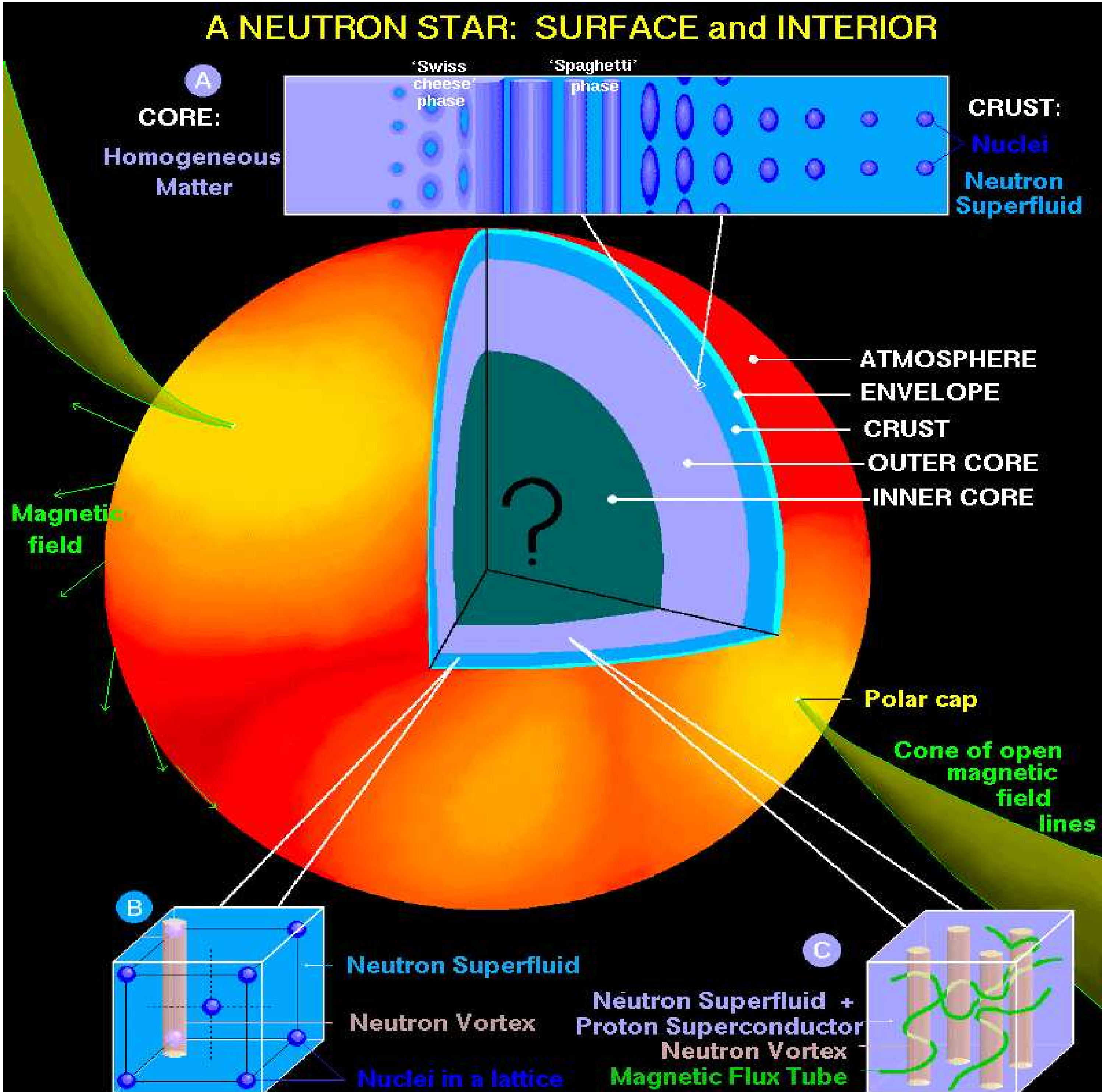
Equation of State Constraints from Neutron Stars

James Lattimer

lattimer@astro.sunysb.edu

Department of Physics & Astronomy

Stony Brook University

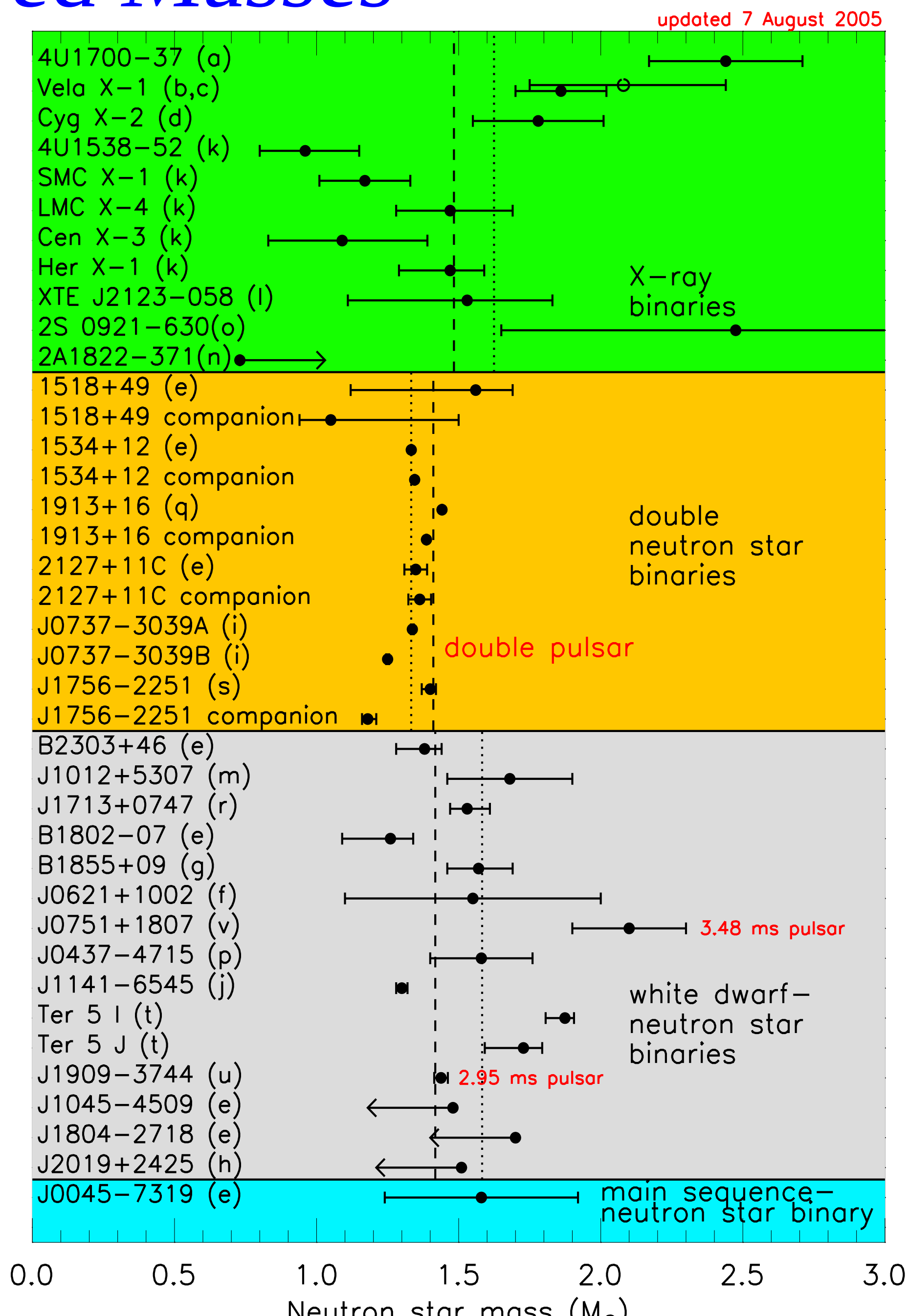


Credit: Dany Page, UNAM

Observable Quantities (1)

- Mass
  - Limits softening from ‘exotica’ (hyperons, Bose condensates, quarks).
  - Limits highest possible density in stars: $\rho < 1.4 \times 10^{16} (\mathrm{M}_\odot / M)^2$ g cm^{-3}.
  - New evidence for $M_{max} > 1.5$ M$_\odot$.

