Analysis of Hidden Stochastic Properties in Images or Curves by Fuzzy Measures and Functions and their Fusion by Fuzzy- or Choquet Integrals

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Abstract

The paper shows the separation of stochastic properties in images or curves by analysis with different methods. A fuzzy measure and a fuzzy function represent these stochastic properties. Related to their importance the such represented stochastic properties are fused by the fuzzy integral. The fuzzy integral can be used as a new fuzzy measure; an iteration is generated until the interesting information is separated. If during the procedure of isolating a selected property the result violates with the original aim, then the search procedure has to be interrupted and another aim has to be traced. This is organised with the help of the Choquet integral. An estimated threshold selects the different stochastic parts.

Keywords: Stochastic properties, Fuzzy integrals, Choquet integrals, Curve analysis, Hidden structures, Fusion of stochastic properties

1. INTRODUCTION

The analysis and the fusion of stochastic properties show a great similarity in the mathematical methods for images and curves. Consequently, both are considered in their generality. The normal basis of many image processing algorithms is the detection and acquisition of information in the form of well-defined features such as edges, contours, structures and skeletons [1, 2, 3]. In many real-world images however, such information is not readily available. The areas in the images have to be distinguished by their stochastic. A similar condition exists for some relationships given as curves [4]. This is the case for curves of signals corrupted by coloured noise, superposed stochastic signals, disturbed time series or noise corrupted stereo lines. For the analysis of such images and curves the stochastic properties are very important and have to be decomposed. Often, in such cases, different stochastic relationships can be identified to describe properties by using fuzzy methods. In this paper methods are given to separate stochastic properties in images and curves. The methods can be applied for time and local dependencies, however the specialities of every case have to be implemented accordingly.

Normally, the stochastic information is given continuously in a time or local interval. This is the foundation for the application of most mathematical methods to analyse the stochastic effects by a system of stochastic differential equations [5]. The measurement however produces only sampled values and furthermore the values are limited in their amplitude and they cannot change suddenly by physical effects. Therefore it is difficult to detect the stochastic in such data. For small regions the central limit theorem (law of large numbers) and the general ergodic theorem are violated under this circumstances. The data have to be analysed for lager areas. The areas are normally so large that we can expect that many kinds of stochastic are superposed in the areas. This means that we have to search for every stochastic component over a large or even the entire region and we have to try to expose different properties by different analysing methods of stochastic. Thus, we cannot fully decompose the superposition. The importance of different stochastic properties obtained by different methods is not the same. This difference of importance related to the applied method gives after their fusion a better decomposition.

Another problem is the representation of different stochastic properties by the same mathematical description. This is a necessary condition for a fusion. The fuzzy measure and the fuzzy function give the possibility to describe different types of stochastic properties in the same manner. We have also the problem, that we do not know all the properties in this system and so we cannot normalise the contributions to a property. This problem is also solved by the fuzzy measure by loss of additivity. This representation gives also the possibility for a fusion in form of the fuzzy integral or more general by a Choquet integral. The isolation of the hidden stochastic properties can be used for the separation by stochastic or for the elimination of stochastic effects.

2. APPLIED METHODS FOR THE SELECTION OF STOCHASTIC PROPERTIES

The application of different methods gives the possibility to find results where the stochastic properties are represented by various degrees of importance. Because the time domain is more familiar to us, we will use the equation in the local dependence as used for images. The stochastic effects are represented by a system of coupled stochastic non-linear differential equations of the form:

$$dy_1(x) = \mathbf{F_1}(y_1, y_2, \cdots, y_n)\,dx + G_1\,n(x)$$

$$dy_2(x) = \mathbf{F_2}(y_1, y_2, \cdots, y_n)\,dx + G_2\,n(x)$$

$$\cdots\cdots\cdot$$

$$dy_m(x) = \mathbf{F_m}(y_1, y_2, \cdots, y_n)\,dx + G_m\,n(x)$$

Here, $y_1, y_2, \cdots, y_m$ are the m components of the stochastic and $\mathbf{F}$ are the m non-linear matrices for the relationships between the components. The m coefficients G are the gain factors for the noise n and x represents the pixel-point or the region in the image. Besides this system for the data exists a stochastic equation for the acquisition of the data. In vector representation the system of stochastic differential equations can be written in a stochastic integral representation of digitised form by:

$$\mathbf{y}(x_{i+1}) - \mathbf{y}(x_i) = \left[\mathbf{F}(\mathbf{y}(x_i)) + \int_{x_i}^{x_{i+1}} G_i(x)\,dW(x)\right](x_{i+1} - x_i)$$

