Extreme Magnetic Field Generation
- large-scale fields in the universe

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Maxwell’s equations

covariant form

$$F^{\alpha\beta}{}_{;\alpha} = \frac{4\pi}{c} j^{\beta}$$

$$F_{\alpha\beta,\gamma} + F_{\gamma\alpha,\beta} + F_{\beta\gamma,\alpha} = F_{\alpha\beta;\gamma} + F_{\gamma\alpha;\beta} + F_{\beta\gamma;\alpha} = 0$$

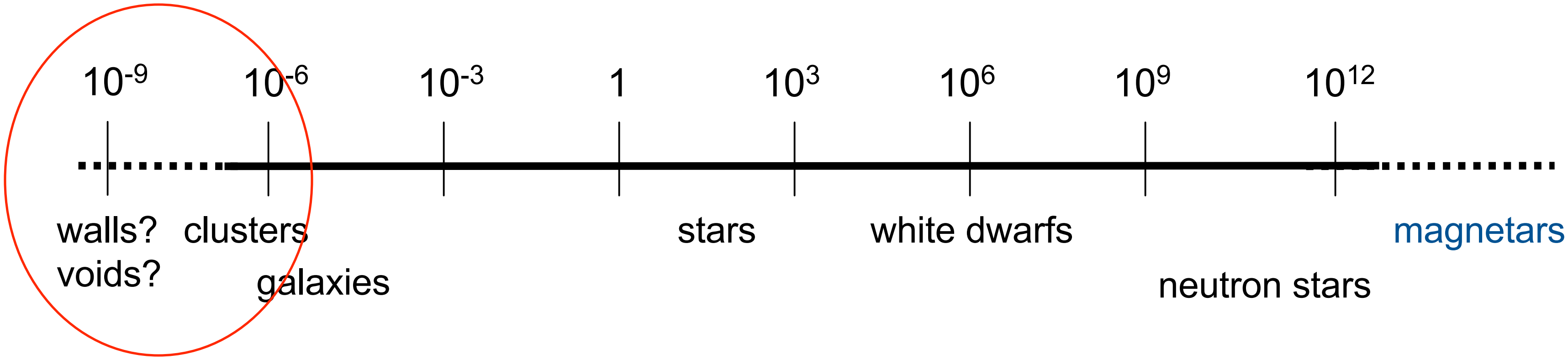
in flat space time

$$\begin{aligned} \nabla \cdot \mathbf{D} &= 4\pi\rho \\ \nabla \times \mathbf{E} &= -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{H} &= -\frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} + \frac{4\pi}{c}\mathbf{j} \end{aligned}$$

Lorentz force equation: $\mathbf{F} = q\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right)$

Astrophysical magnetic fields

magnetic field (gauss)



	walls? voids?	clusters	galaxies	stars	white dwarfs	neutron stars	magnetars
linear size (cm)	$>10^{26}$	10^{25}	10^{23}	10^{10}	10^{9}	10^{6}	10^{6}
mass (solar mass)		10^{13}	10^{11}	~1	1	1	1

Cosmological-scale fields vs stellar fields

“magnetic flux” $\Phi \sim BR^2$

- magnetars	10^{28} Gcm^2
- neutron stars	10^{21} - 10^{25} Gcm^2
- white dwarfs	10^{21} - 10^{25} Gcm^2
- solar-like stars	10^{21} - 10^{23} Gcm^2
- galaxies	~10^{41} Gcm^2
- galaxy clusters	~10^{43} Gcm^2
- superclusters and beyond	???

toplogical properties

boundary conditions

Cosmological magnetic field (I)

- light elements in metal poor halo stars
  ^{6}Li abundance indicating that the galaxy was bombarded by cosmic ray before it was formed (Rollinde et al. 2005)

- pulsar rotational measurement and timing study indicating a dipolar field in the Milky Way (Han et al. 2006)

- Faraday rotation in damped Lyman-α forest systems at $z \sim 2$, suggesting large-scale magnetic field $B \sim 10^{-6}$ G (Blasi, Burles & Olinto 1999)

- in quasars, where MRI operates in accretion disks and jets, at $z > 5$

Cosmological magnetic field (II)

Cosmological magneto-genesis

- fundamental issues

  standard plasma physics or exotic physics ?

  bottom-up process or top-down process ?