Observed Masses

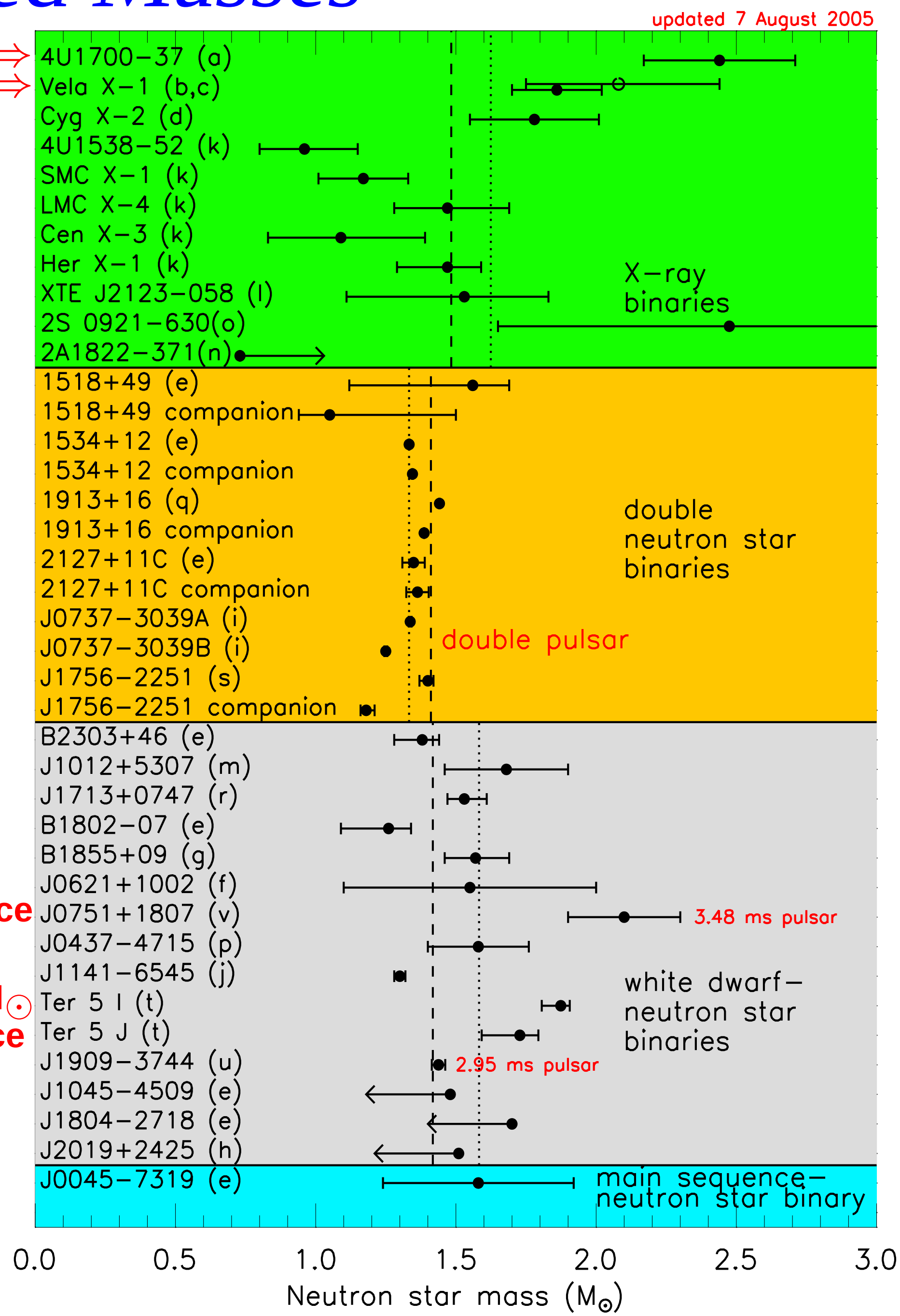


Observed Masses

Black hole? ⇒
Firm lower mass limit? ⇒

$M > 1.6$ $\mathrm{M}_\odot$, **95% confidence**

$M > 1.68$ $\mathrm{M}_\odot$
95% confidence



Maximum Possible Density in Stars

Causality limit for compactness: $R \geq 3GM/c^2$

(Lattimer, Masak, Prakash & Yahil 1990; Glendenning 1992)

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Uniform Density:

$$\rho_{c,UD} = \frac{3M}{4\pi R^3} \leq \frac{3}{4\pi}\left(\frac{c^2}{3G}\right)^3 \frac{1}{M^2} = 5.4 \times 10^{15} \left(\frac{\mathrm{M}_\odot}{M}\right)^2 \mathrm{g\ cm}^{-3}$$

But UD solution violates causality and $\rho_{surface} \neq 0$. No realistic EOS has greater ρ_c for given M than Tolman VII solution ($\rho = \rho_c[1-(r/R)^2]$) (Lattimer & Prakash 2005)

$$\mathrm{Tolman\ \ VII:} \qquad \rho_{c,VII} = \frac{5}{2}\rho_{c,Inc} \leq 13.6 \times 10^{15}\,(\mathrm{M}_\odot/M)^2\,\mathrm{g\ cm}^{-3}$$

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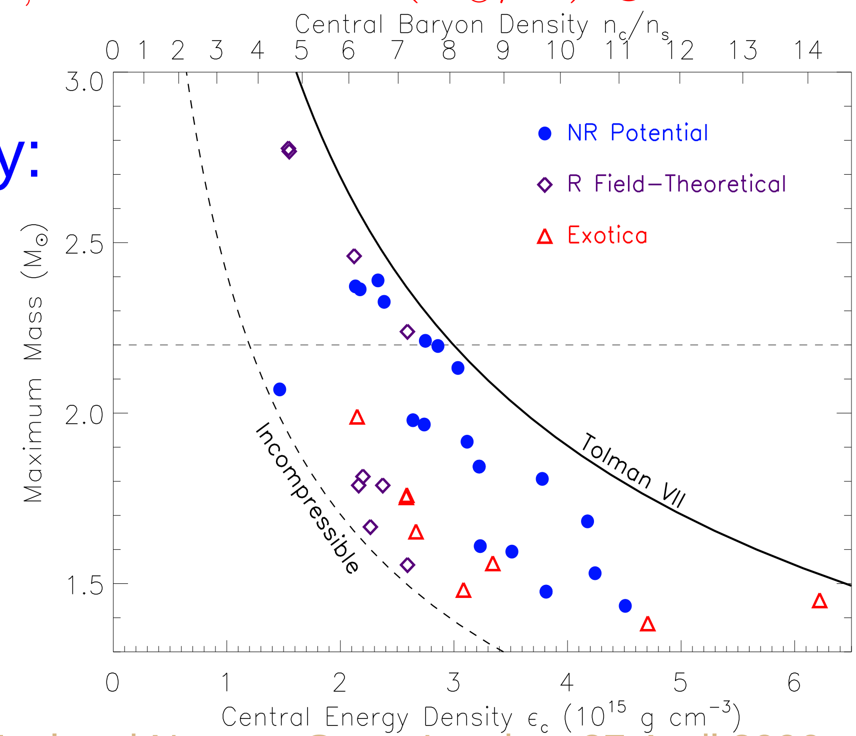
Maximum possible density:

$$2.2\ \mathrm{M}_\odot \Rightarrow$$
$$\rho_{max} < 2.8 \times 10^{15}\mathrm{g\ cm}^{-3}$$



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- Mass
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  - New evidence for $M_{max} > 1.5$ $M_\odot$.
- Radius
  - Limits isospin dependence of EOS (Lattimer & Prakash 2001)

    $$1 < n/n_0 < 2, x \simeq 0 : \quad P(n) \propto [dE_{sym}(n)/dn] \propto R^4_{1-1.5\ M_\odot}$$
  - $P(n_0)$ currently uncertain to factor ~ 6
  - Proposed neutron skin $(R_n - R_p)$ measurements of ^{208}Pb could constrain $E_{sym}(n < n_0)$
  - Involves different physics than M_{max}

Main Classes of Equations of State

- Non-relativistic potential models
  - Momentum- and density-dependent potential
  - Power-series density expansion
  - Density-dependent effective nucleon masses
  - Generally have relatively slowly varying symmetry energies, smaller radii
  - Can become acausal
  - Can be constrained to fit low-density matter properties
- Relativistic field-theoretical models
  - Interactions mediated by bosons (ω, σ, ρ)
  - Implicitly causal
  - Generally have linearly increasing symmetry energies, larger radii
  - Not easily constrained to fit low-density matter properties

  Recently proposed Skyrmion force (Ouyed & Butler 1994, Jaikumar & Ouyed 2005), a variation of RFT, designed to maximize neutron star radii, was fit with incorrect saturation properties.
- Self-bound models
  - Strange quark matter with a lower binding energy than hadronic matter (iron) at zero pressure
  - Radii of high mass stars not necessarily smaller than hadronic stars

