Swfit UV-OT

MULLARD SPACE SCIENCE LABORATORY
UNIVERSITY COLLEGE LONDON
Author: H. Kawakami

Detector System Design III

<< Point source count rate correction by 2-dim approach >>

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Hajime Kawakami

1. Introduction

An imaging photon counting detector with an image intensifier read by a CCD camera achieves high resolution centroiding with a low gain of the intensifier. This provides better lifetime performance of the intensifier. It is, however, not good at observing bright point source image, i.e. a star, due to its slow frame rate (~100 frames/sec) compared with position sensitive detectors with anode readout. To overcome this problem, photo-event loss due to coincidence has to be precisely calculated so that the true incoming rate can be estimated. The target of the brightness is pretty ambitious, 1-2 times of frame rate (i.e. 100-200 counts/sec).

The algorithm of photo-event detection in hardware prohibits picking up more than one event out of 2x2 CCD pixels (140mm x 140mm) at any part of the CCD image sensor. If PSF of optics is far better than 70mm and a star is located at the middle of 2x2 CCD pixel array, all photons fall into the 2x2 CCD. In this case, count rate correction for the star is rather easy, since it treatment is same as a 0-dimension detector.

The wing part of the PSF profile fits better to Lorentzian than Gaussian. Lorentzian profile does not damp as steep as Gaussian does. If a star is located at the centre of a CCD pixel unluckily, the shortest distance to the outside the 2x2 CCD array is 35mm. This suggests significant portion of star light falls outside the 2x2 CCD array. The sampling by 3x3 CCD array offers better photometric accuracy. The shortest distance to the edge of the 3x3 CCD array is 105mm, if the star location is at the centre of a CCD pixel. Even if the star position is at the edge of a CCD pixel, the shortest distance is still 70mm.

The algorithm of photo-event detection, however, does not guarantee single count per frame out of the 3x3 CCD array. It may count 2, and occasionally 3 or even 4 with special events distribution within the array. All possible configurations were taken into account and the expected count rate was formulated, provided 2-dim PSF is known. The equation is not as simple as that of 0-dim detector, but it is possible to calculate a true event rate with a help of a computer.

2. Mathematical Model
Section 2A. 0-dimension approach

We consider the case, in which a star image is sharply focused and all photo-events fall into a single CCD pixel. We assume that "n" events arrived during "N" CCD frames for a long time period of T.

```
                                    ooooooooooooooooooooooooo
                                            n events

<---------------------------   N Frames   ----------------------------->

|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
                                      |
                                      |
                                      |
                        event ? in this Frame
```

Focusing on a particular CCD frame, we consider whether it captures events (doesn't matter single, double, triple,...., events) or not.
The probability of all n-evens falling outside the frame, i.e. falling in the remaining "N-1" frames, is

$$P0 = \left(\frac{N-1}{N} \right)^n$$

This gives the probability that the specific frame does not receive any event.
Probability of at least one of the n evens falling in the specific frame is,

$$P1 = 1 - \left(\frac{N-1}{N} \right)^n = 1 - \left(1 - \frac{1}{N} \right)^{(pN)} \qquad (A.1)$$

$$\text{where,} \quad p = \left(\frac{n}{N}\right) \qquad \text{<--- definition, finite value} \qquad (A.2)$$

We assume integration period T is sufficiently long, hence N and n are large number. In this condition, the above equation is simplified

$$P1 \Longrightarrow 1 - \exp(-p), \quad \text{when } N \longrightarrow \text{infinity} \qquad (A.3)$$

"P1" gives the probability that the specific frame receives events. Now looking at whole "N" frames, "P1" can be expressed by number ratio of event arriving frames to all CCD frames,

$$P1 = \frac{N\_det}{N} = 1 - \exp(-p) \qquad (A.4)$$

i.e.

$$p = \frac{n}{N} = -\ln\left(1 - \frac{N\_det}{N} \right) \qquad (A.5)$$

Imagining a very long integration time, for example T=1E8 sec, numbers used above can be converted to rates.

```
N      Þ FR     x 1E8 sec
N_det  Þ c_det  x 1E8 sec
n      Þ c      x (1.0 - dead time fraction) x 1E8sec          (A.6)

where,     FR: frame rate
        c_det: detected event rate
            c: incoming event rate
```

The dead time correction is expressed by

```
( 1.0 - FT x FR )                                               (A.7)
where,     FT: frame transfer period.
```

Converting the parameters to rates using (A.6) and (A.7), Equation (A.5) becomes

```
                 c_det
c = - ln ( 1 - -----) / (1/FR-FT)                               (A.8).
                  FR
```

Section 2B. 1-dimension approach

We consider the case, in which star image is spread along 3 CCD pixels due to telescope and image tube optics. We assume that "n1" events arrived at Pix_A during "N" frames, "n2" events at Pix_B, and "n3" events at Pix_C.

```
        ooooooo                                n1 events
        ooooooooooooooooooooooooo              n2 events
        oooo                                   n3 events

        <--------------------   N Frames   ---------------------------->

Pix_A  |---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
Pix_B  |---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
Pix_C  |---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
                                    |
                                    |
                     event ? at this Frame in the 3 pixels
```

Focusing on a particular frame, we consider whether the individual 3 CCD pixels capture events (doesn't matter single, double, triple,.... events) or not. There are 8 combinations (= 2^3) in terms of photo-event's arrival ("o") or not ("x") as tabulated in Table 1. The 4th column shows the number of events counted by the photon counting system. Since event detection algorithm of the system does not separate 2 photo-events when falling into 2 adjacent CCD pixels, the system counts as only "one". The extreme example is the 8th raw, in which 3 photo-evens fall into all of the 3 CCD pixels but counted as "one". The 6th row is rare example, in which the 2 photo-events are counted as "two" successfully.

Table 1. Photo-events configuration and counts by system

Pix_A	Pix_B	Pix_C	count
x	x	x	0
o	x	x	1
x	o	x	1
o	o	x	1
x	x	o	1
o	x	o	2
x	o	o	1
o	o	o	1

Note) o: Pixel has events x: Pixel has not event

Probability of no event falling in the 3 CCD pixels in the specific frame (i.e. 1st raw in Table 1) is,

$$P0 = \left(\frac{N-1}{N}\right)^{n1} \left(\frac{N-1}{N}\right)^{n2} \left(\frac{N-1}{N}\right)^{n3}$$

$$= \left(\frac{N-1}{N}\right)^{(n1+n2+n3)}$$

Probability of at least one of the evens falling in the 3 CCD pixels, hence the system counts more than "one" (i.e. 2nd-8th raw in Table 1) is,

$$P1 = \left\{ 1 - \left(\frac{N-1}{N}\right)^{(n1+n2+n3)} \right\} \qquad (B.1)$$

Probability of the 6th raw, when the system counts "two" is,

$$P2 = \left\{ 1 - \left(\frac{N-1}{N}\right)^{n1} \right\} \left(\frac{N-1}{N}\right)^{n2} \left\{ 1 - \left(\frac{N-1}{N}\right)^{n3} \right\} \qquad (B.2)$$

"P1+P2" gives expected number of counts in the specific frame. Now looking at whole "N" frames, "P1+P2" can estimate detected counts over the "N" frames.

$$\frac{N\_det}{N} = P1 + P2$$

$$= 1 + \left(\frac{N-1}{N}\right)^{n2} \left\{ 1 - \left(\frac{N-1}{N}\right)^{n1} - \left(\frac{N-1}{N}\right)^{n3} \right\}$$

$$\Longrightarrow 1 + \exp(-p2) \{ 1 - \exp(-p1) - \exp(-p3) \} \qquad (B.3)$$

where, $p1 = \frac{n1}{N}$, $p2 = \frac{n2}{N}$, $p3 = \frac{n3}{N}$

Or,

$$\frac{N\_det}{N} \Longrightarrow 1 + \exp(-p2) \{ 1 - \exp(-a1\ p2) - \exp(-a3\ p2) \} \qquad (B.4)$$

where, $a1 = \frac{n1}{n2}$, $a3 = \frac{n3}{n2}$ and $p2 = \frac{n2}{N}$

Unlike Eq (A.4)and Eq(A.5), Eq (B.4) has no explicit inverse equation, which can calculate real number of incoming events directly from observed events. But, it is possible to determine "p2" (= n2/N) by Newton-Lapson method with a PC, provided that "a1" and "a3" are known parameters from PSF of optics and the star position within the CCD Pix_B.

Section 2C.1. 2-dimension approach 3x3 CCD sampling

Finally, equation applicable to the practical situation will be provided here.
We consider the case, in which star image is spread among 3x3 CCD pixel array due to telescope and image tube optics.
We assume that "n11" events arrived at Pix_AA during "N" frames,
"n12" events at Pix_AB, "n13" events at Pix_AC,
"n21" events at Pix_BA, "n22" events at Pix_BB, "n23" events at Pix_BC,
"n31" events at Pix_CA, "n32" events at Pix_CB, "n33" events at Pix_CC.