Angular momentum regulation

- proto-star formation in the early universe
- fueling and growing supermassive black holes (quasars at high z …)

Cosmic rays at various z

- acceleration
- propagation
- confinement (?)

Magneto-genesis: Biermann battery (I)

Faraday's Law

$$\frac{\partial \mathbf{B}}{\partial t} = -c\,(\nabla \times \mathbf{E})$$

Ohm's Law (omitting the current for the time being)

$$\mathbf{E} = -\left(\frac{\mathbf{v}}{c} \times \mathbf{B}\right) + \frac{1}{n_\mathrm{e} e}\nabla p_\mathrm{e}$$

The Biermann battery equation

$$\begin{aligned}\frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{c}{n_\mathrm{e}^2 e}(\nabla p_\mathrm{e} \times \nabla n_\mathrm{e}) \\ &= \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{1}{1+\chi}\frac{m_\mathrm{p} c}{\rho^2 e}(\nabla p \times \nabla \rho)\end{aligned}$$

Magneto-genesis: Biermann battery (II)

in co-moving frame

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{1}{a}\nabla \times (\mathbf{v} \times \mathbf{B}) - 2\frac{\dot{a}}{a}\mathbf{B} + \frac{1}{1+\chi}\frac{m_{\mathrm{p}}c}{eB_0 t_0}\frac{(\nabla p \times \nabla \rho)}{\rho^2}$$

barotropic fluid $p = p(\rho)$

- isobaric surface coincide with isopynic surface

$$\nabla p \times \nabla \rho = 0$$

"slow" moving $v\tau \ll L \Rightarrow$ "$\mathbf{v} \to 0$"

flux conservation

$$\frac{\partial \mathbf{B}}{\partial t} \sim -2\frac{\dot{a}}{a}\mathbf{B} \;\Rightarrow\; \Phi = Ba^2 \sim \mathcal{C}$$

Magneto-genesis: bottom up scenario (I)

(1) Biermann battery operations in small scales

- stars, disks, accretion inflows

(2) Dynamo growth

- scaled by rotation rates

- in favour of “compact” objects which are fast rotators

(3) Field expelled from stellar objects

- via supernovae, jets, outflows

(4) Diffusion and connection/combination of small-scale fields

- building up large-scale fields

Magneto-genesis: bottom up scenario (II)

field generation in galaxies via SNR

- suppose in a plerion SNR

$$B \sim 10^{-3}\ \mathrm{G}$$
$$R \sim 1\ \mathrm{pc}$$

- SNR overlapping when $R \sim 60\ \mathrm{pc}$
- reaching pressure equilibrium at $R \sim 15\ \mathrm{pc}$
- diffusion coefficient (via ambipolar process) $D \sim 10^2\ \mathrm{pc\ km\ s}^{-1}$

timescale to fill a galaxy with SNR generated field $\tau \sim 10^{14}\ \mathrm{s} \approx 10^7\ \mathrm{yr}$

expected Strength of the random field $\sqrt{< B^2 >} \sim 3 \times 10^{-7}\ \mathrm{G}$

(Zweibel 2006)

Magneto-genesis: bottom up scenario (III)

the difficulty for field spreading beyond galaxies

- typical size of a rich galaxy cluster $R \sim 1 \text{ Mpc}$
- diffusion coefficient $D \sim 10^6 \text{ pc km s}^{-1}$

timescale for a galactic field to spread over within a galaxy clusters

$$\tau \sim 3 \times 10^{19} \text{ s} \approx 10^{12} \text{ yr} \gg \frac{2}{3H_0}$$

How about magnetic fields in large structures beyond galaxy clusters ???