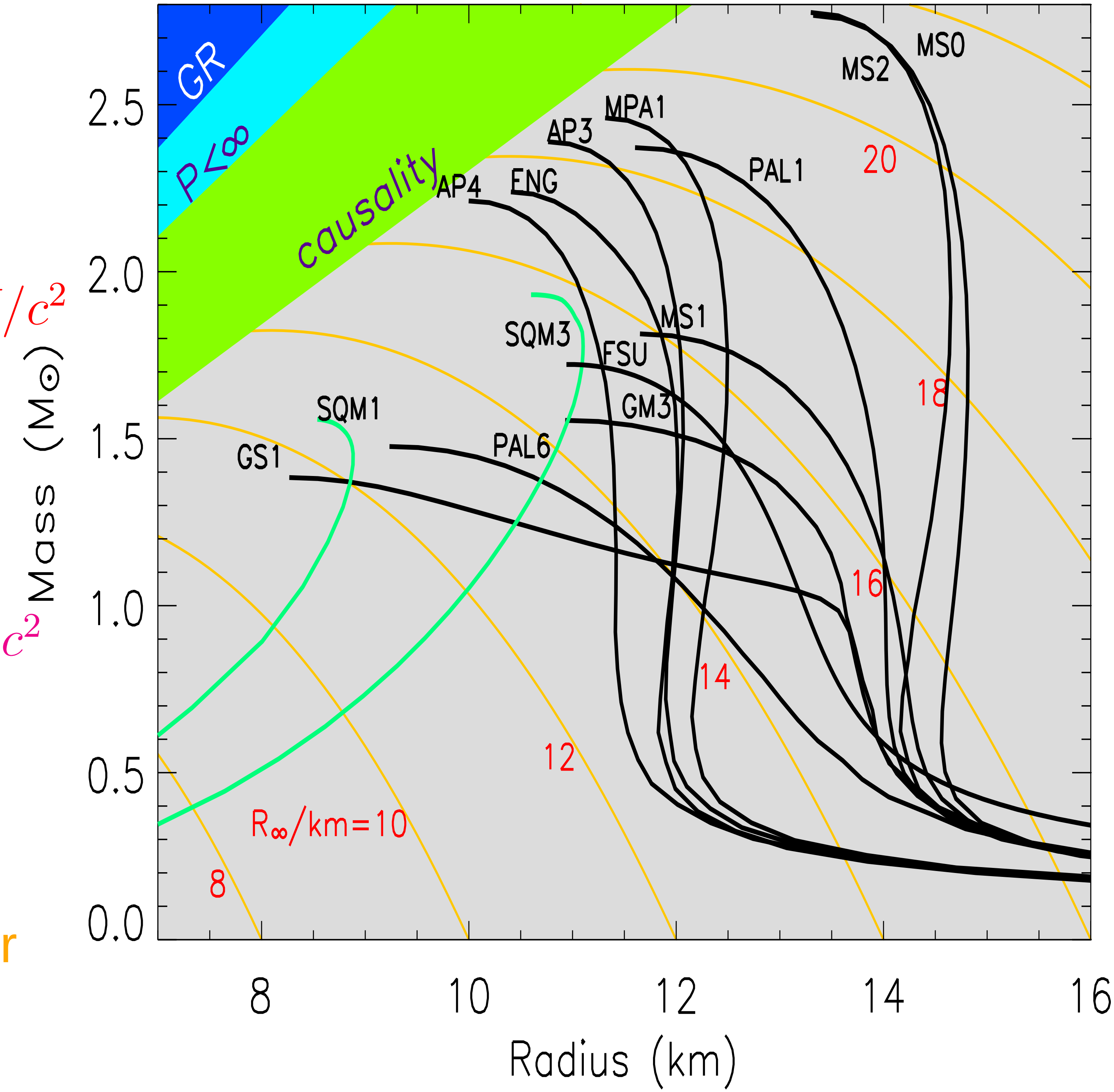
GR:
$R > 2GM/c^2$

$P < \infty$:
$R > (9/4)GM/c^2$

causality:
$R > 2.94GM/c^2$

— normal NS
— SQS

— R_∞ contour



Neutron Star Matter Pressure

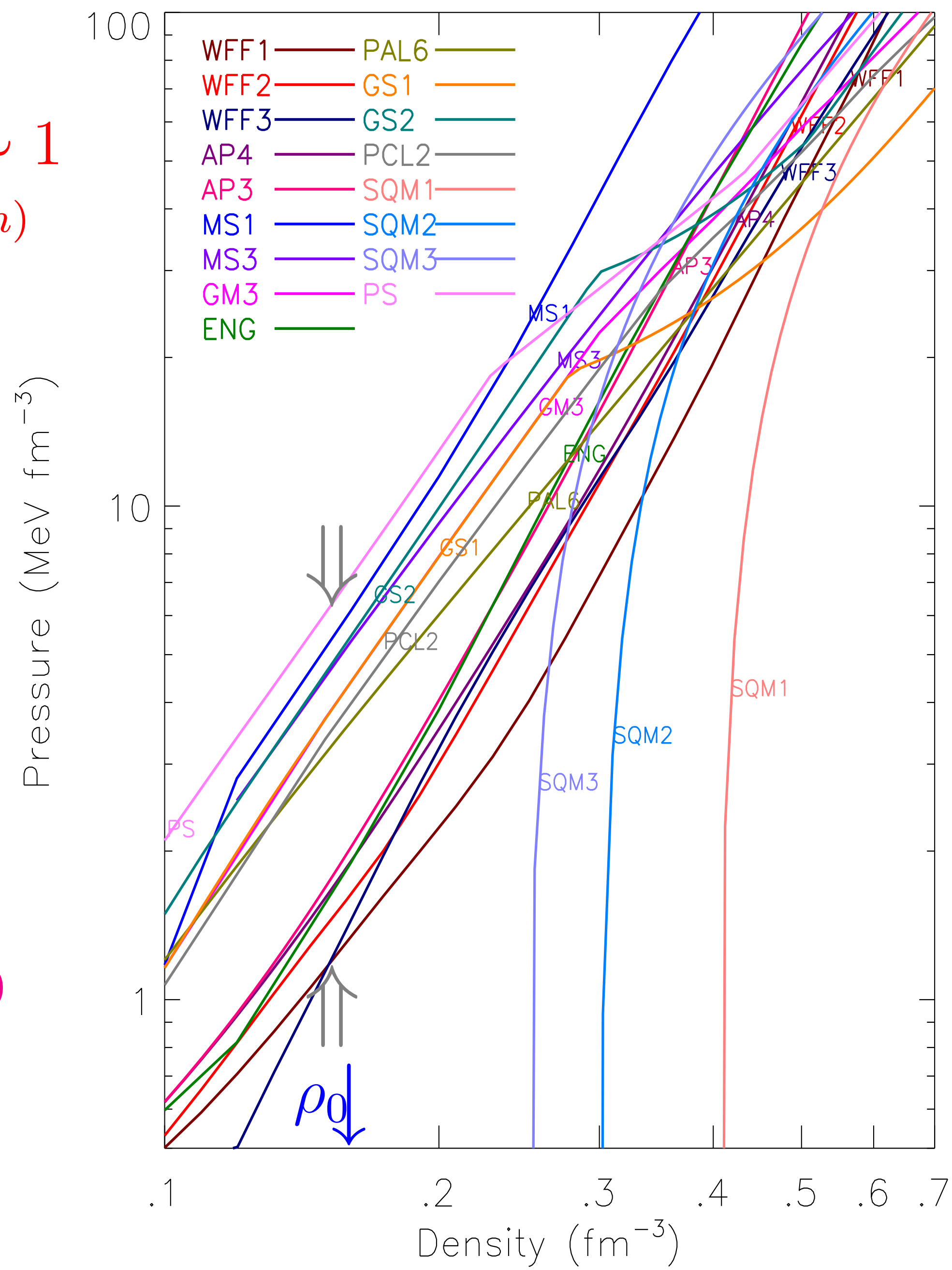
$P \simeq K\rho^{1+1/n}$

$n^{-1} = d\ln P/d\ln\rho - 1 \sim 1$

$R \propto K^{n/(3-n)} M^{(1-n)/(3-n)}$

$R \propto P_*^{1/2}\rho_*^{-1}M^0$

$(1 < \rho_*/\rho_0 < 2)$



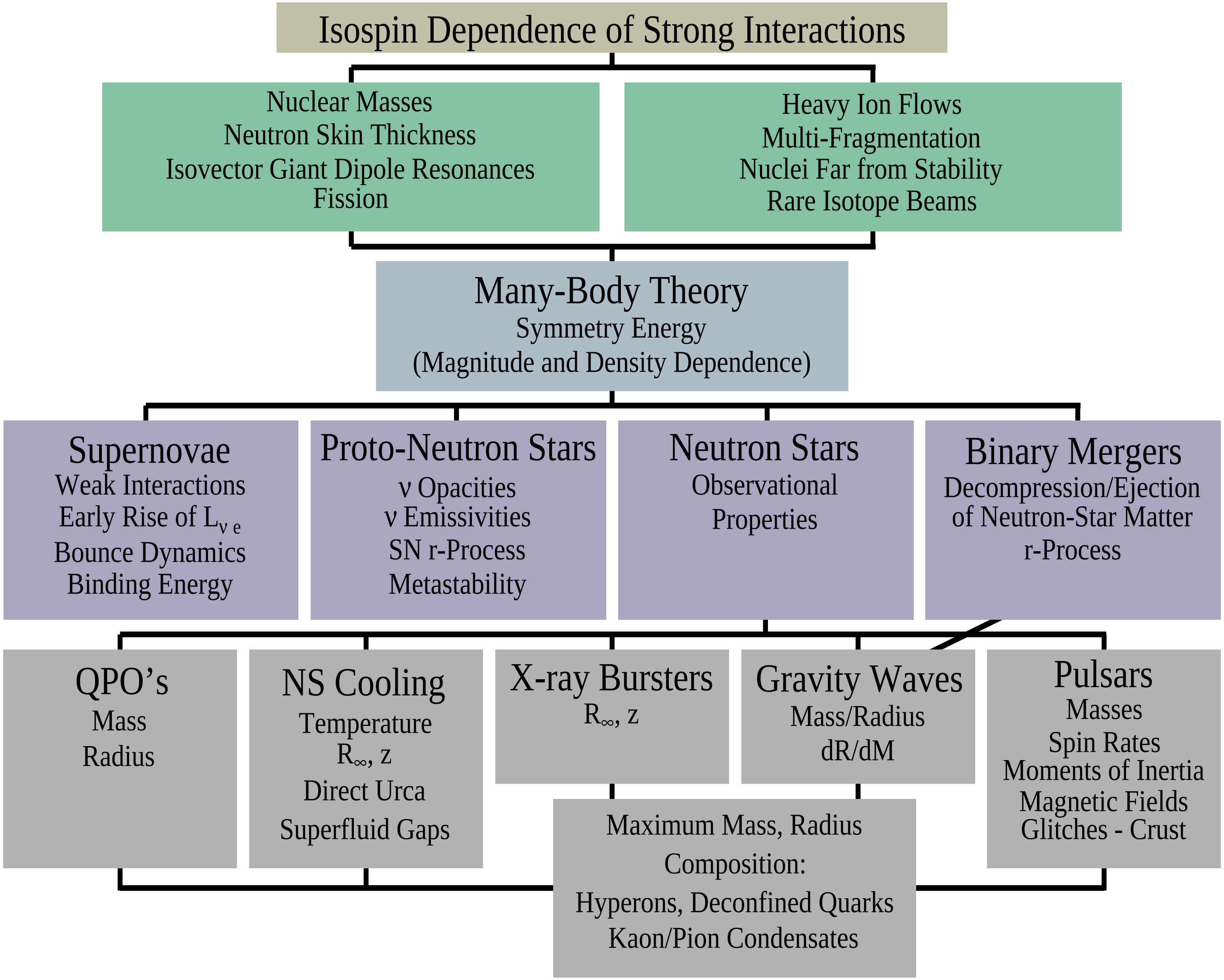
Wide variation:

$1.2 < \frac{P(\rho_0)}{\mathrm{MeV\ \ fm^{-3}}} < 7$

GR phenomenological result (Lattimer & Prakash 2001)

$R \propto P_*^{1/4}\rho_*^{-1/2}$

The Role of the Symmetry Energy



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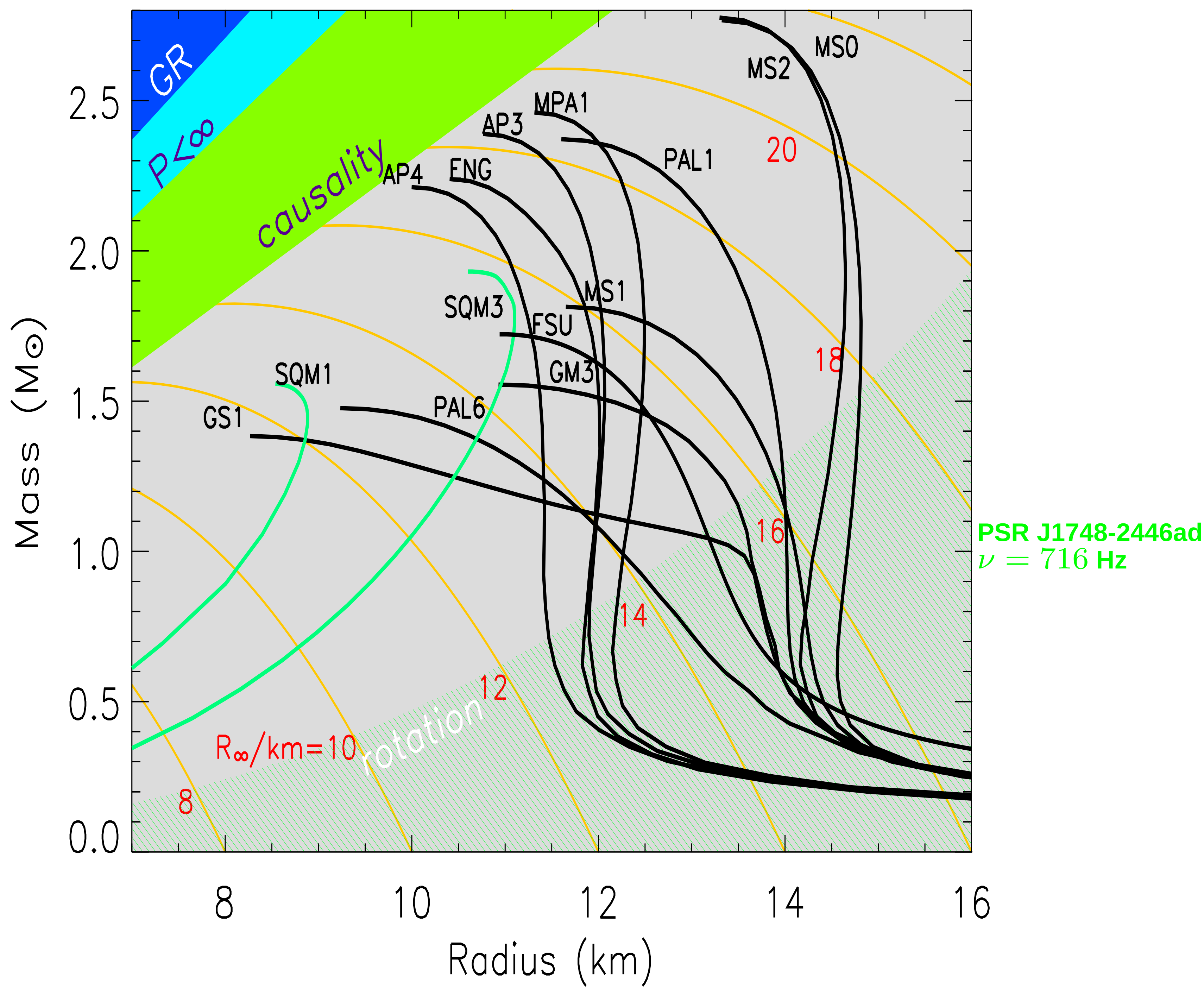
  - $P(n_0)$ currently uncertain to factor ~ 6
  - Proposed neutron skin $(R_n - R_p)$ measurements in ^{208}Pb could constrain $E_{sym}(n < n_0)$
  - Involves different physics than M_{max}.
- ## Spin Frequency
  - Naive Newtonian Roche model (Shapiro & Teukolsky) suggests
  $$P_{min}^{Roche} = 1.0 \left(\frac{R}{10\ \mathrm{km}}\right)^{3/2} \left(\frac{\mathrm{M}_\odot}{M}\right)^{1/2} \ \mathrm{ms}$$
  - Axisymmetric GR calculations (Lattimer & Prakash 2004) yield
  $$P_{min} = 0.96 \pm 0.03 \left(\frac{R}{10\ \mathrm{km}}\right)^{3/2} \left(\frac{\mathrm{M}_\odot}{M}\right)^{1/2} \ \mathrm{ms}$$

  where M and R refer to non-rotating values (EOS-independent relation).



GR
$P<\infty$
causality
rotation
R_∞/km=10
8
12
14
16
18
20
MS0
MS2
MPA1
AP3
PAL1
AP4
ENG
SQM3
MS1
FSU
GM3
SQM1
PAL6
GS1
PSR J1748-2446ad
$\nu = 716$ Hz
Mass ($M_\odot$)
Radius (km)
0.0
0.5
1.0
1.5
2.0
2.5
8
10
12
14
16

Observable Quantities (2)