The information within the pixel $\mathbf{y}(x_{i+1})$ is obtained by the observation process $z(x_k)$. The digitised observation value is given by:

$$z(x_{k+1}) = \int_{x_k}^{x_{k+1}} \mathbf{G}(x)\,\mathbf{y}(x)\,dx + \int_{x_k}^{x_{k+1}} C(x)\,dV(x)$$

W(x) and V(x) are independent Wiener processes. The non-linearities are determined by a priori knowledge. For the solution of the system of non-linear stochastic differential equations the martingale representation [5, 6, 7] is applied. By this method the effect of approximation is clearly to understand because the calculation remains in the same space as in the model given. This is not the case for the most other methods that use the Fokker–Planck equation. In [5] it has been derived, that for a square integrable martingale the process $p_X(z,y)$ can be written in an integral representation of the form

$$p_x(z,y) = p_0 + \int_0^x P_s\,ds + n_x$$

where $N = (n_x, \Re_x)$ is a martingale and P_s is a stochastic process. Because the stochastic differential equations depend only on z, we obtain for the expectation value of $p_x(z,y)$

$$\langle p_x(z,y) \rangle = \mathbf{E}\left[\, p_x(z_x) \,\middle|\, \Re_x^y \,\right].$$

By this method the effect of approximation is clearly to understand because for the calculation the same space is used as for the model.

On the other hand, stochastic properties are eliminated by non-linear transformations such as Walsh transformations, wavelet-transformations and Fourier decompositions. By the choice of the parameters certain characteristics are more important than others. The effect of the changing parameters on the representation of stochastic properties is relevant and will be analysed in advance. The results are used as a priori information. These methods are applied if an a priori knowledge exists of the possible stochastic in the data. If a priori knowledge does not exists then non-parametric methods can find such relationships and also isolate stochastic properties. Methods applied for this are the rank ordering, rank filtering and calculation of distributions of signs. For this the original value is subtracted by a stochastic mean. The values are adapted by the estimation of the best thresholds for the isolation of stochastic properties. These such obtained stochastic properties have to be represented in a special manner to combine many of these for a better estimation.

3. REPRESENTATION OF STOCHASTIC PROPERTIES BY FUZZY MEASURE AND FUZZY FUNCTION

For the fusion of different kinds of stochastic a representation of stochastic properties in the same expression is necessary. The different types of stochastic are collected in a set of properties, where the members of this set are fuzzy in their contribution for the determination of a selected area. The fuzzy measure gives the possibility to describe different types of stochastic properties in the same expression. This is achieved by mapping the properties of different kinds on the closed interval [0,1], which is the area of the fuzzy measure. For an optimised decision all relevant stochastic properties have to be fused. For this fusion stochastic properties are not only described by a fuzzy measure but also by a fuzzy function. If the fuzzy property is more related to a region, then a fuzzy measure is used; if a stochastic property is better described by a particular distribution of the grey values, then this is represented by a fuzzy function.

For a better decision-making the combination of the selected properties related to their importance is used. Considering the importance of a property implicates the loss of additivity. The normalisation as required by the

probability does not exist anymore. By the fuzzy measure the properties described by different kinds of relationships are mapped into the closed interval [0,1]. The fuzzy measure, first defined by Sugeno [8], has, besides the additive terms as a probability measure, a term with the combination of all elementary fuzzy measures multiplied by a factor λ. Fuzzy measures are also described, where only the monotonicity is used [9]. The factor λ has an effect similar to a weight factor for the interaction between the properties. If λ = 0 then the fuzzy measure is equal to the probability measure.
The coupling of the elementary fuzzy measures (densities) $g_1(x_1)$ over the elementary region x_1 with another elementary fuzzy measure $g(x_2)$ over the other elementary region x_2 is defined by [8]:

$$g_l(x_1 \cup x_2) = g_l(x_1) + g_l(x_2) + l\, g_l(x_1) g_l(x_2)$$

where $l = (1 + l\, g(x_1))(1 + l\, g(x_2)) - 1$ is a coupling constant used as a substitution for the loss of additivity. For a set of elements $A = \{x_i\}$ the relationship above can be used recursively and gives:

$$g(A) = \sum_{i=1}^{n} g(x_i) + l \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} g(x_i) g(x_j) + \cdots + l^{n-1} g(x_1) \cdots g(x_n)$$