Table 2. Naming of 3x3 CCC pixels and photon spread

Pix_AA n11	Pix_AB n12	Pix_AC n13
Pix_BA n21	Pix_BB n22	Pix_BC n23
Pix_CA n31	Pix_CB n32	Pix_CC n33

Focusing on a particular frame, we consider whether the individual 9 CCD pixels receive events (doesn't matter single, double, triple,.... events) or not. There are 512 combinations (= 2^9) in terms of photo-event's arrival or not.

Among the all combinations, probability producing more than "one" counts is derived by the same manner as section A.
Probability of no event falling in the 3x3 CCD array in the specific frame is,

$$P0 = \left(\frac{N-1}{N} \right) \hat{}\,(n11+n12+n13+n21+n22+n23+n31+n32+n33)$$

Probability of at least one of the evens falling in the 3x3 CCD pixel, hence the system counts more than "one" is,

$$P1 = \{ 1 - \left(\frac{N-1}{N} \right)^{(n11+n12+n13+n21+n22+n23+n31+n32+n33)} \} \qquad (C.1)$$

$$\Longrightarrow \{ 1 - \exp([-a11-a12-a13-a21-A22-a23-a31-a32-a33]\ p22) \qquad (C.1c)$$

Among the above, system counts "two", "three" and even "four" in special configurations. The 2-dim configurations which produce more than "two" counts, more than "three" counts and more than "four" counts are listed in Table 3.

Table 3. Photo-events configuration and counts by system

----

count >= 2

```
O X o        nc O nc       X X O
 X x X        x x x         X x X
 o X o        o o o         o X o
  (5)          (1)           (6)


nc X o                      o X nc
O x  o        None          X x O
nc x o                      o x nc
  (2)                        (3)

X X X          o X o
X x X         X x X          None
O X o         nc O nc
  (7)           (4)


               nc O nc      nc O nc
               O x x         x x O
               nc x O        O x nc
                (1a)          (1b)

nc X O                       O X nc
 O x x                        X x O
 nc O nc                      nc O nc
  (2a)                         (3a)
```

----

count >= 3

```
O  X O      nc O  nc     X  X  O
X   x X      x  x  x      X  x X
 o X o      O  x O      O  X  O
  (5)         (1)          (6)


nc  X  O                  O  X  nc
O   x  x      None        X  x  O
nc  x  O                  O  x  nc
  (2)                        (3)


             O  X  O
             X  x  X       None
             nc  O  nc
               (4)


O  X  X
X  x  X
O  X  O
  (5a)
```

----

count = 4

```
O  x  O
 x  x  x
O  x  O
```

Note on symbols;
 x: Pixel has not event to avoid bridging other 2 events
X: Pixel has not event to avoid overlap of combination
O: Pixel has events
 o: At least one of pixels marked "o" has events
nc: do not care whether events arrived or not.

??

Explanation of table

count >= 2

(1) picks up all combinations, when Pix_AB received photo-events. The system can separate two events if one of Pix_CA, Pix_CB or Pix_CC received photo-events. Pix_BA, Pix_BB, Pix_BC must be blank to avoid bridge. Pix_AA and Pix_AC do not affect the counts. There are missing configurations in (1). Pix_BA does not bridge Pix_AB and Pix_CC in(1a). Pix_BC does not bridge Pix_AB and Pix_CA in (1b).

(2) picks up all combinations, when Pix_BA received photo-events. All configurations involving Pix_AB were already taken into account in (1), therefore Pix_AB must be blank. The system can separate two events if one of Pix_AC, Pix_BC or Pix_CC received photo-events. Pix_AB, Pix_BB, Pix_CB must be blank to avoid bridge. Pix_AA and Pix_CA do not affect the counts. There is a missing configuration in (2). Pix_CB does not bridge Pix_BA and Pix_AC in(2a).

(3) picks up all combinations, when Pix_BC received photo-events. All configurations involving Pix_AB or Pix_BA were already taken into account in (1) and (2), therefore Pix_AB and Pix_BA must be blank. The system can separate two events if one of Pix_AA, Pix_CA received photo-events. Pix_BB, Pix_CB must be blank to avoid bridge. Pix_AC and Pix_CC do not affect the counts. There is a missing configuration in (3). Pix_CB does not bridge Pix_BC and Pix_AA in (3a).

(4) picks up all combinations, when Pix_CB received photo-events. All configurations involving Pix_AB, Pix_BA or Pix_BC were already taken into account in (1), (2) and (3), therefore Pix_AB, Pix_BA and Pix_BC must be blank. The system can separate two events if one of Pix_AA, Pix_AC received photo-events. Pix_BB must be blank to avoid bridge. Pix_CA and Pix_CC do not affect the counts.

(5) picks up all combinations, when Pix_AA received photo-events. All configurations involving Pix_AB, Pix_BA, Pix_BC or Pix_CB were already taken into account in (1), (2), (3) and (4), therefore Pix_AB, Pix_BA, Pix_BC and Pix_CB must be blank. The system can separate two events if one of Pix_AC, Pix_CA or Pix_CC received photo-events. Pix_BB must be blank to avoid bridge.

(6) picks up all combinations, when Pix_AC received photo-events. All configurations involving Pix_AB, Pix_BA, Pix_BC, Pix_CB or Pix_AA were already taken into account in (1), (2), (3), (4) and (5), therefore Pix_AB, Pix_BA, Pix_BC, Pix_CB and Pix_AA must be blank. The system can separate two events if one of Pix_CA, Pix_CC received photo-events. Pix_BB must be blank to avoid bridge.

(7) picks up all combinations, when Pix_CA received photo-events. All configurations involving Pix_AB, Pix_BA, Pix_BC, Pix_CB, Pix_AA or Pix_AC were already taken into account in (1), (2), (3), (4), (5) and (6), therefore Pix_AB, Pix_BA, Pix_BC, Pix_CB, Pix_AA and Pix_AC must be blank. The system can separate two events if Pix_CC received photo-events. Pix_BB must be blank to avoid bridge.

count >= 3
(1) picks up all combinations, when Pix_AB received photo-events. The system can separate the three events if Pix_CA, Pix_CC received photo-events. Pix_BA, Pix_BB, Pix_BC, Pix_CB must be blank to avoid bridge. Pix_AA and Pix_AC do not affect the counts.

(2) picks up all combinations, when Pix_BA received photo-events. All configurations involving Pix_AB were already taken into account in (1), therefore Pix_AB must be blank. The system can separate the tree events if Pix_AC, Pix_CC received photo-events. Pix_BB, Pix_CB, Pix_BC must be blank to avoid bridge. Pix_AA and Pix_CA do not affect the counts.

(3) picks up all combinations, when Pix_BC received photo-events. All configurations involving Pix_AB or Pix_BA were already taken into account in (1) and (2), therefore Pix_AB and Pix_BA must be blank. The system can separate the three events if Pix_AA, Pix_CA received photo-events. Pix_BB, Pix_CB must be blank to avoid bridge. Pix_AC and Pix_CC do not affect the counts.

(4) picks up all combinations, when Pix_CB received photo-events. All configurations involving Pix_AB, Pix_BA or Pix_BC were already taken into account in (1), (2) and (3), therefore Pix_AB, Pix_BA and Pix_BC must be blank. The system can separate the three events if Pix_AA, Pix_AC received photo-events. Pix_BB must be blank to avoid bridge. Pix_CA and Pix_CC do not affect the counts.

(5) picks up all combinations, when Pix_AA received photo-events. All configurations involving Pix_AB, Pix_BA, Pix_BC or Pix_CB were already taken into account in (1), (2), (3) and (4), therefore Pix_AB, Pix_BA, Pix_BC and Pix_CB must be blank. The system can separate the three events if Pix_AC and one of Pix_CA, Pix_CC received photo-events. Pix_BB must be blank to avoid bridge. There is a missing configuration in (5). Pix_AC can be blank, when if Pix_CA and Pix_CC received photo-events in(5a).

(6) picks up all combinations, when Pix_AC received photo-events. All configurations involving Pix_AB, Pix_BA, Pix_BC, Pix_CB or Pix_AA were already taken into account in (1), (2), (3), (4) and (5), therefore Pix_AB, Pix_BA, Pix_BC, Pix_CB and Pix_AA must be blank. The system can separate the three events if Pix_CA, Pix_CC received photo-events. Pix_BB must be blank to avoid bridge.

count = 4
The system can separate the four events Pix_AB, Pix_AC, Pix_CA and Pix_CC received photo-events. Pix_BA, Pix_BB, Pix_BC, Pix_AB, Pix_CB must be blank to avoid bridge.
??