- Radiation Radius
  - Combination of flux and temperature measurements yield

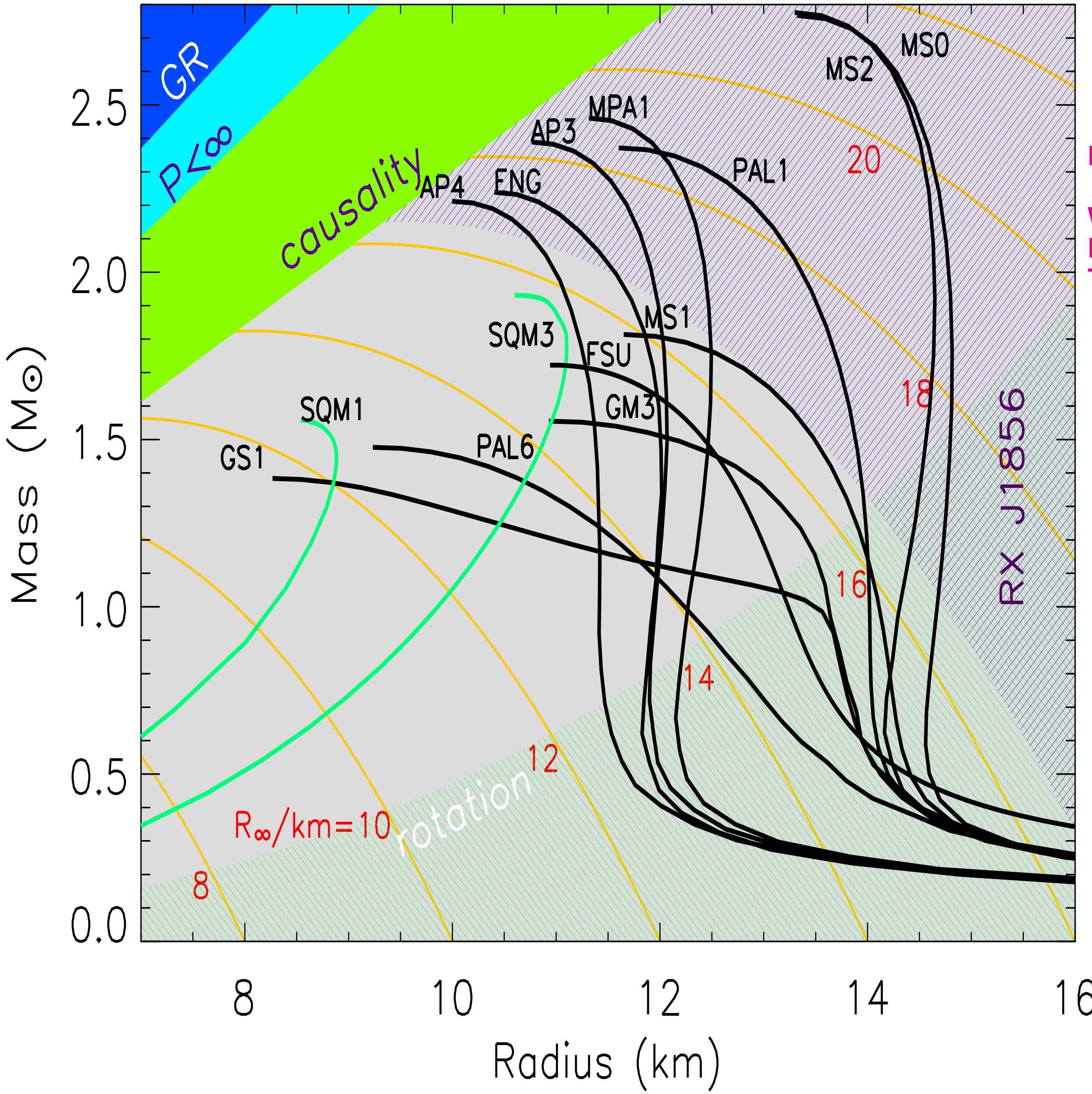
$$R_\infty = R/\sqrt{1-2GM/Rc^2}$$

  - Uncertainties include distance ($R_\infty \propto d$), interstellar H absorption (hard UV and X-rays), atmospheric composition
  - Best chances are from
    - Nearby isolated neutron stars (parallax measurable)
    - Quiescent X-ray binaries in globular clusters (reliable distances, low B H-atmosperes)

Optical flux on Rayleigh-Jeans tail 5-7 times extrapolated X-ray BB and $T_{opt} \sim T_X/2$:

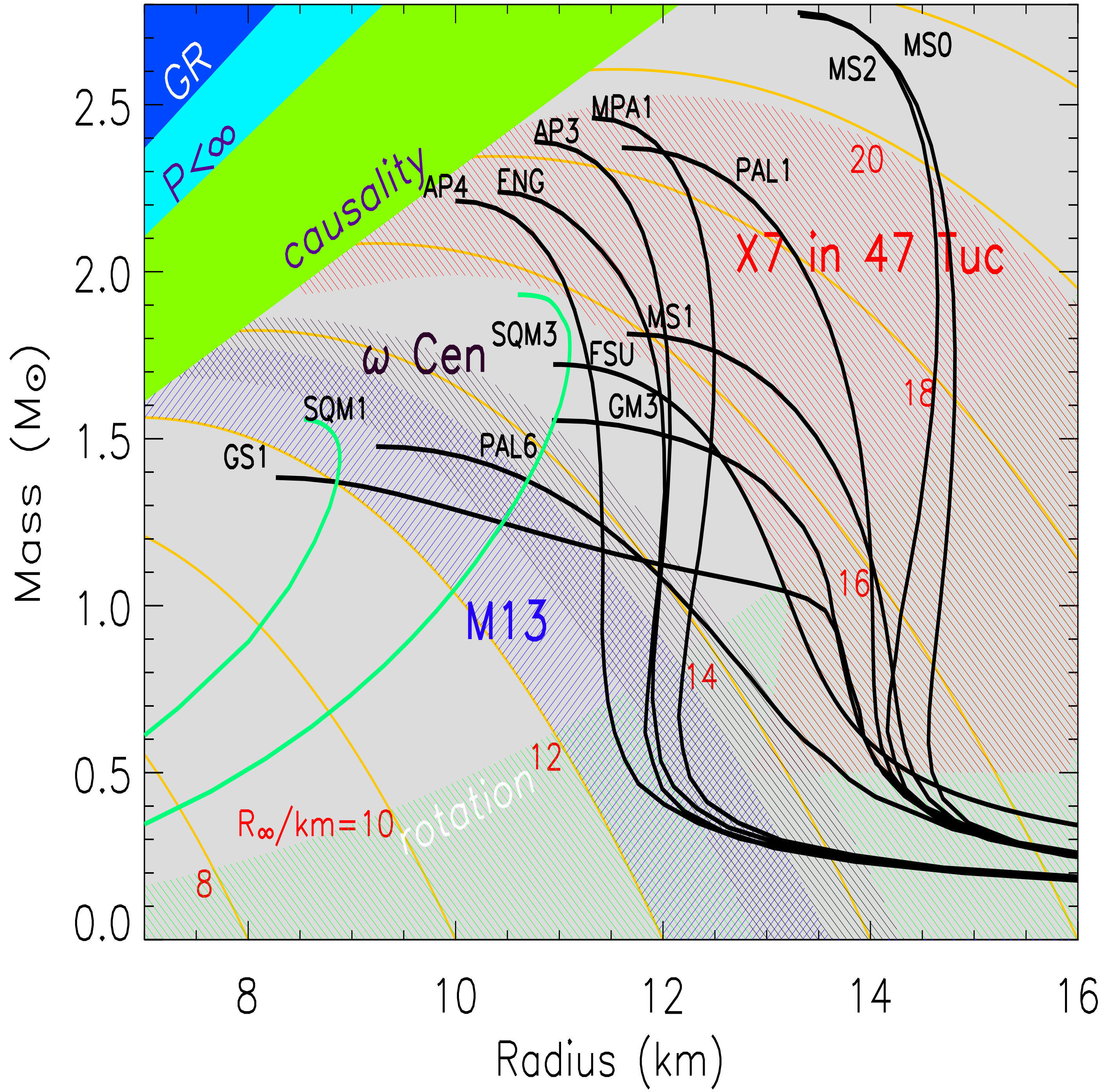
$$F_{opt} = 4\pi \left(\frac{R_{opt}}{d}\right)^2 T_{opt} = 4\pi f \left(\frac{R_X}{d}\right)^2 T_X$$

$$R = \sqrt{R_{opt}^2 + R_X^2} = \sqrt{1+2f}R_X$$



RX J1856-3754:

Walter & Lattimer 2002
Braje & Romani 2002
Truemper 2005
D=120 pc



X7 in 47 Tucanae:

Rybicki, Heinke, Narayan & Grindlay 2005

ω Cen:

Gendre, Barret & Webb 2003a

M13:

Gendre, Barret & Webb 2003b

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$$R_\infty = R/\sqrt{1 - 2GM/Rc^2}$$
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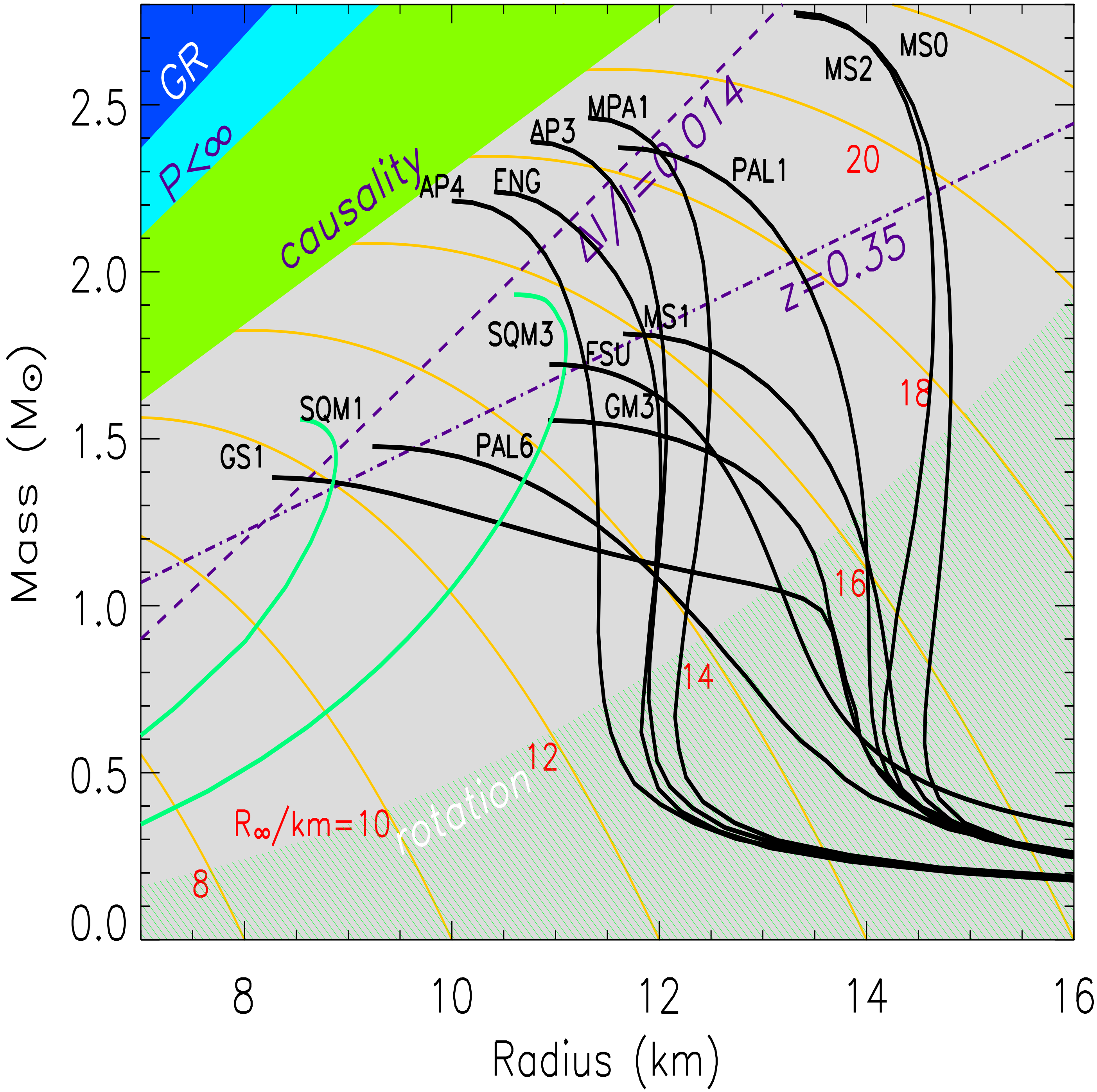
    - Quiescent X-ray binaries in globular clusters (reliable distances, low B H-atmosperes)
- Redshift $z = (1 - 2GM/Rc^2)^{-1/2} - 1$
  - Possible lines from active X-ray bursters XTE J1814-338 $z < 0.38$, 4U1820-30 $0.20 < z < 0.30$, EXO 0748-676 $z \simeq 0.35$ (Cottam, Paerels & Mendez 2002)
  - $R = R_\infty(1+z)^{-1}, \quad M = R_\infty c^2(1+z)^{-1}[1-(1+z)^{-2}]/2G$
- Pulsar Glitches
  - Global transfer of angular momentum, possibly from weak coupling between crustal n superfluid and star.
$$I_{crust}/I_{star} > 0.014 \propto P_t R^4 M^{-2}$$
  - $P_t < 0.65$ MeV fn^{-3} core-crust interface pressure
  - Establishes lower limit to R^2/M (Link, Epstein & Lattimer 1999)



EXO 0748-676:
$z = 0.35$

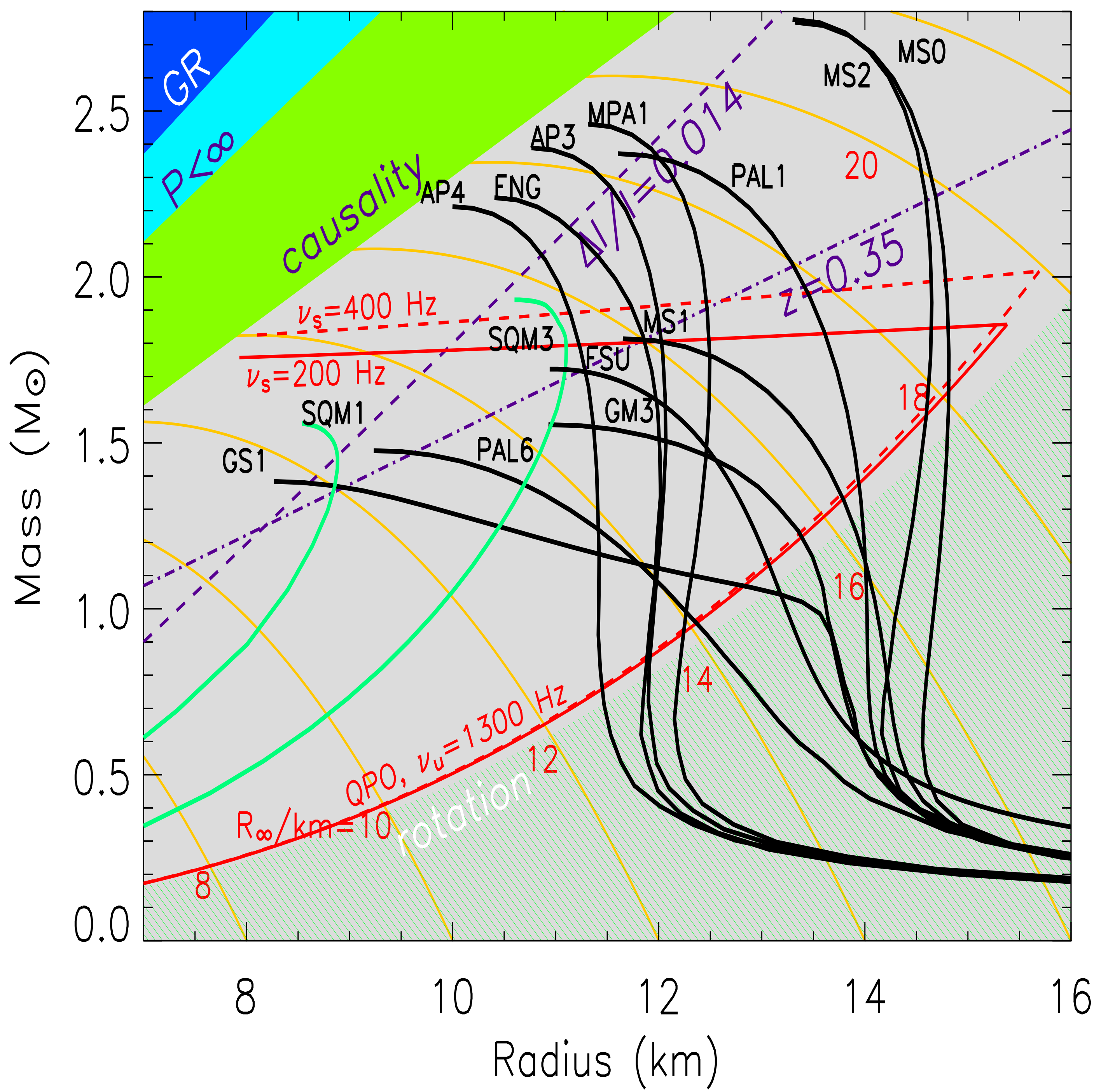
Cottam, Paerels & Mendez 2002

Vela:
$\Delta I/I > 0.014$

Link, Epstein & Lattimer 1999

Observable Quantities (3)

- QPOs
  - Possible association of observed frequency with inner radius of accretion disk $R_A > 6GM/c^2$ and $R_A > R$ (Lamb & Miller 2000)


GR
P<∞
causality
ΔI/I=0.014
z=0.35
νs=400 Hz
νs=200 Hz
QPO, νu=1300 Hz
R∞/km=10
rotation
MS0
MS2
MPA1
AP3
PAL1
AP4
ENG
MS1
SQM3
FSU
GM3
SQM1
PAL6
GS1
20
18
16
14
12
8
Mass (M⊙)
Radius (km)
2.5
2.0
1.5
1.0
0.5
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10
12
14
16

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- Burst sources limited by Eddington luminosity
  $F_{edd} \propto Md^2/\kappa$
- Neutron star oscillations
  $\Delta R/R \simeq 0.1$ (Watts & Strohmayer)

$$\frac{GM}{Rc^2} \simeq \frac{1}{2} - \frac{\Delta R}{R}\frac{\mathcal{H}}{2}(\frac{\Delta R}{R} + \mathcal{H} - 1)^{-1} \tag{1}$$

$\mu_n - \mu_{n0} \simeq -7 + \frac{K}{18}u_c(3u_c - 4) + S_v(u_c) + \left(\frac{dS_v}{d\ln\rho}\right)_c$

$u_c \simeq 2/3$

$\mathcal{H} = e^{2(\mu_n - \mu_{n0})/m_B} \simeq 1.03$

$\frac{GM}{Rc^2} \simeq 0.15$

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- Moment of Inertia
  - Spin-orbit coupling of same magnitude as post-post-Newtonian effects (Barker & O'Connell 1975, Damour & Schaeffer 1988)
  - Precession alters inclination angle and periastron advance
  - More EOS sensitive than R: $I \propto MR^2$
  - Double pulsar PSR J0737-3037 is candidate

Moments of Inertia and Precession

- Spin-orbit coupling: $\dot{\vec{S}}_A = -\dot{\vec{L}} = \frac{G(4M_A+3M_B)}{2M_A a^3 c^2}\vec{L} \times \vec{S}_A$

