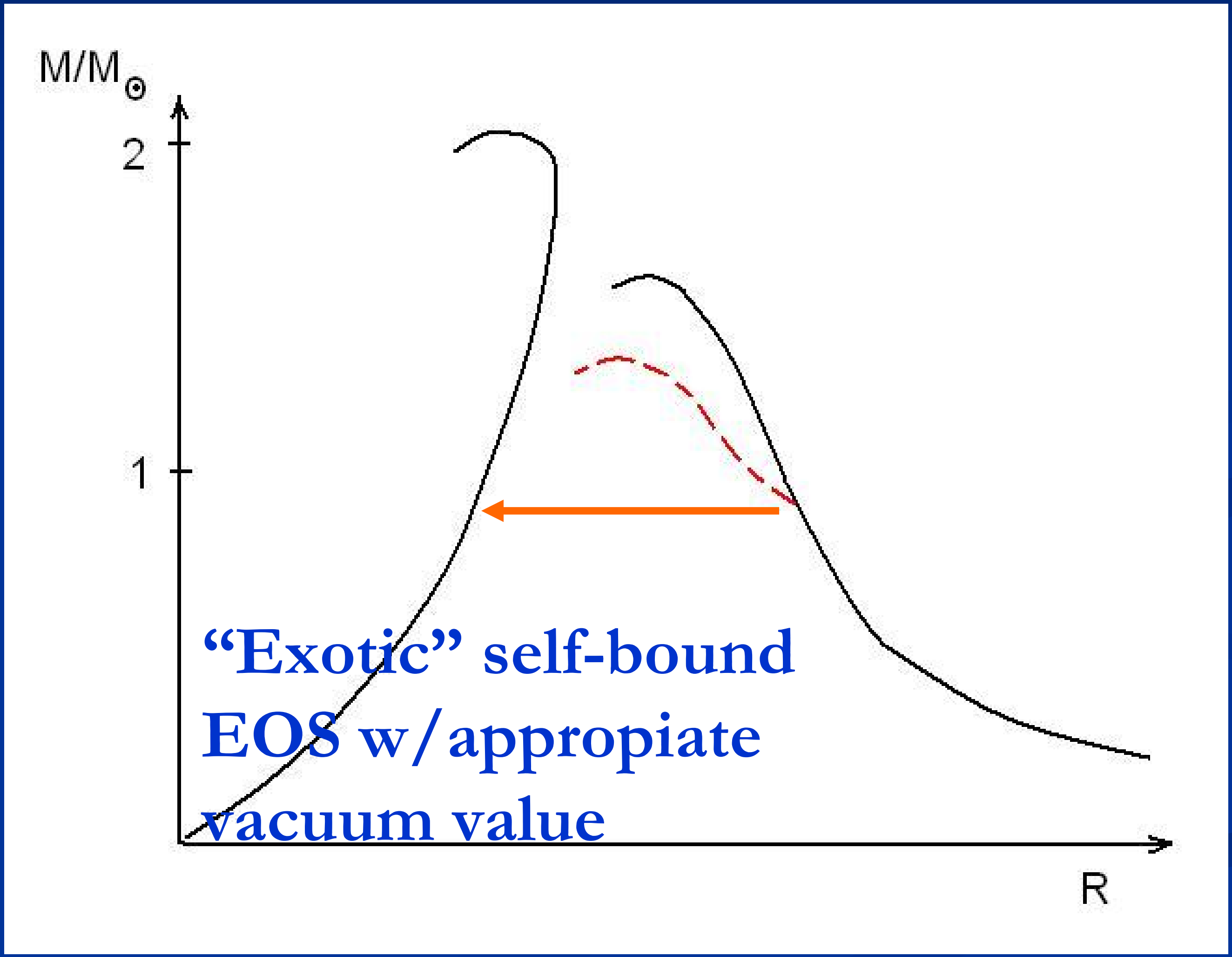
Why mass determinations around $2\,M_{\odot}$ and well below $1.4\,M_{\odot}$ are so important ?