This can be written as a product

$$g(X) = \frac{1}{l}\left[\prod_{x_j \in A} (1 + l\, g(x_i) - 1)\right] \qquad where \quad l \neq 0$$

The coupling parameter λ will be obtained by solving the equation

$$1 + l = \left[\prod_{x_j \in X} \left(1 + l g(x_i)\right)\right]$$

This shows the mathematical concept for calculating the measure for the coupled elementary properties for small areas. If the non-linear equation for λ is too difficult or very time-consuming to solve, then a good approximation is an iterated coupling with two properties, where one of them is the result before. Properties especially capable for the representation by a fuzzy measure are:

- logical functions of some bitmaps of data within an interval,
- estimated values of non-linear filtering higher/lower as a threshold
- regions given by the retransform of a wavelet representation with adapted coefficients
- maxima/minima regions of a curve

For the fusion the stochastic properties are also represented by fuzzy functions. The fuzzy functions are mostly a collection of values over a number of single pixel points. The values of the neighbour pixels are of stochastic nature and often not directly correlated with this value. Normally, these fuzzy functions are described by a characterisation over a threshold. Outside of such a characteristic threshold the values depend very weakly on the real value. Inside the interval the values generate fuzzy properties for the adapted condition. For the fuzzy function properties are used such as:

- differences of data values in different distances
- the weak change of the data values in related areas
- the difference of the stochastic in different directions
- the number of ranks related to different distances
- the stochastic values obtained by the subtraction of the original and estimated value
- the values obtained by wavelet transformation of a selected parameter

These fuzzy functions are also normalised and mapped on an interval given by a boundary. Whereas the fuzzy measure is better adapted for effects represented in special regions, the fuzzy function characterise the stochastic change over a region of a fuzzy measure. The values are combined in the similar manner as with the fuzzy measure. For example, the image values can be divided in bitmaps. The highest and the two lowest bitmaps can be combined logically in a non-addditive case with help of a given function. A similar case is the combination of histogram values over very small regions obtained with different intervals and combined non-additively by some parameters. More demonstratively spoken, a more functional property is represented by a fuzzy function $h(x_{k,l})$ over a region of a fuzzy measure $g(x_{k,l})$.

4. FUSION OF STOCHASTIC PROPERTIES BY A FUZZY INTEGRAL

The values over the possible region of a stochastic, represented by a fuzzy measure $g(x_{k,l})$, are connected with the values $h(x_{k,l})$ over the pixels representing the strength of a stochastic property. The value $h(x_{k,l})$ is described by a fuzzy function where the values are normalised to 1. The functional relationship between the fuzzy measure and the fuzzy function is represented by the fuzzy integral. For the fuzzy integral the old definition of the fuzzy integral of Sugeno [8] will be used, because it is well adapted to the problem of detection of stochastic properties. With Sugeno [8] the fuzzy measure is combined with the fuzzy function in the form (written as a stylised f)

$$f_A h_a(x) \oplus dg = \sup_{a \in [0,1]} \left\{ \min \left[a, g\left(A \cap H_a \right) \right] \right\},$$

with $H_a = \left\{ x \mid h_a \geq a \right\}$

Here h_a is the cut of h at the constant α. For $h_a(x)$ the values at the pixel-points are used, representing a stochastic property. α is the threshold where the assumption is fulfilled, that the property is used in the minimal condition. The region A is given as the data region where surely a specific stochastic is expected. It may be also the whole image for the pixel region and the whole possible range for a fuzzy function.