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$$P_p = \frac{2(M_A + M_B) a c^2}{G M_B (4M_A + 3M_B)} P(1 - e^2) \simeq 74.9 \text{ years}$$

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$$P_p = \frac{2(M_A+M_B)ac^2}{GM_B(4M_A+3M_B)}P(1-e^2) \simeq 74.9 \text{ years}$$

- Precession Amplitude:

$$\delta_i = \frac{|\vec{S_A}|}{|\vec{L}|}\sin\theta \simeq \frac{I_A(M_A+M_B)}{a^2 M_A M_B}\frac{P}{P_A}\sin\theta \simeq (3.6-7.2)\sin\theta\times 10^{-5}$$

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- Delay in Time-of-Arrival:

$$\Delta t = \left(\frac{M_B}{M_A + M_B}\right) \frac{a}{c} \delta_i \cos i \approx 0.4 - 4.0 \ \sin\theta \ \mu\text{s}$$

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- Periastron Advance $\propto \vec{S}_A \cdot \vec{L}$: $A_P/A_{PN} =$

$$\frac{2\pi I_A}{P_A} \left(\frac{2 + 3M_B/M_A}{3M_A^2 + 3M_B^2 + 2M_A M_B} \right) \sqrt{\frac{M_A + M_B}{Ga}} \cos\theta \simeq (2.2 - 4.3) \times 10^{-4} \cos\theta$$

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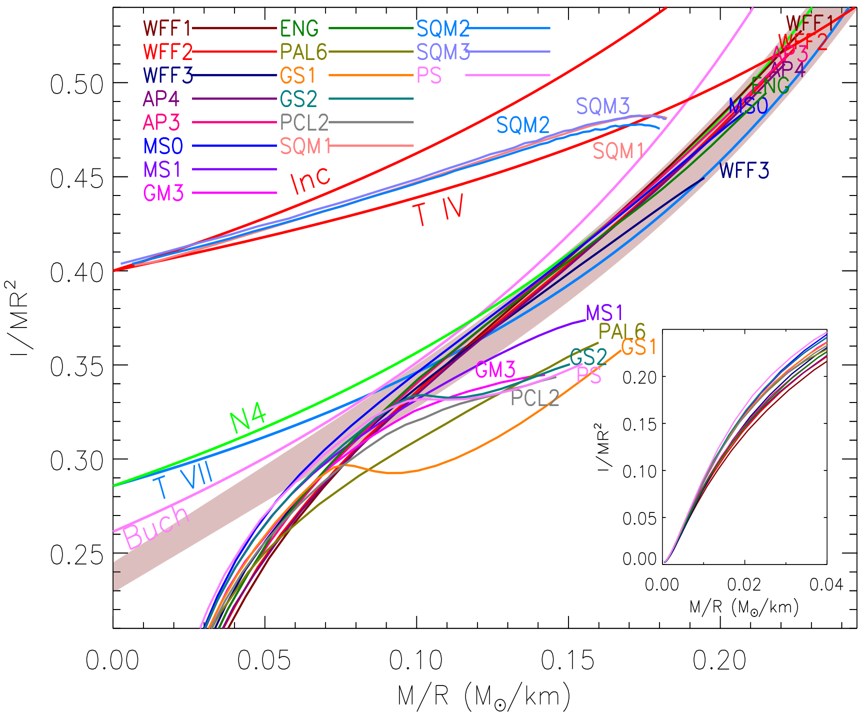
- Moment of Inertia – Mass – Radius

$$I \simeq (0.237\pm 0.008)MR^2\left[1+4.2\frac{M\text{ km}}{R\text{ M}_\odot}+90\left(\frac{M\text{ km}}{R\text{ M}_\odot}\right)^4\right]$$

Moments of Inertia



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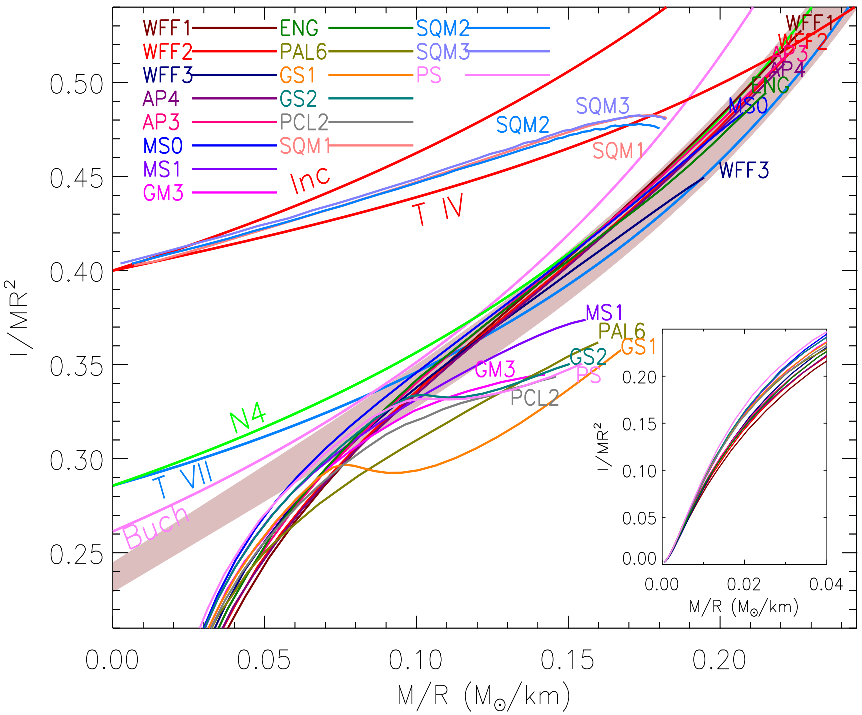