Two examples:

PSR J0751+1807 $2.1 \pm 0.2\,M_{\odot}$ Nice et al. 2005

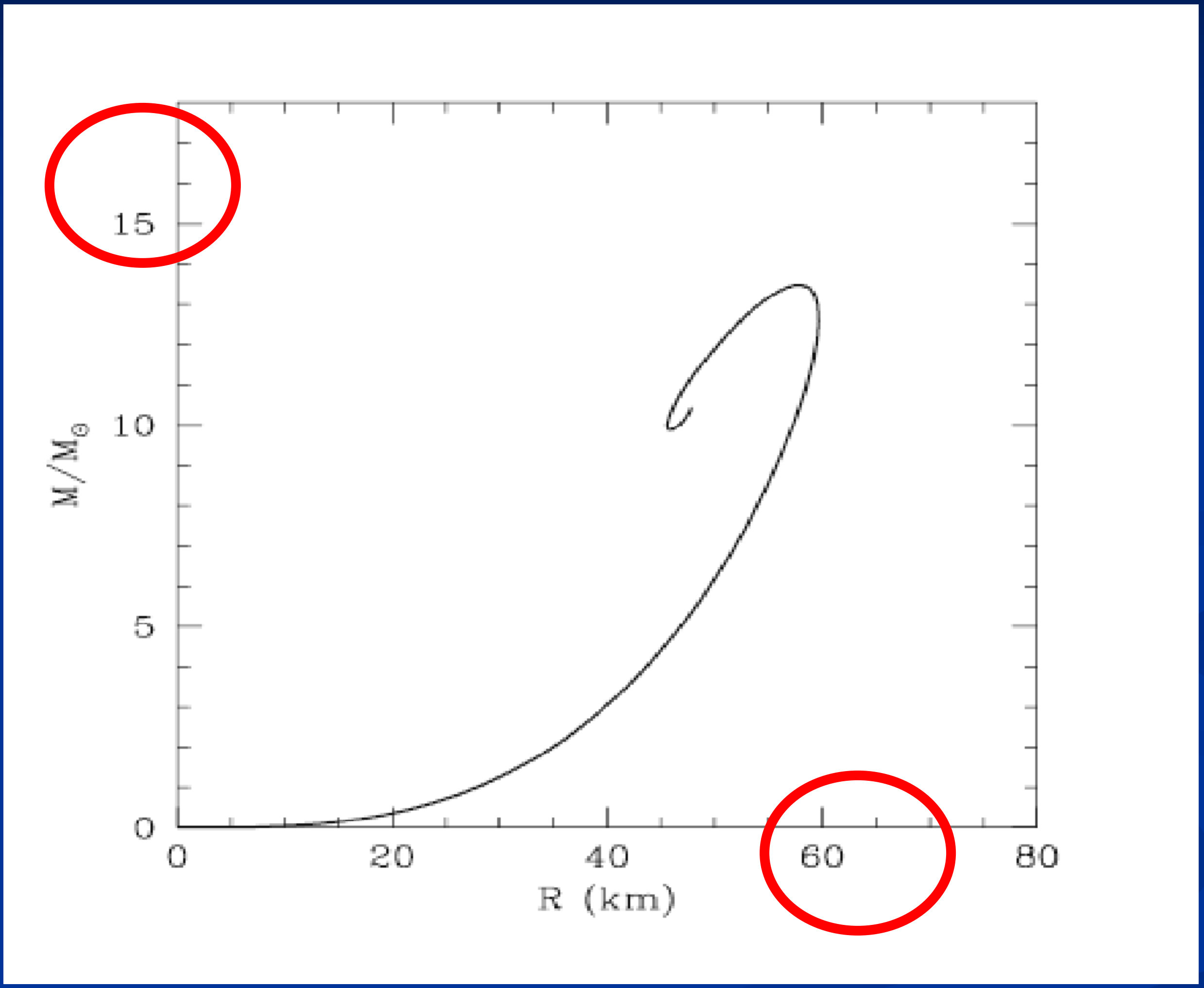
SMC X-1 $0.91 \pm 0.08\,M_{\odot}$ Baker, Norton & Quaintrell 2005

What do these determinations mean and how are these objects formed?



(J.E.H. & I. Bombaci, in preparation)

The importance of masses and radii together with other constraints (Lattimer)…



(J.C. Miller, T. Shahbaz & L.A. Nolan, astro-ph/9708065 Q-stars)

Conclusions

- If we understand the vacuum, the main uncertainty in the EOS will be gone. "Soft" or "hard" EOS are mainly a consequence of the vacuum (+condensation)
- Hyperons do exist, and therefore either they are not present inside stars, or they have very repulsive interactions with nucleons to create models w/ $M \geq 2\,M_{\Theta}$ In this way, high masses may be indicating exotics, rather than excluding them
- Still within this tentative scenario, small masses must show small radii, but only for $M \approx 0.5\,M_{\Theta}$, not $M \approx 1 M_{\Theta}$

There are more things in Heaven and Earth, Horatio
Than are dreamt of in your philosophy Hamlet, Act I