The important property of a fuzzy measure is that its value is mapped on the closed interval [0,1]. This is given by the calculated value of the fuzzy integral. This gives the possibility to use the result of a fuzzy integral as a new fuzzy measure g_2

$$g_{2_{i,j}}(A) = f_A h_a(x) \oplus dg_1$$

This newly produced fuzzy measure is linked with the region obtained by another stochastic property so that f_2 obtained

$$f_2 = f_A h'_{a'}(x) \oplus dg_2$$

In the following step the next region of another property is combined with this fuzzy function f_2. In such a way a set of fuzzy functions $\{ f_1, f_2, \cdots, f_n \}$ is obtained by fuzzy measures $\{ g_1, g_2, \cdots, g_n \}$. The summation of all combinations of fuzzy measures with fuzzy functions makes sure that all possible properties in all combinations, which should be considered, are used. In such a way an image is obtained, where the (grey) values represent a measure for the membership to the stochastic. This is a normal number, not a fuzzy number. With the help of a threshold we decide, which pixel of the data belongs to the existence of a selected stochastic.

5. CHOQUET INTEGRAL FOR THE REVISION OF DECISION MAKING DURING THE PROCESS

The fuzzy function $h_a(x)$ can be represented by the quantity of the similarity between an assumed stochastic part in the image and the real stochastic part in the image or curve. The fuzzy measure can be described as the area where this stochastic is expected. For the fusion of information by the fuzzy integral it is assumed that the fuzzy measure is monotone. If the properties are contradictory and it is not clear at the beginning which kind of information is more important, then the result obtained from the first part of the image or curve can be changed to another one in the second part of the image or curve. In this case the incompatibility is greater as the synergy between the fuzzy sets B and C so that the monotonicity is violated. With the help of the Choquet integral [10] a measure theory for non-monotonic fuzzy measures can be constructed. The Choquet integral can be applied for the detection of stochastic properties [11] if the monotonicity for the detection of a stochastic property is violated.

With respect to the non-monotonic fuzzy measures a functional of bounded variation can be represented as a Choquet integral. Thus the non-monotonic fuzzy measure χ for the properties s_1 and s_2, given by $c(s_1 \cup s_2) \leq c(s_1)$, can be combined with a fuzzy function of properties by the Choquet integral. The Choquet integral, noted by $(C)\int$, is defined by:

$$(C)\int w(A \cap H_a)\, dc$$

with $H_a = \left\{ x \mid h_a(x) \geq a \right\}$

For three ordered functions $f(s_1) \leq f(s_2) \leq f(s_3)$ coupled by non-monotonic measures a sum can be written as:

$$f(s_1)\, c\left(\{s_1, s_2, s_3\}\right) + \left[f(s_2) - f(s_1) \right] c\left(s_2, s_3\right) + \left[f(s_3) - f(s_2) \right] c\left(s_3\right)$$

Here the contribution of $f(s_1)$ in the first term can be less then the contribution in the second term. Such a summation can be used for the representation of a Choquet integral in discrete form:

$$D_m = (C)\int f\, dc = \sum_{i=1}^{m} \left(f(s_i) - f(s_{i-1}) \right) c(s_i, s_{i+1}, \cdots, s_m)$$

With the estimation value y_m of the pixel x_m a series of Choquet integrals D_m can be built. With Choquet integrals, ordered as a rank with the threshold K, the stochastic properties can be summed in the form

$$D_m = D_{m-1} + \{u(y_m - K) - u(y_{m-1} - K)\} * c\{u(y_m - K), |y_m - y_{m-1}|, |y_m - y_{m-2}|\}$$

with $D_0 = 0$

The constant K is important for characterising the properties. K can be obtained by analysing the non-linear filtered curves of the histograms of special parts of the image.

For the combination of different properties, which may be also violated in parts, the Choquet integral is used. For processing the curve the Choquet integral can be written in discrete form by

$$D_m = \sum_{i=1}^{m} \left(s(x_i) - s(x_{i-1})\right) c(S_i) \quad \text{with}$$

$$S_i = \bigcup_{k=i}^{n} s(x_k) \ ,$$

where $s(x_i)$ are the values of the properties of the pixel points x_i. To understand this principle let us consider an example analysing a curve with a disturbed maximum. For the fuzzy measure the maxima may be more important than the gradient, so it can be assumed

$$c_{\max} = 0.55 \quad c_{Min} = 0.45 \quad \text{and} \quad c_{grad} = 0.3 \ .$$

The existence of a maximum is a condition for the existence of a minimum and so the relation between both is given as

$$\chi_{\max,\min} = 0.5 \quad < \chi_{\max} + \chi_{\min} = 0.95 \ .$$

The existence of gradient and maximum or minimum is a good feature and gives the condition

$$\chi_{\max,\text{ grad.}} = \chi_{\min,\text{ grad.}} = 0.9 \ > \ \chi_{\max} + \chi_{\text{grad.}} = 0.75$$