The fist 3x3 and the following 4 combinations in the above diagram show all possible cases resulting in detected count of more than "2" out of the 3x3 CCD array. These contain the combinations to be counted as "3" and "4".
The next 3x3 combinations show all possible combinations resulting in detected count of more than "3". These contain the combination to be counted as "4".
The final case shows the combination counted as "4".

Probability of the 1st 3x3 combinations and the following 4 combinations, hence the system counts more than "two" is,

$$P2 = \left\{ 1 - \left(\frac{N-1}{N} \right)^{n12} \right.$$
$$\times \left. \left(\frac{N-1}{N} \right)^{n21} \right\} \left(\frac{N-1}{N} \right)^{n22} \left(\frac{N-1}{N} \right)^{n23}$$
$$\times \left\{ 1 - \left(\frac{N-1}{N} \right)^{(n31+n32+n33)} \right\}$$

$$+ \left(1- \left(\frac{N-1}{N} \right)^{n21} \right\}$$
$$\times \left(\frac{N-1}{N} \right)^{n12} \left(\frac{N-1}{N} \right)^{n22} \left(\frac{N-1}{N} \right)^{n32}$$
$$\times \left\{ 1 - \left(\frac{N-1}{N} \right)^{(n13+n23+n33)} \right\}$$

$$+ \left(1- \left(\frac{N-1}{N} \right)^{n23} \right\}$$
$$\times \left(\frac{N-1}{N} \right)^{n12} \left(\frac{N-1}{N} \right)^{n22} \left(\frac{N-1}{N} \right)^{n32} \left(\frac{N-1}{N} \right)^{n21}$$
$$\times \left\{ 1 - \left(\frac{N-1}{N} \right)^{(n11+n31)} \right\}$$

$$+ \left(1- \left(\frac{N-1}{N} \right)^{n32} \right\}$$
$$\times \left(\frac{N-1}{N} \right)^{n21} \left(\frac{N-1}{N} \right)^{n22} \left(\frac{N-1}{N} \right)^{n23} \left(\frac{N-1}{N} \right)^{n12}$$
$$\times \left\{ 1 - \left(\frac{N-1}{N} \right)^{(n11+n13)} \right\}$$

$$+ \left(1- \left(\frac{N-1}{N} \right)^{n11} \right\}$$
$$\times \left(\frac{N-1}{N} \right)^{n21} \left(\frac{N-1}{N} \right)^{n22} \left(\frac{N-1}{N} \right)^{n23} \left(\frac{N-1}{N} \right)^{n12} \left(\frac{N-1}{N} \right)^{n32}$$
$$\times \left\{ 1 - \left(\frac{N-1}{N} \right)^{(n13+n31+n33)} \right\}$$

$$+ \left(1- \left(\frac{N-1}{N} \right)^{n13} \right\}$$
$$\times \left(\frac{N-1}{N} \right)^{n11} \left(\frac{N-1}{N} \right)^{n12} \left(\frac{N-1}{N} \right)^{n21} \left(\frac{N-1}{N} \right)^{n22} \left(\frac{N-1}{N} \right)^{n23} \left(\frac{N-1}{N} \right)^{n32}$$
$$\times \left\{ 1 - \left(\frac{N-1}{N} \right)^{(n31+n33)} \right\}$$

$$+ \left(1- \left(\frac{N-1}{N} \right)^{n31} \right\} \left(1- \left(\frac{N-1}{N} \right)^{n33} \right\}$$
$$\times \left(\frac{N-1}{N} \right)^{n11} \left(\frac{N-1}{N} \right)^{n12} \left(\frac{N-1}{N} \right)^{n13} \left(\frac{N-1}{N} \right)^{n21} \left(\frac{N-1}{N} \right)^{n22} \left(\frac{N-1}{N} \right)^{n23}$$

$$\times\left(\frac{N-1}{N}\right)^{n32}$$

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$$+\left\{1-\left(\frac{N-1}{N}\right)^{n12}\right\}$$

$$\times\left(\frac{N-1}{N}\right)^{n22}\left(\frac{N-1}{N}\right)^{n23}\left(\frac{N-1}{N}\right)^{n32}$$

$$\times\left\{1-\left(\frac{N-1}{N}\right)^{n21}\right\}\left\{1-\left(\frac{N-1}{N}\right)^{n33}\right\}$$

$$+\left\{1-\left(\frac{N-1}{N}\right)^{n12}\right\}$$

$$\times\left(\frac{N-1}{N}\right)^{n21}\left(\frac{N-1}{N}\right)^{n22}\left(\frac{N-1}{N}\right)^{n32}$$

$$\times\left\{1-\left(\frac{N-1}{N}\right)^{n23}\right\}\left\{1-\left(\frac{N-1}{N}\right)^{n31}\right\}$$

$$+\left\{1-\left(\frac{N-1}{N}\right)^{n21}\right\}$$

$$\times\left(\frac{N-1}{N}\right)^{n12}\left(\frac{N-1}{N}\right)^{n22}\left(\frac{N-1}{N}\right)^{n23}$$

$$\times\left\{1-\left(\frac{N-1}{N}\right)^{n13}\right\}\left\{1-\left(\frac{N-1}{N}\right)^{n32}\right\}$$

$$+\left\{1-\left(\frac{N-1}{N}\right)^{n23}\right\}$$

$$\times\left(\frac{N-1}{N}\right)^{n12}\left(\frac{N-1}{N}\right)^{n21}\left(\frac{N-1}{N}\right)^{n22}$$

$$\times\left\{1-\left(\frac{N-1}{N}\right)^{n11}\right\}\left\{1-\left(\frac{N-1}{N}\right)^{n32}\right\}$$

$$=\left\{1-\left(\frac{N-1}{N}\right)^{n12}\right\}\left(\frac{N-1}{N}\right)^{(n21+n22+n23)}$$

$$\times\left\{1-\left(\frac{N-1}{N}\right)^{(n31+n32+n33)}\right\}$$

$$+\left\{1-\left(\frac{N-1}{N}\right)^{n21}\right\}\left(\frac{N-1}{N}\right)^{(n12+n22+n32)}$$

$$\times\left\{1-\left(\frac{N-1}{N}\right)^{(n13+n23+n33)}\right\}$$

$$+ \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{23}} \right\} \left(\frac{N-1}{N} \right)^{(n_{12}+n_{22}+n_{32}+n_{21})} \times \left\{ 1 - \left(\frac{N-1}{N} \right)^{(n_{11}+n_{31})} \right\}$$

$$+ \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{32}} \right\} \left(\frac{N-1}{N} \right)^{(n_{21}+n_{22}+n_{23}+n_{12})} \times \left\{ 1 - \left(\frac{N-1}{N} \right)^{(n_{11}+n_{13})} \right\}$$

$$+ \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{11}} \right\} \left(\frac{N-1}{N} \right)^{(n_{21}+n_{22}+n_{23}+n_{12}+n_{32})} \times \left\{ 1 - \left(\frac{N-1}{N} \right)^{(n_{13}+n_{31}+n_{33})} \right\}$$

$$+ \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{13}} \right\} \left(\frac{N-1}{N} \right)^{(n_{11}+n_{12}+n_{21}+n_{22}+n_{23}+n_{32})} \times \left\{ 1 - \left(\frac{N-1}{N} \right)^{(n_{31}+n_{33})} \right\}$$

$$+ \left(1 - \left(\frac{N-1}{N} \right)^{n_{31}} \right\} \left(1 - \left(\frac{N-1}{N} \right)^{n_{33}} \right\} \times \left(\frac{N-1}{N} \right)^{(n_{11}+n_{12}+n_{13}+n_{21}+n_{22}+n_{23}+n_{32})}$$

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$$+ \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{12}} \right\} \left(\frac{N-1}{N} \right)^{(n_{22}+n_{23}+n_{32})} \times \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{21}} \right\} \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{33}} \right\}$$

$$+ \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{12}} \right\} \left(\frac{N-1}{N} \right)^{(n_{21}+n_{22}+n_{32})} \times \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{23}} \right\} \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{31}} \right\}$$

$$+ \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{21}} \right\} \left(\frac{N-1}{N} \right)^{(n_{12}+n_{22}+n_{23})} \times \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{13}} \right\} \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{32}} \right\}$$

$$+ \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{23}} \right\} \left(\frac{N-1}{N} \right)^{(n_{12}+n_{21}+n_{22})} \times \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{11}} \right\} \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{32}} \right\}$$