Magneto-genesis: top down scenario (I)

behind the photon last scattering surface

- the media are opaque to “light”
- photons, electrons and protons are strongly coupled

Thomson cross sections of charged particles

$$\sigma_{\mathrm{t}} = \left(\frac{e^2}{mc^2}\right)^2$$

$$m_{\mathrm{e}} \ll m_{\mathrm{p}} \Rightarrow \sigma_{\mathrm{t}}^{(\mathrm{e})} \gg \sigma_{\mathrm{t}}^{(\mathrm{p})}$$

near the last scattering surface

- photons preferentially scatter off electrons rather than protons
- velocity difference between charged particles, i.e. electric currents
- anisotropy in the photon radiation field leading to “micro-rotation”

Magneto-genesis: top down scenario (II)

- Faraday induction equation
- Ohm's law
- Euler equation
- metric for space-time structure

magnetic field evolution equation (Ichiki et al. 2007)

$$\begin{aligned}\frac{d}{dt}(a^2 B^i) &= \frac{4a}{3}\frac{\sigma_{\rm th}}{e}\rho^{(0)}\epsilon^{ijk} \\ &\times\left[\left(\frac{\rho^{(1)}_{\gamma,k}}{\rho^{(0)}} - 2\phi^{(1)}_{,j}\right)\left(v^{(1)}_{{\rm e}k} - v^{(1)}_{\gamma k}\right) - 2{h^{(1)l}}_{j,k}\left(v^{(1)}_{{\rm e}l} - v^{(1)}_{\gamma l}\right) - \frac{1}{4}\left(v^{(1)}_{{\rm e}l}{\Pi^{(1)}}^{l}{}_{\gamma,j}\right)_{,k} - \left(v^{(2)}_{{\rm e}j,k} - v^{(2)}_{\gamma j,k}\right)\right]\end{aligned}$$

scalar perturbation

tensor perturbation

vorticity

photon density contrast

baryon-photon slip

stress anisotropy

Magneto-genesis: top down scenario (III)

for scale invariant density perturbation

$$S(k) \sim k^{-7/3} C_1 + k^5 C_2$$

slip term   stress anisotropy term

$$B \propto \sqrt{k^3 S(k)} \quad \propto$$

$k^{7/2}$	> 100 Mpc
$k^{1/3}$	~ 10 - 100 Mpc
k	~ 10^{-3} pc - 10 Mpc
~ 0	< 10^{-3} pc

expected field strength $B \sim 10^{-21} - 10^{-16}\ \mathrm{G}$

which is larger than $B_{\mathrm{seed}} \sim 10^{-30} - 10^{-20}\ \mathrm{G}$, the required seed field for dynamo amplification to micro-gauss field in IGM and ICM

Neutrinos

standard model for particle physics

- lepton family

ν_e	ν_μ	ν_τ
e	μ	τ

\+ their anti-particles

- neutrinos have non-zero mass
- neutrino oscillations/mixing during propagation
- neutrinos interact with their leptons via weak interaction

