Modelling the dynamics of superfluid neutron stars

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Overall context

As $T \to 0$ matter either freezes to a solid or becomes superfluid.

(He^3, Bose-Einstein condensates, fermion Cooper pairing etc.)

Mature neutron stars are thought to contain: superfluid neutrons, superconducting protons, superfluid hyperons and possibly also colour superconducting quarks.

Want to understand the dynamics of these phases.

- "glitches"

  What is the mechanism?

- precession

  What about vortex pinning?

- oscillations

  Imprint for asteroseismology?

- gravitational waves

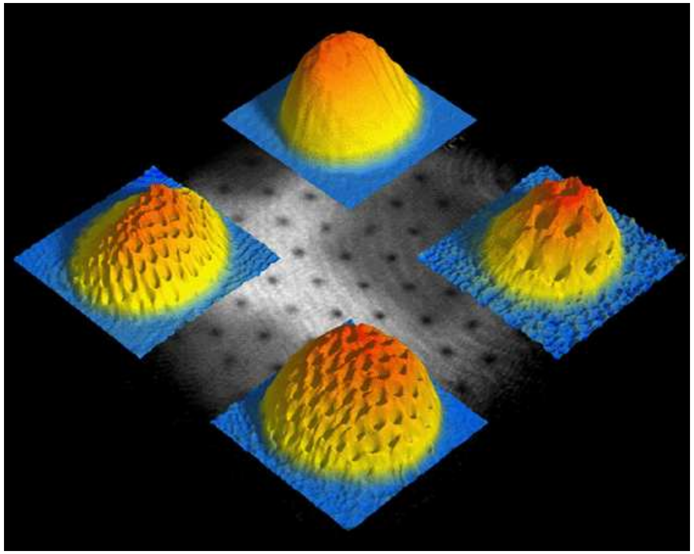
  Does viscosity stabilise r-modes?

  Is mutual friction the key damping agent?

  Hyperon bulk viscosity?

- two-stream instability

  Can it operate in neutron stars?



Shear viscosity

Below the neutron superfluid transition temperature the dominant contribution to the shear viscosity is made by the scattering of relativistic electrons.

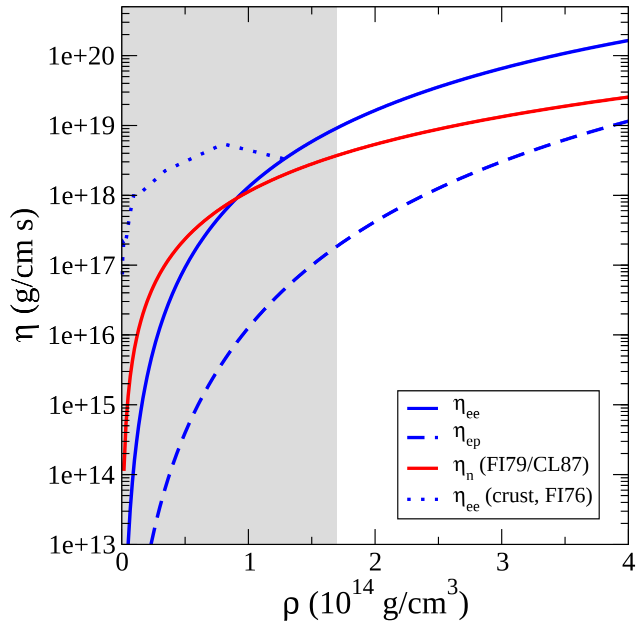
Against intuition, this leads to a stronger shear viscosity.

Above proton transition temperature:

$$\eta_{\rm ep} \approx 1.8 \times 10^{18} \left(\frac{x_{\rm p}}{0.01}\right)^{13/6} \rho_{15}^{13/6} T_8^{-2} \text{ g/cm s}$$

When protons are superconducting:

$$\eta_{\rm ee} \approx 4.4 \times 10^{19} \left(\frac{x_{\rm p}}{0.01}\right)^{3/2} \rho_{15}^{3/2} T_8^{-2} \text{ g/cm s}$$



The protons play the key role. Individual scattering processes add like “parallel resistors”;

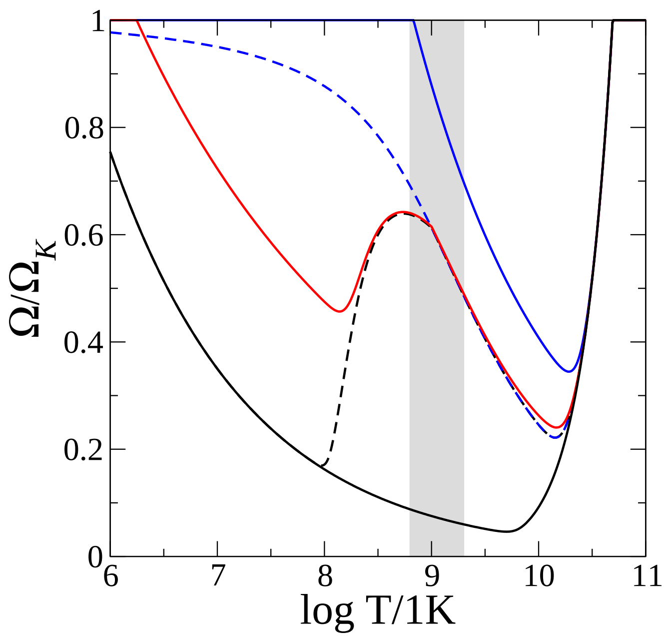
$$\tau = \left[\frac{1}{\tau_{\rm ee}} + \frac{1}{\tau_{\rm ep}}\right]^{-1} \qquad \text{where} \qquad \tau_{\rm ee} >> \tau_{\rm ep}$$