The fuzzy function of the presence of a maximum is given by $f_{\max}$, for the minimum by $f_{\min}$ and by f_{grad} for the gradient. Three parts of the curve may be used to find the correspondence with another curve. These three parts are defined by the above values and for the Choquet integrals they are obtained as:

curve	$f_{\max}$	$f_{\min}$	f_{grad}	*value of weighting*	*Choquet integral*
A	0.9	0.8	0.5	0.76	2.21
B	0.5	0.6	0.9	0.64	1.95
C	0.7	0.7	0.75	0.75	2.57

It can be decided by the maximum of the Choquet integral, which points are to be used as corresponding points. The Choquet integral suggests the right decision C, whereas the weighting suggests the part A.

6. Conclusions and Applications

The method gives the possibility to separate various types of stochastic by a representation of the stochastic properties by their importance applying different statistical methods and estimation procedures. By a representation of the stochastic properties in form of the fuzzy measure and a fuzzy function the properties can be fused by the fuzzy integral. This algorithm is working decision-directed. If the aim is changing and the a priori assumption is wrong then the algorithm can be changed automatically by the application of the Choquet integral for the process of combination of various kinds of properties.

The method to characterise areas by their stochastic properties is applied to images of different kinds. The stochastic structures are detected in images with treetops, surfaces with corrosion, material surfaces with texture structures and medical tissues of pathological structures such as cancer. The application of stochastic methods for the separation of combined stochastic properties does not give enough information to separate the textures. If the isolation of the relevant information by iterative application of the fuzzy integral is applied then the information is better. The optimum is obtained if the stochastic information is fused by the fuzzy integral, whereas the cutting threshold is also to be estimated. It is good to see that decisions improve with the combination of more information. Nevertheless, a priori knowledge is very important to get good results, because this method works decision-directed and if the direction of the decision is not clear enough, then the obtained solutions are not very precise. If nearly nothing is known a priori, then the non-parametric method is used to find the hidden stochastic in the image. In this way the rank ordering is an effective method for finding contributions for the fuzzy algorithm.

Examples of disturbances by sensors are SAR- (synthetic aperture radar) or SONAR images with speckle generated by stochastic back scattering. Here, the stochastic is very dominant and this method is applied on this.

An example of analysing curves disturbed by many stochastic influences is the retrieval of profiles obtained from molecular spectra. Weak signals give a big problem, because only the pressure broadening of the line can be used to estimate the profile and the line shape is to be estimated from disturbed spectra. The baseline is not known and the neighbouring lines superpose the line of interest. For the analysis many sources of the incoming noise and the non-linear linkage by the receiver in the GHz range are used. By application of the methods described above, the profiles of very weak signals could be retrieved [12]. Another example for a successfully application is the analysis of lines of a stereo image with disturbances and obstacles hidden in one of the stereo image [13]. The main problem is to find the corresponding points even if the noise is big and the stereo points are detected automatically. By estimating the shape of the curves and analysing the neighbourhood of an event and the combination of all noisy effects the right corresponding points can be found.