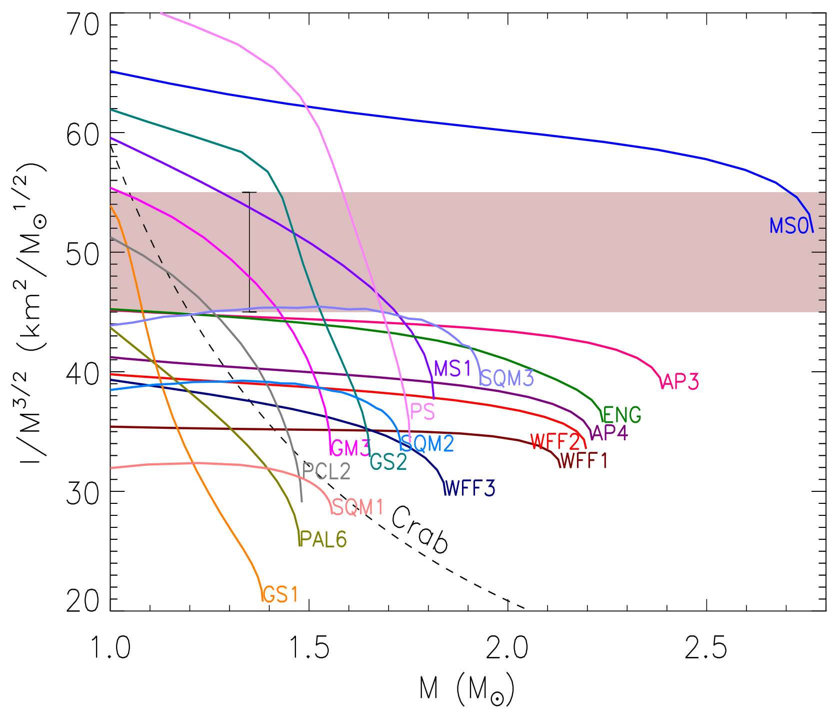

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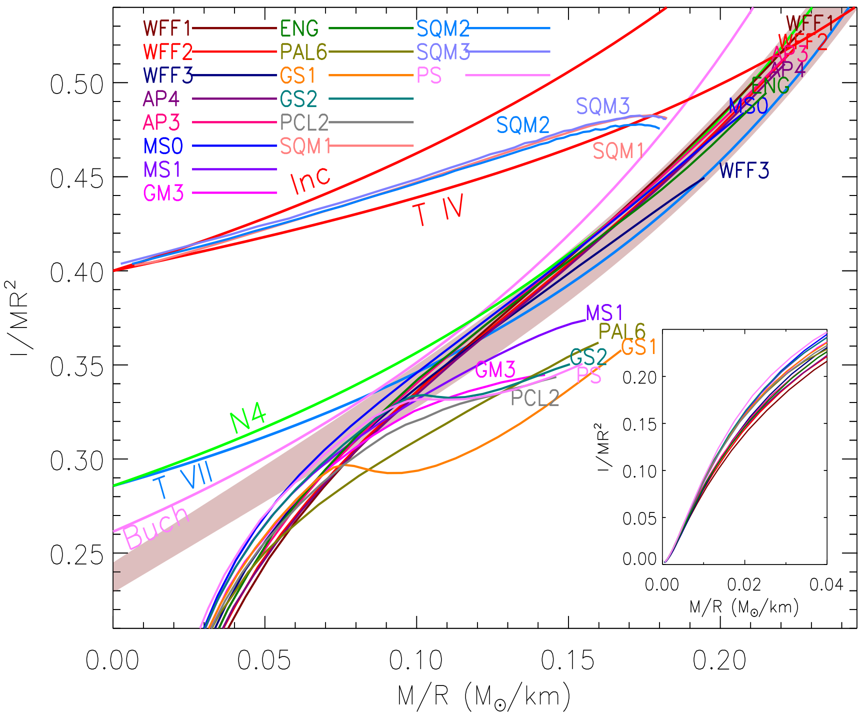
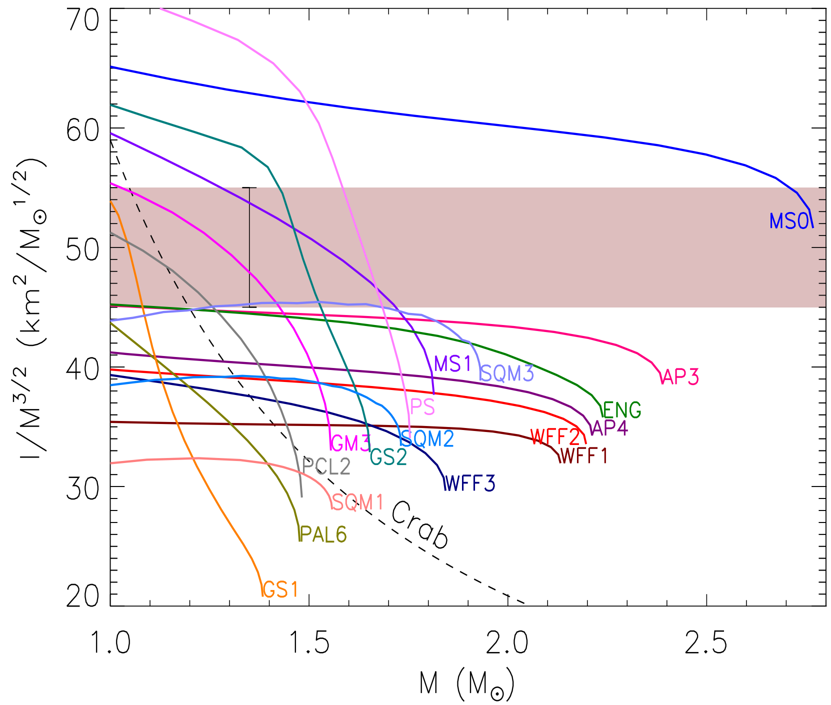
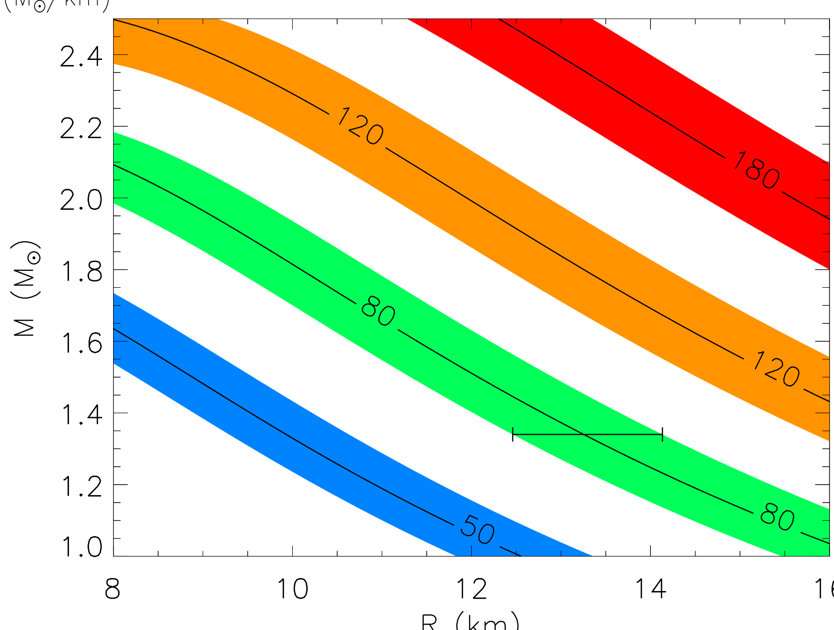






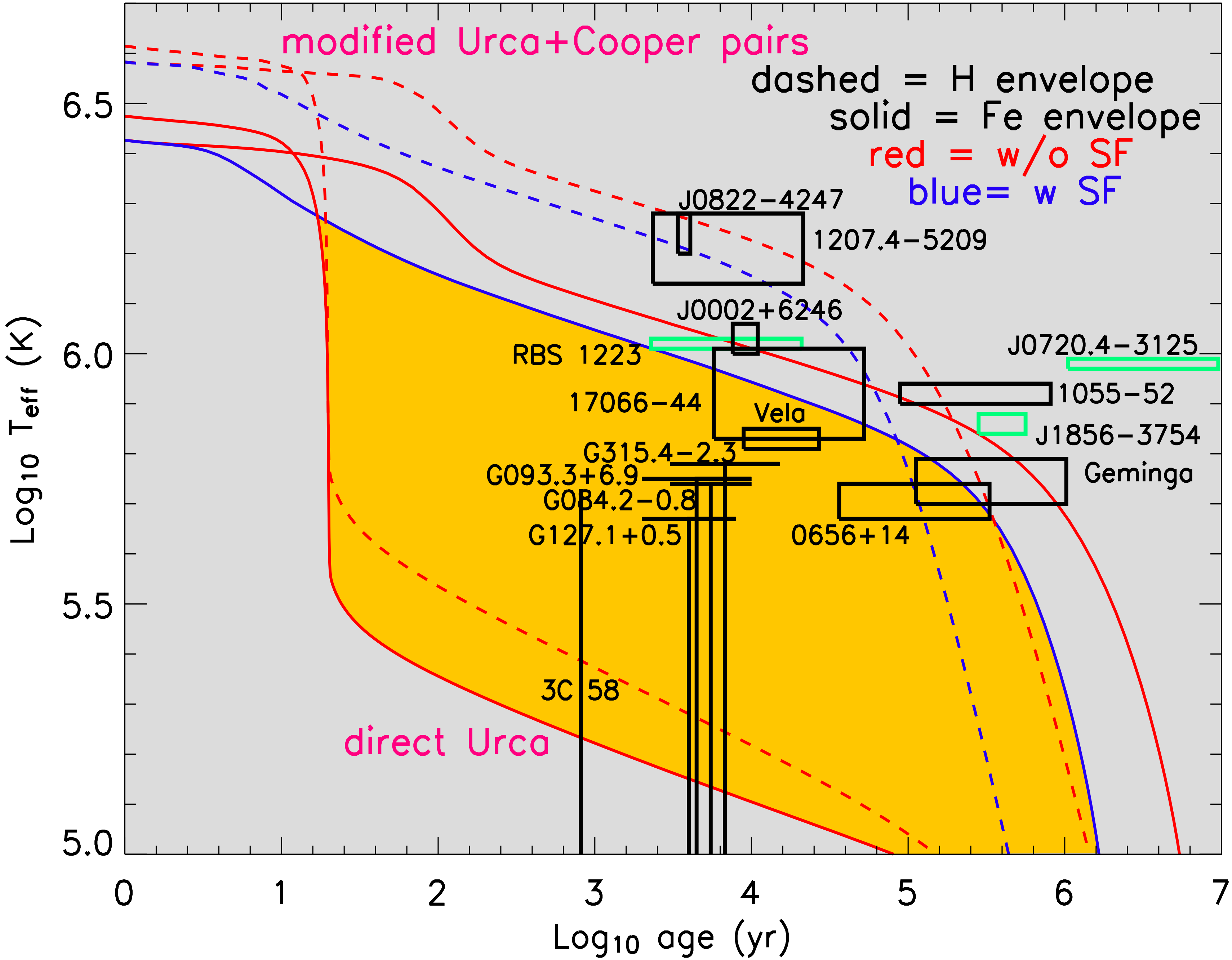
Lattimer & Schutz (2005)

Observable Quantities (4)

- Temperature and Age
  - Growing database of cooling neutron stars.
  - Atmosphere composition, magnetic field strength, distance are major uncertainties.
  - Hints that some neutron stars may exhibit 'rapid' (*i.e.,* direct Urca) cooling: Vela (Pavlov et al. 2001), supernova remnants 3C58 (Slane et al. 2004), G084.2-0.8, G093.3+6.9, G127.1+0.5 & G315.4-2.3 (Kaplan et al. 2004)
  - Probes superfluid properties of neutron star interior

Neutron Star Cooling

Page, Lattimer, Prakash & Steiner (2004)



Conclusions

- Masses may have a wide range.
- M_{max} might be > 1.7 $M_\odot$, limits EOS at high density.
- Radii remain elusive, but some R_∞ measurements imply relatively large values.
- R measurement gives direct information about the EOS near the saturation density for extremely asymmetric matter, *i.e.,* the density derivative of the symmetry energy.
- Other observables (redshifts, QPOs, Eddington-limited fluxes from accreting sources, moments of inertia) could yield mass/radius measurements in near future.
- Neutron star cooling is rich source of information about interior composition, hints that rapid cooling necessary in a few cases.