The most important contribution comes from the most frequent scattering process.

$\boxed{\text{Need:}}$ Superfluid suppression factors for finite temperature (cf. cooling)

Hyperon viscosity

Bulk viscosity due to non-leptonic interactions may suppress the gravitational-wave driven r-mode instability. In particular, the hyperon bulk viscosity is relevant at low temperatures (the coefficient scales as T^{-2} rather than T^6).



But, in order not to be in conflict with cooling data, the hyperons must be superfluid.

Then the nuclear reactions that lead to the bulk viscosity are suppressed...

... and the effect on the instability may not be so great after all.

Most recent estimates suggest that r-modes may be unstable in LMXBs, and could possibly provide persistent gravitational-wave signal.

$\boxed{\text{Need:}}$ Better hyperon gap estimates and multifluid models for n, p, Λ, Σ^- etcetera.

Modelling multifluids

The equations describing a two-fluid model for superfluid neutron stars can be derived from an energy functional $E(n_{\rm n}, n_{\rm p}, w^2)$ where $w_i^{\rm yx} \equiv v_i^{\rm y} - v_i^{\rm x}$ (${\rm x}, {\rm y}$ are constituent indices);

$$dE = \sum_{\rm x=n,p} \mu_{\rm x} dn_{\rm x} + \alpha dw^2 \longrightarrow \tilde{\mu}_{\rm x} = \frac{1}{m_{\rm B}} \frac{\partial E}{\partial n_{\rm x}} \text{ and } \alpha = \frac{\partial E}{\partial w^2}$$

In the conservative case this leads to

$$\partial_t n_{\rm x} + \nabla_i (n_{\rm x} v^i_{\rm x}) = 0$$

and

$$n_{\rm x} \left(\partial_t + \pounds_{v_{\rm x}} \right) p_i^{\rm x} + n_{\rm x} \nabla_i \left(\mu_{\rm x} - \frac{1}{2} m v_{\rm x}^2 \right) = 0$$

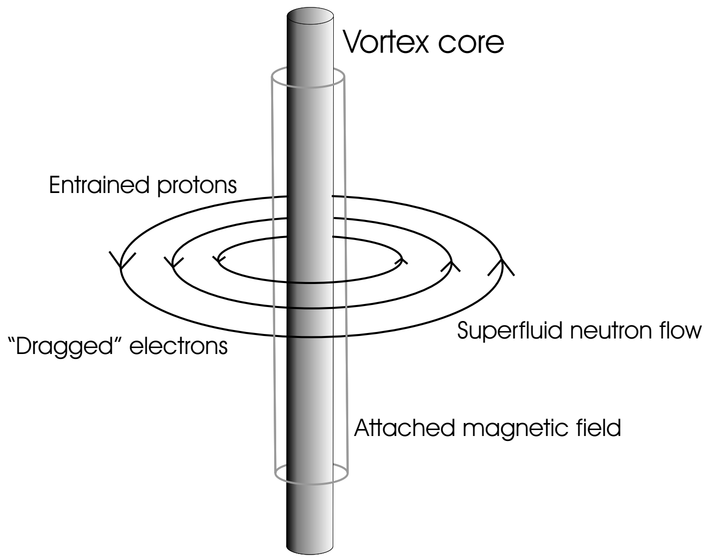
The momentum $p_i^{\rm x} = m(v_i^{\rm x} + \varepsilon_{\rm x} w_i^{\rm yx})$, where $\varepsilon_{\rm x} = 2\alpha / m_{\rm B} n_{\rm x}$, encodes the entrainment effect.

Note: The entrainment can be represented by an effective mass $m_{\rm p}^*$.

$\boxed{\text{Need:}}$ Entrainment parameter at finite temperature (EoS “out of equilibrium”)

Mutual friction

The presence of vortices leads to mutual friction between interpenetrating superfluids (neutrons and protons).