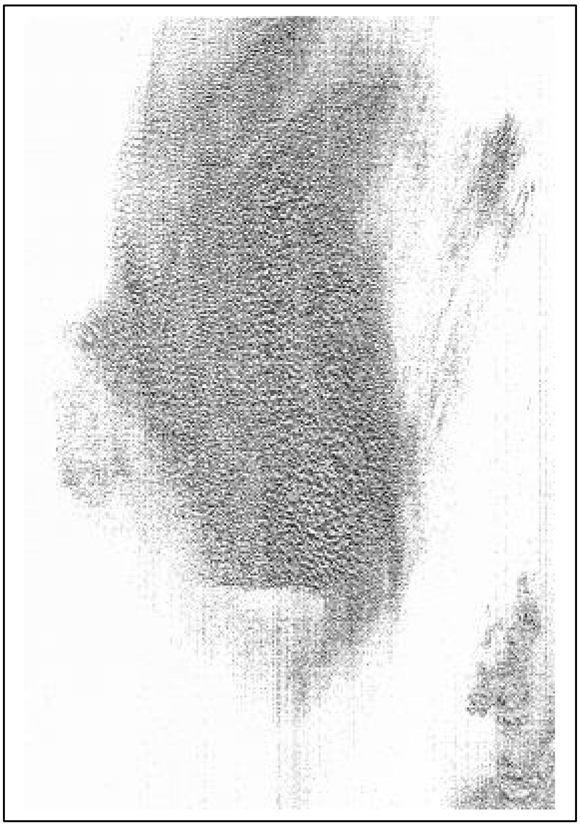

Fig .1 Original image of superposed cirrus clouds

For the detection of clouds the stochastic is very important. The cirrus clouds are mostly determined by their stochastic. Similar problems occur with contrails, especially if they are aged. The aged contrails have a similar brightness as the cirrus clouds and can be distinguished by the difference in their stochastic.

Fig. 1 shows an image of cirrus clouds on top of each other. By the analysis of the stochastic by fuzzy measures and fusion with the fuzzy integral the hidden stochastic structures can be separated. The top cirrus cloud is given in Fig. 2 whereas the upper is represented in Fig. 3. In Fig. 2 the extension of the stochastic, represented in waves, is to be seen.

Fig. 4 shows the separation of hidden contributions of a spectra for the very weak signal of ClO in the sub-millimetre range. The received spectrum with noise on it is the dotted line. The thick line in Fig. 4 is the non-linearly filtered spectrum. By elimination of parts with different kinds of stochastic and subtraction of the original spectra, a separation of the different contributions is achieved. Fig. 5 shows the adaptation of the noise curve by an assumed spectrum.

Fig. 2 Lower cirrus cloud separated by their stochastic

Fig . 3 Upper cirrus cloud separated by their stochastic

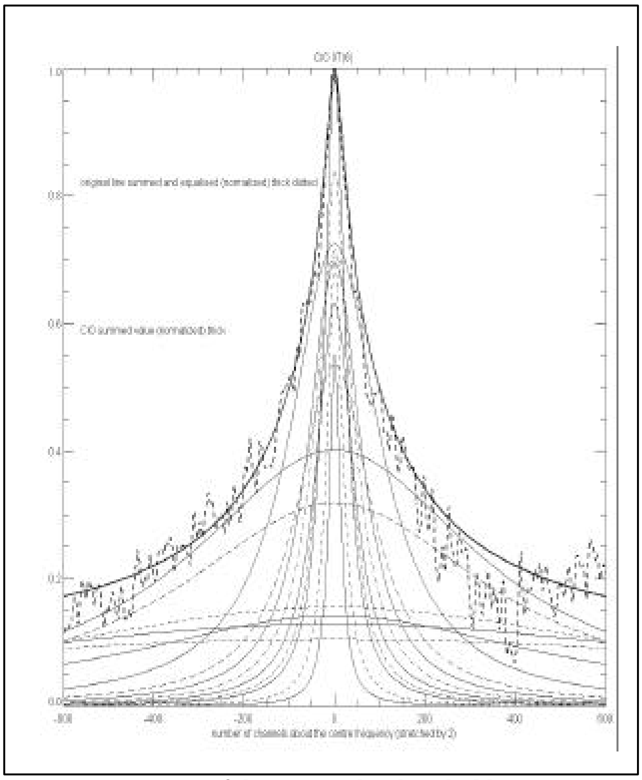

Fig. 4 Separation of the spectra by their hidden parts

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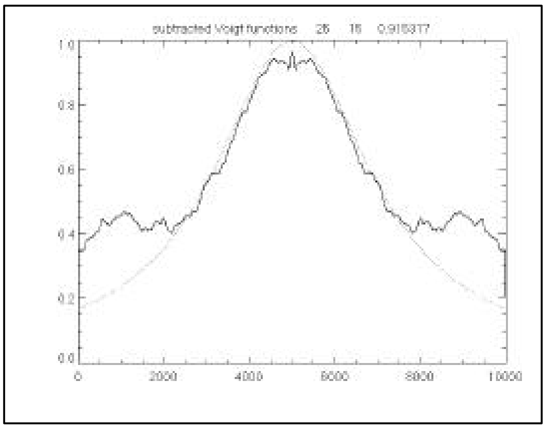

Fig. 5 Example of extracting a hidden spectrum

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