Assuming N and n are large number,
P2 ====>

```
+ { 1 - exp(-p12) } exp(-p21-p22-p23) { 1 - exp(-p31-p32-p33) }

+ { 1 - exp(-p21) } exp(-p12-p22-p32) { 1 - exp(-p13-p23-p33) }

+ { 1 - exp(-p23) } exp(-p12-p22-p32-p21) { 1 - exp(-p11-p31) }

+ { 1 - exp(-p32) } exp(-p21-p22-p23-p12) { 1 - exp(-p11-p13) }

+ { 1 - exp(-p11) } exp(-p21-p22-p23-p12-p32) { 1 - exp(-p13-p31-p33) }

+ { 1 - exp(-p13) } exp(-p11-p12-p21-p22-p23-p32) { 1 - exp(-p31-p33) }

+ { 1 - exp(-p31) } exp(-p11-p12-p13-p21-p22-p23-p32) { 1 - exp(-p33) }
Misc-4
+ { 1 - exp(-p12) } { 1 - exp(-p21) } ( 1 - exp(-p33) ) exp(-p22-p23-p32)

+ { 1 - exp(-p12) } { 1 - exp(-p23) } { 1 - exp(-p31) } exp{-p22-p21-p32)

+ { 1 - exp(-p21) } { 1 - exp(-p13) } { 1 - exp(-p32) } exp{-p12-p22-p23)

+ { 1 - exp(-p23) } { 1 - exp(-p11) } { 1 - exp(-p32) } exp{-p12-p22-p21)

                                                                  (C.2a)
```

where, $p11 = \frac{n11}{N}$, $p12 = \frac{n12}{N}$, $p13 = \frac{n13}{N}$,

$p21 = \frac{n21}{N}$, $p22 = \frac{n22}{N}$, $p23 = \frac{n23}{N}$,

$p31 = \frac{n31}{N}$, $p32 = \frac{n32}{N}$, $p33 = \frac{n33}{N}$.

Or,

P2 ===>

```
  {1- exp(-a12 p22)} exp([-a12-A22-a23] p22) {1 - exp( [-a31-a32-a33] p22)}

+ {1- exp(-a12 p22)} exp([-a12-a22-a32] p22) {1 - exp( [-a13-a23-a33] p22)}

+ {1- exp(-a23 p22)} exp([-a12-A22-a32-a21] p22) {1 - exp( [-a11-a31] p22)}

+ {1- exp(-a12 p22)} exp([-a21-A22-a23-a12] p22) {1 - exp( [-a11-a13] p22)}

+ {1- exp(-a11 p22)} exp([-a21-A22-a23-a12-a32] p22)
x {1- exp([-a13-a31-a33] p22)}

+ {1 - exp(-a13 p22)} exp([-a11-a12-a21-A22-a23-a32] p22)
x {1- exp([-a31-a33] p22)}

+ {1 - exp(-a31 p22)} exp([-a11-a12-a13-a21-A22-a23-a32] p22)
x {1 - exp(-a33 p22)}

Misc-4
+ { 1 - exp(-a12 p22) } { 1 - exp(-a21 p22) } ( 1 - exp(-a33 P22) )
x exp( (-A22-a23-a32) p22 )
```

```
+ { 1 - exp(-a12 p22) } { 1 - exp(-a23 p22) } { 1 - exp(-a31 P22) }
x exp{ (-A22-a21-a32) p22 }

+ { 1 - exp(-a21 p22) } { 1 - exp(-a13 p22) } { 1 - exp(-a32 P22) }
x exp{ (-a12-A22-a23) p22 }

+ { 1 - exp(-a11 p22) } { 1 - exp(-a23 p22) } { 1 - exp(-a32 P22) }
x exp{ (-a12-A22-a21) p22 }

                                                                  (C.2b)
                n11          n12          n13
where,   a11= ---,     a12=---,     a13=---,
                n22          n22          n22

                n21                       n23
         a21= ---,                  a23=---,
                n22                       n22

                n31          n32          n33
         a31= ---,     a32=---,     a33=---
                n22          n22          n22     .
```

Probability of the next 3x3 combinations, hence the system counts more than "three" is,

```
             N-1
P3 = { 1 - ( --- )^n12 }
              N
          N-1             N-1          N-1          N-1
      x ( --- )^n21 }   ( --- )^n22  ( --- )^n23  ( --- )^n32
           N               N            N            N
                N-1                 N-1
      x { 1 - ( --- )^n31 }   { 1 - ( --- )^n33 }
                 N                   N

             N-1
   + { 1 - ( --- )^n21 }
              N
          N-1             N-1          N-1          N-1
      x ( --- )^n12 }   ( --- )^n22  ( --- )^n32  ( --- )^n23
           N               N            N            N
                N-1                 N-1
      x { 1 - ( --- )^n13 }   { 1 - ( --- )^n33 }
                 N                   N

             N-1
   + { 1 - ( --- )^n23 }
              N
          N-1             N-1          N-1          N-1
      x ( --- )^n21 }   ( --- )^n22  ( --- )^n23  ( --- )^n21
           N               N            N            N
                N-1                 N-1
      x { 1 - ( --- )^n11 }   { 1 - ( --- )^n31 }
                 N                   N

             N-1
   + { 1 - ( --- )^n32 }
              N
          N-1             N-1          N-1          N-1
      x ( --- )^n21 }   ( --- )^n22  ( --- )^n23  ( --- )^n12
           N               N            N            N
                N-1                 N-1
```

$$\times \left\{ 1 - \left(\frac{}{N} \right)^{n_{11}} \right\} \left\{ 1 - \left(\frac{}{N} \right)^{n_{13}} \right\}$$

$$+ \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{11}} \right\} \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{13}} \right\}$$

$$\times \left(\frac{N-1}{N} \right)^{n_{21}} \left(\frac{N-1}{N} \right)^{n_{22}} \left(\frac{N-1}{N} \right)^{n_{23}} \left(\frac{N-1}{N} \right)^{n_{12}} \left(\frac{N-1}{N} \right)^{n_{32}}$$

$$\times \left\{ 1 - \left(\frac{N-1}{N} \right)^{(n_{31}+n_{33})} \right\}$$

$$+ \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{13}} \right\} \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{31}} \right\} \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{33}} \right\}$$

$$\times \left(\frac{N-1}{N} \right)^{n_{11}} \left(\frac{N-1}{N} \right)^{n_{12}} \left(\frac{N-1}{N} \right)^{n_{21}} \left(\frac{N-1}{N} \right)^{n_{22}} \left(\frac{N-1}{N} \right)^{n_{23}} \left(\frac{N-1}{N} \right)^{n_{32}}$$

$$= \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{12}} \right\} \left(\frac{N-1}{N} \right)^{(n_{21}+n_{22}+n_{23}+n_{32})}$$

$$\times \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{31}} \right\} \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{33}} \right\}$$

$$+ \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{21}} \right\} \left(\frac{N-1}{N} \right)^{(n_{12}+n_{22}+n_{32}+n_{23})}$$

$$\times \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{13}} \right\} \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{33}} \right\}$$

$$+ \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{23}} \right\} \left(\frac{N-1}{N} \right)^{(n_{21}+n_{22}+n_{23}+n_{21})}$$

$$\times \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{11}} \right\} \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{31}} \right\}$$

$$+ \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{32}} \right\} \left(\frac{N-1}{N} \right)^{(n_{21}+n_{22}+n_{23}+n_{12})}$$

$$\times \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{11}} \right\} \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{13}} \right\}$$

$$+ \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{11}} \right\} \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{13}} \right\} \left(\frac{N-1}{N} \right)^{(n_{21}+n_{22}+n_{23}+n_{12}+n_{32})}$$

$$\times \left\{ 1 - \left(\frac{N-1}{N} \right)^{(n_{31}+n_{33})} \right\}$$

$$+ \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{13}} \right\} \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{31}} \right\} \left\{ 1 - \left(\frac{N-1}{N} \right)^{n_{33}} \right\}$$

$$\times \left(\frac{N-1}{N} \right)^{(n_{11}+n_{12}+n_{21}+n_{22}+n_{23}+n_{32})} \tag{C.3}$$

Assuming N and n are large number,

P3 ====>

$$
\begin{aligned}
&\{1-\exp(-p_{12})\}\ \exp(-p_{21}-p_{22}-p_{23}-p_{32})\ \{1-\exp(-p_{31})\}\ \{1-\exp(-p_{33})\}\\
+&\{1-\exp(-p_{21})\}\ \exp(-p_{12}-p_{22}-p_{32}-p_{23})\ \{1-\exp(-p_{13})\}\ \{1-\exp(-p_{33})\}\\
+&\{1-\exp(-p_{23})\}\ \exp(-p_{21}-p_{22}-p_{23}-p_{21})\ \{1-\exp(-p_{11})\}\ \{1-\exp(-p_{31})\}\\
+&\{1-\exp(-p_{32})\}\ \exp(-p_{21}-p_{22}-p_{23}-p_{12})\ \{1-\exp(-p_{11})\}\ \{1-\exp(-p_{13})\}\\
+&\{1-\exp(-p_{11})\}\ \{1-\exp(-p_{13})\}\ \exp(-p_{21}-p_{22}-p_{23}-p_{12}-p_{32})\ \{1-\exp(-p_{31}-p_{33})\}\\
+&\{1-\exp(-p_{13})\}\ \{1-\exp(-p_{31})\}\ \{1-\exp(-p_{33})\}\ \exp(-p_{11}-p_{12}-p_{21}-p_{22}-p_{23}-p_{32})
\end{aligned}
\tag{C.3a}
$$