- superfluid neutron <u>momentum</u> vorticity is quantised ($\kappa^i = \epsilon^{ijk}\nabla_j p^{\rm n}_k$)
- induces flow in part of the protons because of entrainment
- magnetic fields form on the vortices

$$B \approx 2\times 10^{14}{\rm G}\,\left(\frac{x_{\rm p}}{0.05}\right)\left(\frac{\rho}{10^{14}{\rm g/cm}^3}\right)\left|\frac{m_p - m_p^*}{m_p^*}\right|$$

- electrons scatter dissipatively off these magnetic fields

Balancing the Magnus force to force due to electrons scattering off of the vortex

$$f_i^{\rm M} = \rho_{\rm n}\epsilon_{ijk}\kappa^j w^k_{\rm nL} = \mathcal{R}(v_i^{\rm p} - v_i^{\rm L}) = f_i^{\rm e}$$

one can infer that the mutual friction force acting on the neutrons is

$$f^i_{\rm mf} = \mathcal{B}\rho_n n_v \underbrace{\epsilon^{ijk}\hat{\kappa}_j\epsilon_{klm}\kappa^l w^m_{\rm pn}}_{{\rm projects}\perp\kappa^i} + \mathcal{B}'\rho_{\rm n}n_v\underbrace{\epsilon^{ijk}\kappa_j w_k^{\rm pn}}_{{\rm acts}\perp w_k^{\rm pn}}$$

$\boxed{\text{Need:}}$ Understanding of possible vortex "clusters".

Dynamical coupling

The dimensionless coefficients are $\mathcal{B}' = \mathcal{B}^2$ and

$$\mathcal{B} = \frac{\mathcal{R}}{\rho_{\rm n}\kappa} \approx 4\times 10^{-4} \left(\frac{m_{\rm p}-m_{\rm p}^*}{m_{\rm p}}\right)^2 \left(\frac{m_{\rm p}}{m_{\rm p}^*}\right)^{1/2} \left(\frac{x_{\rm p}}{0.05}\right)^{7/6} \left(\frac{\rho}{10^{14}{\rm g/cm}^3}\right)^{1/6}$$

Working out dynamical coupling timescale we find

$$\left.\begin{array}{l} n_{\rm n}\partial_t p_i^{\rm n} + \ldots = f_i^{\rm mf} \\ n_{\rm p}\partial_t p_i^{\rm p} + \ldots = -f_i^{\rm mf} \end{array}\right\} \longrightarrow \frac{m_{\rm p}^*}{m_{\rm p}}\partial_t w_i^{\rm np} + \ldots \approx -\frac{\mathcal{B}\kappa n_v}{x_{\rm p}} w_i^{\rm np}$$

That is, the timescale on which the two fluids are (locally) coupled can be estimated as

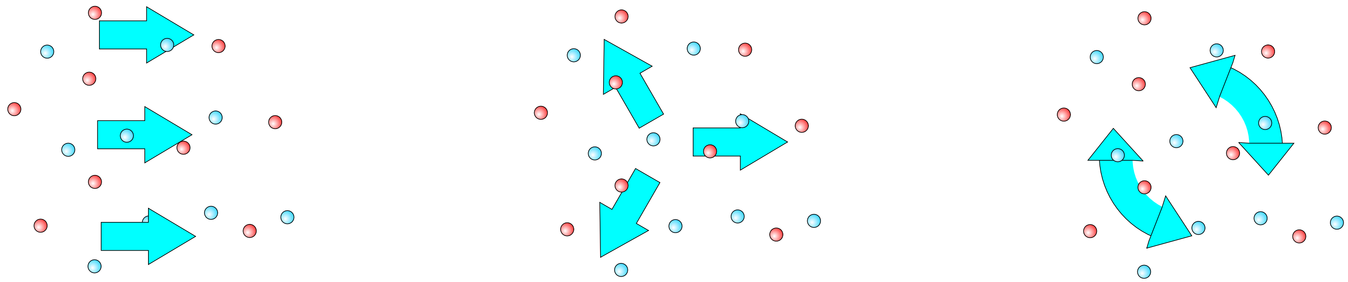
$$\tau_d \approx \frac{m_{\rm p}^*}{m_{\rm p}}\frac{x_{\rm p}}{\mathcal{B}\kappa n_v} \approx 10P(s) \left(\frac{m_{\rm p}^*}{m_{\rm p}-m_p^*}\right)^2 \left(\frac{x_{\rm p}}{0.05}\right)^{-1/6} \left(\frac{\rho}{10^{14}{\rm g/cm}^3}\right)^{-1/6}$$

This is about 1 order of magnitude <u>faster</u> than the estimate by Alpar and Sauls (1988).
Usually taken as evidence that glitches must originate in the crust superfluid.

$\boxed{\text{Need:}}$ Understanding of “pinning” and role of turbulence (vortex tangles).

Current to-do list

- develop a flux-conservative formalism for multi-fluid systems, including dissipation and entrainment.



Note: The $\boxed{\text{simplest model}}$ for neutron stars, starting with four fluids and reducing to two degrees of freedom, requires 19 dissipation coefficients.

- consider relativistic models for superfluids coexisting with an elastic crust.
  This should allow us to build “quantitative” models for glitch relaxation.
- worry about $T \neq 0$ (probably always relevant!).
  How many “fluids” do you need? How account for excitations?
- think about superfluid “turbulence”.
  Is it relevant for neutron stars? If so, how does it manifest itself?
- learn more about “exotica”.
  How does colour superconductivity work?