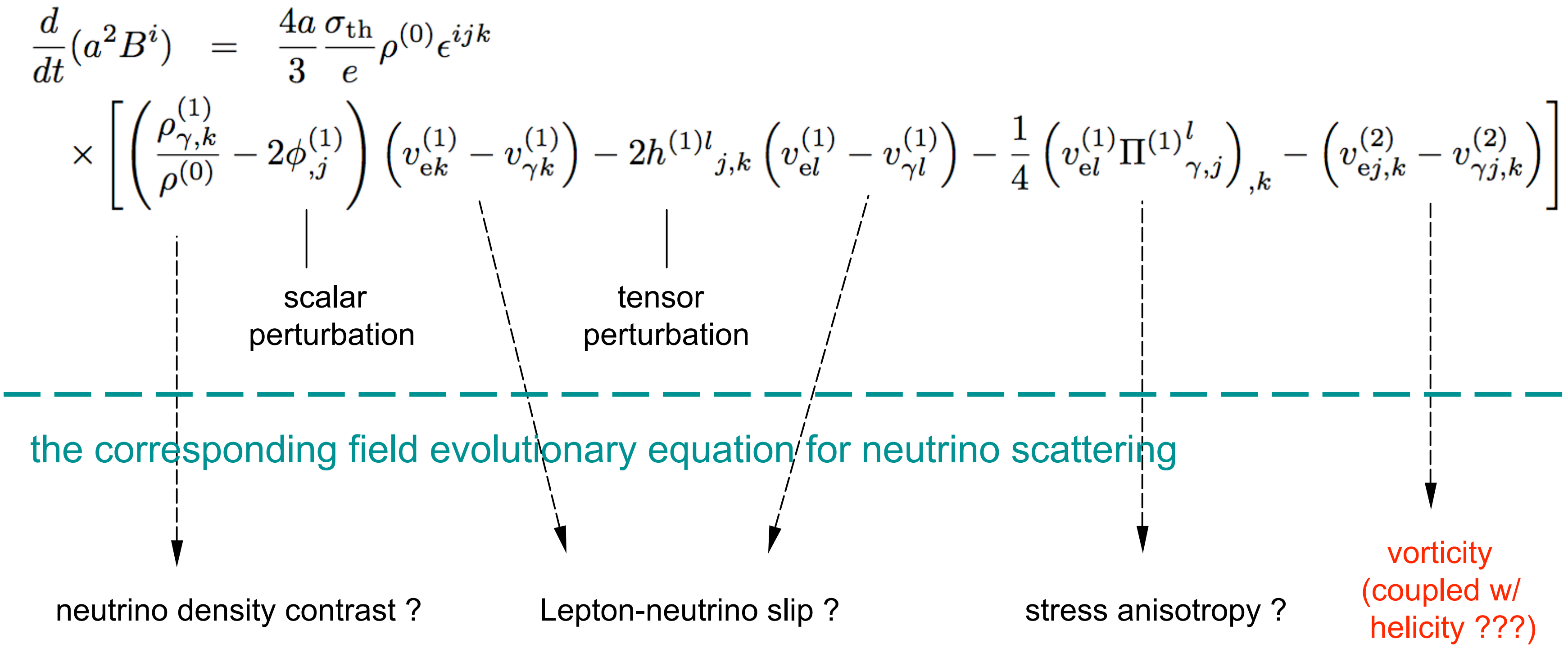
$$k^\alpha \frac{\partial \mathcal{I}^i(k^\beta)}{\partial x^\alpha} + \Gamma^\alpha_{\beta\gamma} k^\beta k^\gamma \frac{\partial \mathcal{I}(k^\beta)}{\partial k}$$

$$= -u^\delta k^\delta \left[\sum_j \chi_0^{ij}(x^\beta, k^\beta) \mathcal{I}^i(k^\beta) + \eta_0^i(x^\beta, k^\beta) + \int d^4k' \sum_j \sigma_0^{ij}(x^\beta, k^\beta, k'^\beta) \mathcal{I}^j(k'^\beta) \right]$$

destruction/conversion creation scattering

Cosmological fields (I)

- neutrino last scattering surfaces (CNB!)
- 3 scattering surfaces instead of 1 only surface
- charge particle slip
- in dense early universe



$$\frac{d}{dt}(a^2B^i) = \frac{4a}{3}\frac{\sigma_{\rm th}}{e}\rho^{(0)}\epsilon^{ijk}$$

$$\times\left[\left(\frac{\rho^{(1)}_{\gamma,k}}{\rho^{(0)}} - 2\phi^{(1)}_{,j}\right)\left(v^{(1)}_{\mathrm{e}k}-v^{(1)}_{\gamma k}\right) - 2{h^{(1)l}}_{j,k}\left(v^{(1)}_{\mathrm{e}l}-v^{(1)}_{\gamma l}\right) - \frac{1}{4}\left(v^{(1)}_{\mathrm{e}l}{\Pi^{(1)l}}_{\gamma,j}\right)_{,k} - \left(v^{(2)}_{\mathrm{e}j,k}-v^{(2)}_{\gamma j,k}\right)\right]$$

scalar perturbation

tensor perturbation

the corresponding field evolutionary equation for neutrino scattering

neutrino density contrast ?

Lepton-neutrino slip ?

stress anisotropy ?

vorticity (coupled w/ helicity ???)

Cosmological fields (II)

- primordial neutrino generated fields are not necessarily weak

- diluted as the universe expanded

- magnetic fields are topological
  - not so easy to be destroyed
  - could retain coherent signatures when emerged from behind the last neutrino and photon scattering surfaces
  - largest scale cosmic magnetic field ???

More thoughts later ……