Or,

P3 ====>

$$
\begin{aligned}
&\{1-\exp(-a_{12}\,p_{22})\}\ \exp([-a_{21}-A_{22}-a_{23}-a_{32}]\,p_{22})\ \{1-\exp(-a_{31}\,p_{22})\}\\
\times&\{1-\exp(-a_{33}\,p_{22})\}\ \{1-\exp([-a_{31}-a_{32}-a_{33}]\,p_{22})\}\\
+&\{1-\exp(-a_{21}\,p_{22})\}\ \exp([-a_{12}-A_{22}-a_{32}-a_{23}]\,p_{22})\ \{1-\exp(-a_{13}\,p_{22})\}\\
\times&\{1-\exp(-a_{33}\,p_{22})\}\\
+&\{1-\exp(-a_{23}\,p_{22})\}\ \exp([-a_{21}-A_{22}-a_{23}-a_{21}]\,p_{22})\ \{1-\exp(-a_{11}\,p_{22})\}\\
\times&\{1-\exp(-a_{31}\,p_{22})\}\\
+&\{1-\exp(-a_{32}\,p_{22})\}\ \exp([-a_{21}-A_{22}-a_{23}-a_{22}]\,p_{22})\ \{1-\exp(-a_{11}\,p_{22})\}\\
\times&\{1-\exp(-a_{13}\,p_{22})\}\\
+&\{1-\exp(-a_{11}\,p_{22})\}\ \{1-\exp(-a_{13}\,p_{22})\}\ \exp([-a_{21}-A_{22}-a_{23}-a_{12}-a_{32}]\,p_{22})\\
\times&\{1-\exp([-a_{31}-a_{33}]\,p_{22})\}\\
+&\{1-\exp(-a_{13}\,p_{22})\}\ \{1-\exp(-a_{31}\,p_{22})\}\ \{1-\exp(-a_{33}\,p_{22})\}\\
\times&\exp([-a_{11}-a_{12}-a_{21}-A_{22}-a_{23}-a_{32}]\,p_{22})
\end{aligned}
\tag{C.3b}
$$

Probability of the final case, i.e. the system counts "four" is,

$$
\begin{aligned}
P4 = &\left\{1-\left(\frac{N-1}{N}\right)^{n11}\right\}\left(\frac{N-1}{N}\right)^{n12}\left\{1-\left(\frac{N-1}{N}\right)^{n13}\right\}\\
&\times\left(\frac{N-1}{N}\right)^{n21}\Big\}\left(\frac{N-1}{N}\right)^{n22}\left(\frac{N-1}{N}\right)^{n23}\\
&\times\left\{1-\left(\frac{N-1}{N}\right)^{n31}\right\}\left(\frac{N-1}{N}\right)^{n32}\left\{1-\left(\frac{N-1}{N}\right)^{n33}\right\}\\
= &\left\{1-\left(\frac{N-1}{N}\right)^{n11}\right\}\left\{1-\left(\frac{N-1}{N}\right)^{n13}\right\}
\end{aligned}
$$

```
              N-1                        N-1
   x { 1 - ( --- )^n31 }    { 1 - ( --- )^n33 }
              N                          N
        N-1
   x ( --- ) ^(n12+n21+n22+n23+n32)
        N                                                         (C.4)
```

Assuming N and n are large number,

P4 ====>

```
  { 1 - exp(-p11) } { 1 - exp(-p13) } { 1 - exp(-p31) } { 1 - exp(-p33) }
  x exp(-p12-p21-p22-p23-p32)                                    (C.4a)
```

Or,

P4 ====>

```
  { 1 - exp(-a11 p22) } { 1 - exp(-a13 p22) }
x { 1 - exp(-a31 p22) } { 1 - exp(-a33 p22) }
x exp( [-a12-a21-A22-a23-a32] p22 )                              (C.4b)
```

Expected count rate out of the 3x3 CCD array is,

```
N_det
----- = P1 + P2 + P3 + P4
 N

      ====> 1.0-exp((-a11-a12-a13-a21-A22-a23-a31-a32-a33)*p22)
```

```
c    Double count
      1    + ( 1 - exp(-a12*p22) ) * exp( (-a21-A22-a23)*p22 )
      1    * ( 1 - exp( (-a31-a32-a33) *P22 ) )
      2    + ( 1 - exp(-a21*p22) ) * exp( (-a12-A22-a32)*p22 )
      2    * ( 1 - exp( (-a13-a23-a33)*p22 ) )
      3    + ( 1 - exp(-a23*p22) ) * exp( (-a12-A22-a32-a21)*p22 )
      3    * ( 1 - exp( (-a11-a31)*p22 ) )
      4    + ( 1 - exp(-a32*P22) ) * exp( (-a21-A22-a23-a12)*P22 )
      4    * ( 1 - exp( (-a11-a13)*P22 ) )
      5    + ( 1 - exp(-a11*P22) ) * exp( (-a21-A22-a23-a12-a32)*P22 )
      5    * ( 1 - exp( (-a31-a33-a13)*P22 ) )
      6    + ( 1 - exp(-a13*P22) ) * exp( (-a21-A22-a23-a12-a32-a11)*P22)
      6    * ( 1 - exp( (-a31-a33)*P22 ) )
      7    + ( 1 - exp(-a31*P22) ) * exp( (-a11-a12-a13-a21-A22-a23-a32)
      7    *P22 ) * ( 1 - exp( (-a33)*P22 ) )
c     Misc 4 combinations
      1a   + ( 1 - exp(-a12*p22) ) * ( 1 - exp(-a21*p22) )
      1a   * ( 1 - exp(-a33*P22) ) * exp( (-A22-a23-a32)*p22 )
      1b   + ( 1 - exp(-a12*p22) ) * ( 1 - exp(-a23*p22) )
      1b   * ( 1 - exp(-a31*P22) ) * exp( (-A22-a21-a32)*p22 )
      2a   + ( 1 - exp(-a21*p22) ) * ( 1 - exp(-a13*p22) )
      2a   * ( 1 - exp(-a32*P22) ) * exp( (-a12-A22-a23)*p22 )
      3a   + ( 1 - exp(-a11*p22) ) * ( 1 - exp(-a23*p22) )
      3a   * ( 1 - exp(-a32*P22) ) * exp( (-a12-A22-a21)*p22 )
```

```
c    Triple counts
      1    + ( 1 - exp(-a12*P22) ) * exp( (-a21-A22-a23-a32)*P22 )
      1    * ( 1 - exp(-a31*P22) ) * ( 1 - exp(-a33*P22 ) )
      2    + ( 1 - exp(-a21*P22) ) * exp( (-a12-A22-a32-a23)*P22 )
      2    * ( 1 - exp(-a13*P22) ) * ( 1 - exp(-a33*P22 ) )
```

```
3    + ( 1 - exp(-a23*P22) ) * exp( (-a12-a21-A22-a32)*P22 )
3    * ( 1 - exp(-a11*P22 ) )   * ( 1 - exp(-a31*P22 ) )
4    + ( 1 - exp(-a32*P22) ) * exp( (-a12-a21-A22-a23)*P22 )
4    * ( 1 - exp(-a11*P22 ) )   * ( 1 - exp(-a13*P22 ) )
5    + ( 1 - exp(-a11*P22) ) * ( 1 - exp(-a13*P22) )
5    * exp( (-a12-a21-A22-a23-a32)*P22 )
5    * ( 1 - exp( (-a31-a33)*P22 ) )
6    + ( 1 - exp(-a13*P22) ) * ( 1 - exp(-a31*P22) )
6    * ( 1 - exp(-a33*P22) ) * exp((-a11-a12-a21-A22-a23-a32)*P22)
5a   + ( 1 - exp(-a11*P22) ) * ( 1 - exp(-a31*P22) )
5a   * ( 1 - exp(-a33*P22) ) * exp((-a12-a13-a21-A22-a23-a32)*P22)
```

c Quadra count

```
1    + ( 1 - exp(-a11*P22) ) * ( 1 - exp(-a13*P22) )
1    * ( 1 - exp(-a31*P22) ) * ( 1 - exp(-a33*P22) )
1    * exp( (-a12-a21-A22-a23-a32)*P22 )                  (C.5)
```

Unlike Eq (A.4)and Eq(A.5), Eq (C.5) has no inverse equation, which can calculate real rate of incoming events from observed events. But, it is possible to determine "n22/N" (=p22) by Newton-Lapson method with a cheap PC. "a11" "a12" "a13" "a21" "a23" "a31" "a32" and "a33" should be known from PSF of optics and the star position within the CCD Pix_BB.

Section 2C.2. 2-dimension approach 5x5 CCD sampling

Photon spread of a very bright star sometimes cannot be covered by 3x3 CCD array, especially when the star is located at the cornour of CCD pixel. Here is probability equation for 5x5 CCD sampling, i.e. in the case that all photons from the star fall within a 5x5 CCD array.
Focusing on a particular frame, we consider whether the individual 25 CCD pixels receive events (doesn't matter single, double, triple,.... events) or not. There are 33 million combinations (= 2^25) in terms of photo-event's arrival or not. It is no longer possible to list up all possible configurations to be counted as "two", "three", "four",...... from the whole 5x5 array. In stead, we will investigate whether an event falling in the 5x5 square frame can add a count to the whole 5x5 array. Counts produced from the inner 3x3 array were already taken into account in the previous section. The event in the 5x5 square frame does not add an extra count to the whole array when adjacent CCD pixels in the 3x3 square frame receive events, because the event is bridged to the 3x3 array. The bridge to the 3x3 array sometimes happens via several trains of events at the 5x5 square frame. The bridge does not only happen to the 3x3 array but also to other pixels at the 5x5 square frame. Length of the train can be up to 11, but probability is extremely low in a long bridge. Bridge length of up to 3 only is taken into account in this document.

Table 4. Extended naming of 5x5 CCD array and PSF

Pix_DD a00	Pix_DA a01	Pix_DB a02	Pix_DC a03	Pix_DE a04
Pix_AD a10	Pix_AA a11	Pix_AB a12	Pix_AC a13	Pix_AE a14
Pix_BD a20	Pix_BA a21	Pix_BB A22	Pix_BC a23	Pix_BE a24
Pix_CD a30	Pix_CA a31	Pix_CB a32	Pix_CC a33	Pix_CE a34
Pix_ED a40	Pix_EA a41	Pix_EB a42	Pix_EC a43	Pix_EE a44

Table 5. Photo-events configuration for 5x5 array and counts by system

---

To be counted in

O X nc nc nc	nc O X nc nc	nc nc O nc nc	nc nc X O nc	nc nc nc X O
X x nc nc nc	nc x x nc nc	nc x x x nc	nc nc x x nc	nc nc nc x X
nc nc nc nc nc	nc nc nc nc nc	nc nc nc nc nc	nc nc nc nc nc	nc nc nc nc nc
nc nc nc nc nc	nc nc nc nc nc	nc nc nc nc nc	nc nc nc nc nc	nc nc nc nc nc
nc nc nc nc nc	nc nc nc nc nc	nc nc nc nc nc	nc nc nc nc nc	nc nc nc nc nc
(13)	(2)	(1)	(3)	(14)
nc X nc nc nc				nc nc nc X nc
O x nc nc nc				nc nc nc x O
X x nc nc nc	None	None	None	nc nc nc x X
nc nc nc nc nc				nc nc nc nc nc
nc nc nc nc nc				nc nc nc nc nc
(8)				(11)
nc nc nc nc nc				nc nc nc nc nc
nc x nc nc nc				nc nc nc x nc
O x nc nc nc	None	None	None	nc nc nc x O
nc x nc nc nc				nc nc nc x nc
nc nc nc nc nc				nc nc nc nc nc
(7)				(10)
nc nc nc nc nc				nc nc nc nc nc
nc nc nc nc nc				nc nc nc nc nc
X x nc nc nc	None	None	None	nc nc nc x X
O x nc nc nc				nc nc nc x O
nc X nc nc nc				nc nc nc X nc
(9)				(12)
nc nc nc nc nc	nc nc nc nc nc	nc nc nc nc nc	nc nc nc nc nc	nc nc nc nc nc
nc nc nc nc nc	nc nc nc nc nc	nc nc nc nc nc	nc nc nc nc nc	nc nc nc nc nc
nc nc nc nc nc	nc nc nc nc nc	nc nc nc nc nc	nc nc nc nc nc	nc nc nc nc nc
X x nc nc nc	nc x x nc nc	nc x x x nc	nc nc x x nc	nc nc nc x X
O X nc nc nc	nc O X nc nc	nc nc O nc nc	nc nc X O nc	nc nc nc X O
(15)	(5)	(4)	(6)	(16)

Bridge distance = 2 Pixels

None	nc O X nc nc o x x nc nc nc o nc nc nc nc nc nc nc nc nc nc nc nc nc (2B)	None	nc nc X O nc nc nc x x o nc nc nc o nc nc nc nc nc nc nc nc nc nc nc (3B)	None
None	None	None	None	None
nc o nc nc nc o x nc nc nc O x nc nc nc o x nc nc nc nc o nc nc nc (7B)	None	None	None	nc nc nc o nc nc nc nc x o nc nc nc x O nc nc nc x o nc nc nc o nc (10B)
None	None	None	None	None
None	nc nc nc nc nc nc nc nc nc nc nc o nc nc nc o x x nc nc nc O X nc nc (5B)	None	nc nc nc nc nc nc nc nc nc nc nc nc nc o nc nc nc x x o nc nc X O nc (6B)	None

----

Bridge distance = 3 Pixels

<table>
<tr><td>None</td><td>nc O X nc nc
o x x nc nc
o X nc nc nc
nc o nc nc nc
nc nc nc nc nc
(2B')</td><td>nc o O o nc
o x x x o
nc o nc o nc
nc nc nc nc nc
nc nc nc nc nc
(1B')</td><td>nc nc X O nc
nc nc x x o
nc nc nc X o
nc nc nc o nc
nc nc nc nc nc
(3B')</td><td>None</td></tr>
<tr><td>None</td><td>None</td><td>None</td><td>None</td><td>None</td></tr>
<tr><td>None</td><td>None</td><td>None</td><td>None</td><td>None</td></tr>
<tr><td>None</td><td>None</td><td>None</td><td>None</td><td>None</td></tr>
<tr><td>None</td><td>nc nc nc nc nc
nc o nc nc nc
o X nc nc nc
o x x nc nc
nc O X nc nc
(5B')</td><td>nc nc nc nc nc
nc nc nc nc nc
nc o nc o nc
o x x x o
nc o O o nc
(4B')</td><td>nc nc nc nc nc
nc nc nc o nc
nc nc nc X o
nc nc x x o
nc nc X O nc
(6B')</td><td>None</td></tr>
</table>

Note on symbols;
x: Pixel has not event to avoid bridging other 2 events
X: Pixel has not event to avoid overlap of combination
O: Pixel has events
o: Pixel has events to bridge CCD pixels
nc: do not care whether events arrived or not.

??

Explanation of table

(1) picks up all configurations, in which Pix_DB can add a count to the whole 5x5 array. Pix_AA, Pix_AB, Pix_AC must be blank to separate Pix_DB from the inner 3x3 array. There are over counting in the configuration (1). Pix_DB bridges into Pix_BA via Pix_DA and Pix_AD as shown in (1B). Pix_DB also bridges into Pix_BC via Pix_DC and Pix_AE .

(2) picks up all configurations, in which Pix_DA can add a count to the whole 5x5 array. Adjacent Pix_DB must be blank, since all its contributions were already counted in (1). Pix_AA, Pix_AB must be blank to separate Pix_DA from the inner 3x3 array. There are over counting in the configuration (2). Pix_DA bridges into Pix_BA via Pix_AD as shown in (2B). Pix_DA also bridges into Pix_CA via Pix_AD and Pix_BD as shown in (2B').
Pix_BA must be blank to avoid overlapping with (2B).

(3) picks up all configurations, in which Pix_DC can add a count to the whole 5x5 array. Adjacent Pix_DB must be blank, since all its contributions were already counted in (1).
Pix_AC, Pix_AB must be blank to separate Pix_DC from the inner 3x3 array. There are over counting in the configuration (3). Pix_DC bridges into Pix_BC via Pix_AE as shown in (3B). Pix_DC also bridges into Pix_CC via Pix_AE and Pix_BE as shown in (3B'). Pix_BC must be blank to avoid overlapping with (3B).

(4) picks up all configurations, in which Pix_EB can add a count to the whole 5x5 array. Pix_CA, Pix_CB, Pix_CC must be blank to separate Pix_EB from the inner 3x3 array. There are over counting in the configuration (4). Pix_EB bridges into Pix_BA via Pix_EA and Pix_CD as shown in (4B). Pix_DB also bridges into Pix_BC via Pix_EC and Pix_CE .

(5) picks up all configurations, in which Pix_EA can add a count to the whole 5x5 array. Adjacent Pix_EB must be blank, since all its contributions were already counted in (4). Pix_CA, Pix_CB must be blank to separate Pix_EA from the inner 3x3 array. There are over counting in the configuration (5). Pix_EA bridges into Pix_BA via Pix_CD as shown in (5B). Pix_EA also bridges into Pix_AA via Pix_CD and Pix_BD as shown in (5B'). Pix_BA must be blank to avoid overlapping with (5B).

(6) picks up all configurations, in which Pix_EC can add a count to the whole 5x5 array. Adjacent Pix_EB must be blank, since all its contributions were already counted in (4). Pix_CB, Pix_CC, must be blank to separate Pix_EC from the inner 3x3 array. There are over counting in the configuration (6). Pix_EC bridges into Pix_BC via Pix_CE as shown in (6B). Pix_EA also bridges into Pix_AC via Pix_CE and Pix_BE as shown in (6B'). Pix_BC must be blank to avoid overlapping with (6B).

(7) picks up all configurations, in which Pix_BD can add a count to the whole 5x5 array. Pix_AA, Pix_BA, Pix_CA must be blank to separate Pix_BD from the inner 3x3 array. There are over counting in the configuration (7). Pix_BD bridges into Pix_DA via Pix_AD as shown in (7B). Pix_BD also bridges into Pix_EA via Pix_CD.

(8) picks up all configurations, in which Pix_AD can add a count to the whole 5x5 array. Adjacent Pix_BD and Pix DA must be blank, since all their contributions were already counted in (2) and (7). Pix_AA, Pix_BA must be blank to separate Pix_AD from the inner 3x3 array.

(9) picks up all configurations, in which Pix_CD can add a count to the whole 5x5 array.

Adjacent Pix_BD and Pix EA must be blank, since all their contributions were already counted in (5) and (7). Pix_BA, Pix_CA must be blank to separate Pix_CD from the inner 3x3 array.

(10) picks up all configurations, in which Pix_BE can add a count to the whole 5x5 array. Pix_AC, Pix_BC, Pix_CC must be blank to separate Pix_BE from the inner 3x3 array. There are over counting in the configuration (10). Pix_BE bridges into Pix_DC via Pix_AE as shown in (10B). Pix_BE also bridges into Pix_EC via Pix_CE. There are over counting in the configuration (10). Pix_BE bridges into Pix_DC via Pix_AE as shown in (10B). Pix_BE also bridges into Pix_EC via Pix_CE.

(11) picks up all configurations, in which Pix_AE can add a count to the whole 5x5 array. Adjacent Pix_BE and Pix DC must be blank, since all their contributions were already counted in (3) and (10). Pix_AC, Pix_BC must be blank to separate Pix_AE from the inner 3x3 array.

(12) picks up all configurations, in which Pix_CE can add a count to the whole 5x5 array. Adjacent Pix_BE and Pix EC must be blank, since all their contributions were already counted in (6) and (10). Pix_BC, Pix_CC must be blank to separate Pix_CE from the inner 3x3 array.

(13) picks up all configurations, in which Pix_DD can add a count to the whole 5x5 array. Adjacent Pix_DA and Pix AD must be blank, since all their contributions were already counted in (2) and (8). Pix_AA must be blank to separate Pix_DD from the inner 3x3 array.

(14) picks up all configurations, in which Pix_DE can add a count to the whole 5x5 array. Adjacent Pix_DC and Pix AE must be blank, since all their contributions were already counted in (3) and (11). Pix_AC must be blank to separate Pix_DE from the inner 3x3 array.

(15) picks up all configurations, in which Pix_ED can add a count to the whole 5x5 array. Adjacent Pix_EA and Pix CD must be blank, since all their contributions were already counted in (5) and (9). Pix_CA must be blank to separate Pix_ED from the inner 3x3 array.

(16) picks up all configurations, in which Pix_EE can add a count to the whole 5x5 array. Adjacent Pix_EC and Pix CE must be blank, since all their contributions were already counted in (6) and (12). Pix_CC must be blank to separate Pix_EE from the inner 3x3 array.

???

For expected count rate out of the 5x5 CCD array, the following terms must be added to Eq (C.5)

```
CC   5x5 CCD SAMPLING
      1    + ( 1 - exp(-a02*P22) ) * exp( (-a11-a12-a13)*P22 )
      2    + ( 1 - exp(-a01*P22) ) * exp( (-a11-a12-a02)*P22 )
      3    + ( 1 - exp(-a03*P22) ) * exp( (-a12-a13-a02)*P22 )
      4    + ( 1 - exp(-a42*P22) ) * exp( (-a31-a32-a33)*P22 )
      5    + ( 1 - exp(-a41*P22) ) * exp( (-a31-a32-a42)*P22 )
      6    + ( 1 - exp(-a43*P22) ) * exp( (-a32-a33-a42)*P22 )
      7    + ( 1 - exp(-a20*P22) ) * exp( (-a11-a21-a31)*P22 )
      8    + ( 1 - exp(-a10*P22) ) * exp( (-a11-a21-a20-a01)*P22 )
      9    + ( 1 - exp(-a30*P22) ) * exp( (-a21-a31-a20-a41)*P22 )
     10    + ( 1 - exp(-a24*P22) ) * exp( (-a13-a23-a33)*P22 )
     11    + ( 1 - exp(-a14*P22) ) * exp( (-a13-a23-a24-a03)*P22 )
     12    + ( 1 - exp(-a34*P22) ) * exp( (-a23-a33-a24-a43)*P22 )
     13    + ( 1 - exp(-a00*P22) ) * exp( (-a01-a11-a10)*P22 )
     14    + ( 1 - exp(-a04*P22) ) * exp( (-a03-a13-a14)*P22 )
     15    + ( 1 - exp(-a40*P22) ) * exp( (-a41-a31-a30)*P22 )
     16    + ( 1 - exp(-a44*P22) ) * exp( (-a43-a33-a34)*P22 )

C    Bridge distance=2
     2B    - ( 1 - exp(-a01*P22) ) * exp( (-a11-a12-a02)*P22 )
     2B    * ( 1 - exp(-a10*P22) ) * ( 1 - exp(-a21*P22) )
     3B    - ( 1 - exp(-a03*P22) ) * exp( (-a12-a13-a02)*P22 )
     3B    * ( 1 - exp(-a14*P22) ) * ( 1 - exp(-a23*P22) )
     5B    - ( 1 - exp(-a41*P22) ) * exp( (-a31-a32-a42)*P22 )
     5B    * ( 1 - exp(-a30*P22) ) * ( 1 - exp(-a21*P22) )
     6B    - ( 1 - exp(-a43*P22) ) * exp( (-a32-a33-a42)*P22 )
     6B    * ( 1 - exp(-a34*P22) ) * ( 1 - exp(-a23*P22) )

     7B    - ( 1 - exp(-a20*P22) ) * exp( (-a11-a21-a31)*P22 )
     7B    * ( 1 - (1 - (1-exp(-a10*P22))*(1-exp(-a01*P22)))
     7B           *(1 - (1-exp(-a30*P22))*(1-exp(-a41*P22))) )
    10B    - ( 1 - exp(-a24*P22) ) * exp( (-a13-a23-a33)*P22 )
    10B    * ( 1 - (1 - (1-exp(-a14*P22))*(1-exp(-a03*P22)))
    10B           *(1 - (1-exp(-a34*P22))*(1-exp(-a43*P22)))

C    Bridge distance=3
     2B    - ( 1 - exp(-a01*P22) ) * exp( (-a11-a12-a02)*P22 )
     2B    * ( 1 - exp(-a10*P22) ) * exp(-a21*P22)
     2B    * ( 1 - exp(-a20*P22) ) * ( 1 - exp(-a31*P22) )
     3B    - ( 1 - exp(-a03*P22) ) * exp( (-a12-a13-a02)*P22 )
     3B    * ( 1 - exp(-a14*P22) ) * exp(-a23*P22)
     3B    * ( 1 - exp(-a24*P22) ) * ( 1 - exp(-a33*P22) )
     5B    - ( 1 - exp(-a41*P22) ) * exp( (-a31-a32-a42)*P22 )
     5B    * ( 1 - exp(-a30*P22) ) * exp(-a21*P22)
     5B    * ( 1 - exp(-a20*P22) ) * ( 1 - exp(-a11*P22) )
     6B    - ( 1 - exp(-a43*P22) ) * exp( (-a32-a33-a42)*P22 )
     6B    * ( 1 - exp(-a34*P22) ) * exp(-a23*P22)
     6B    * ( 1 - exp(-a24*P22) ) * ( 1 - exp(-a13*P22) )

     1B    - ( 1 - exp(-a02*P22) ) * exp( (-a11-a12-a13)*P22 )
     1B    * (1 -(1 -(1-exp(-a01*P22))*(1-exp(-a10*P22))*(1-exp(-a21*P22)))
     1B         *(1 -(1-exp(-a03*P22))*(1-exp(-a14*P22))*(1-exp(-a23*P22))) )
     4B    - ( 1 - exp(-a42*P22) ) * exp( (-a31-a32-a33)*P22 )
     4B    * (1 -(1 -(1-exp(-a41*P22))*(1-exp(-a30*P22))*(1-exp(-a21*P22)))
     4B         *(1 -(1-exp(-a43*P22))*(1-exp(-a34*P22))*(1-exp(-a23*P22))) )
```

(C.6)

3. Coincidence correction curve for 3 types of PSF

Coincidence correction curve given by Eqs (C.5) and (C.6) look awful complicated, but the curve itself is smooth and it is quick to calculate with the help of PC, provided PSF is known. There are 3 causes of image spreading in OM image, i.e.;

1) Airy pattern - telescope diffraction	: J1(r) / r
2) Photocathode gap effect	: (Lorentzan)^4
3) Optics aberration or contrast	: (Lorentzan)^2 .

The sizes of photon spreading due to these mechanisms are compared in Fig. 1. Since diffraction limited telescope produces only 8mm full width, its diffraction rings are far smaller than a CCD pixel. So, 3x3 CCD sampling is absolutely enough. The situation is pretty different with x4 magnifier, though. The significant fraction of light spread out of a CCD pixel. The 3x3 CCD sampling may not be enough.
Photocathode gap causes spread of photoelectron emitted from photocathode surface. The spread is larger in the shorter wavelengths. The detail is described in Appendix. There is absolute maximum on the displacement of photo-electron. It must not be larger than 50um (D=100um) at any wavelengths when 400V applied to the photocathode gap. Since CCD pixel size is 74um, the 3x3 sampling is enough for the photocathode gap effect. The profile does changed with the x4 magnifier.
A PSF due to optics was derived from a number of stars in LMC. Telescope aberration and contrast of optics causes extended wing around a star. The wing part is most significant among the 3 causes, hence it requires 5x5 CCD sampling. With x4 magnifier, the wing spreads even further. PSF due to the optics should be same at all wavelengths.
Fig. 2 and Fig. 3 show the slice of the 3 types of PSFs. The star image due to optical aberration does not damp as fast as photocathode gap effect as seen in Fig. 3.

For a systematic characterization of XMM-flight detector, best-fitted analytic functions were used in the following calculations. Photocathode gap effect was represented by (Lorentzan)^4 as shown in Fig. 4. The function was multiplied by "cos(r)" at the wing, so that the profile is forced to be zero beyond the limit (~50um). Optical aberration was represented by (Lorentzan)^2 as shown in Fig. 5. Airy pattern was expressed by the well-known 1st Bessel function.

Equations for coincidence correction contain many terms corresponding many combinations. There is a chance of missing combinations or overlapping. There are also chances of mistyping. Therefore, it is essential to check the validity of all terms.
The equations were derived in asymmetric manner. For instance, the 7th term of double counting in Eq(C.5) does not have partner term for symmetry. The correction curve, however, must show symmetric nature after adding all terms. Symmetric profile functions with various widths were placed at 4 symmetric positions. The 1st set was at top left cornour of CCD pixel, top right, bottom right and bottom left. The 2nd set was at top boundary of CCD pixel, right, bottom and left. All four in the 1st set showed exactly same curves between 0% -1000% input rate for Eq(C.5) and Eq(C.6). All four in the 2nd set also showed exactly same curves. To eliminate the possibility of miss typing, text of the equations in this draft was directly copied to Fortran source list and then complied for the above calculations. Actually, several mistyping and missing combinations were discovered by this approach.

Fig. 6 shows the correction curve for the diffraction pattern. Since the PSF is small enough for photons to fall into 2x2 CCD pixels, 0-dim equation is sufficient. Star position does not affect the curve. Fig. 7 shows for the diffraction pattern with the x4 magnifier. It requires 5x5 CCD sampling. Its spread seems to be too big even for the 5x5 sampling, as the curve changes with star position within a CCD pixel. This behavior is not easy to handle. Fig. 8 expresses the importance of 2-Dim approach. When incoming rate is 150%, it is miss-calibrated as 180% with the 0-Dim equation. Extraordinary example is at 200% incoming rate, in which 0-Dim equation gives infinity.

Fig. 9 shows the correction curve for photocathode gap effect. Because of the fast damp at wing part, 0-dim equation is sufficient for FWHM=25um, which corresponds to at optical wavelengths. There is, however, a very little difference for FWHM=40um, which corresponds to at UV wavelengths as shown in Fig. 10. Width 5x5 CCD sampling, the count rate is corrected in great accuracy, independent of the FWHM and star positions.

Optical aberration is the most dominant source for the image spreading. Figs.11 and 12 show correction curves for the optical aberration. It requires the 5x5 CCD sampling. The curve changes with profile width but does not much with star position within a CCD pixel. This is good new for actual calibration. The 0-Dim equation is again out of question for this wide spread wing. Figs.13 and 14 show the correction curves with the x4 magnifier.
The curve largely changes from width of 80um to 100um. But, they are not much sensitive to star positions, which is extremely good new for actual calibration. Again, the 0-Dim equation is far away from real correction curve as seen in Fig. 14..

4. 1 CCD sampling when photons spread to 3x3 CCD array

In actual situation, a star is sometimes located under high background, for example against nebulosity or zodiacal light. Even if the spread of photon from the star is known to be more than 3x3 CCD array, expanding the sampling window often costs too much because of the bright background. For example, if the telescope is pointed to zodiacal light dominated area and the clear filter is used, 76 events/sec of sky background will arrive in the 5x5 CCD array. It is ideal if we could determine the star brightness with 3x3 CCD array or even with 1 CCD array. This is particularly problem in high time resolution mode, since acquired photons are not sufficient to subtract the background effect. In this section, mathematical models are given when the sampling window is smaller than the size of the PSF.

Section 4D. 1. 1 CCD sampling when photons spread to 3 CCD pixels (1-dimension)

We consider the case, in which star image is spread along 3 CCD pixels but we sample the counts from the central CCD pixel only. We assume that "n1" events arrived at Pix_A during "N" frames, "n2" events at Pix_B, and "n3" events at Pix_C.

```
         ooooooo                     n1 events
         oooooooooooooooooooooooo    n2 events
         oooo                        n3 events

         <--------------------  N Frames  -------------------------->
Pix_A   |---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
Pix_B   |---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
Pix_C   |---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
                                  |
                                  |
                                  |
                   How many events ? at this Frame in the 3 pixels
```

with narrow PDH and narrow event width

Focusing on a particular frame, we consider whether the individual 3 CCD pixels capture events (This time, it is the matter whether single, double, triple,.... events) or not. There are 8 combinations (= 2^3) in terms of photo-event's arrival ("o") or not ("x") as tabulated in Table 6. The 4th column shows the number of events allocated to the Pix_B. Count " 1 " is secured in the 3rd raw, but not in the 4th raw since the merged event may shift to the Pix_A. If both of Pix_B and Pix_A received one event in the frame, the chance to be allocated to Pix_B is 50:50. If Pix_B received 2 events while Pix_A one event, Pix_B will get " 1 " count. If Pix_B received 3 events while Pix_A 2 events, again Pix_B will get " 1 " count.

Table 6. Photo-events configuration and counts by system

Pix_A	Pix_B	Pix_C	count
x	x	x	0
o	x	x	0
x	o	x	1
o	o	x	1/0
x	x	o	0
o	x	o	0
x	o	o	1/0
o	o	o	1/0

Note) o: Pixel has events   x: Pixel has not event

Table 7. The cases Pix_B can hold a count

Pix_B	o	o	o	o	o	o	o	o	
events	o	o	o	o	o	o	o		
6	o	o	o	o	o	o		x	
5	o	o	o	o	o		x	x	
4	o	o	o	o		x	x	x	
3	o	o	o		x	x	x	x	
2	o	o		x	x	x	x	x	
1	o		x	x	x	x	x	x	
	0	1	2	3	4	5	6	events	Pix_A

I will list up the probabilities that Pix_A steals count from Pix_B in a frame.
When Pix_A received **1 event** and Pix_B 1event, Pix_A steals a count from Pix_B with 50% chance.

$$0.5 \cdot \{ p1 \cdot \exp(-p1) \}\{ p2 \cdot \exp(-p2) \},$$

where, $p1 = \frac{n1}{N}$, $p2 = \frac{n2}{N}$, $p3 = \frac{n3}{N}$

When Pix_A received **2 events** and Pix_B 1 event, Pix_A steals a count from Pix_B.

$$\{ p1^2 \cdot \exp(-p1) \}\{ p2 \cdot \exp(-p2) \}$$

When Pix_A received 2 events and Pix_B 2 events, Pix_A steals a count from Pix_B with 50% chance.

$$0.5 \cdot \{ p1^2 \cdot \exp(-p1) \}\{ p2^2 \cdot \exp(-p2) \},$$

When Pix_A received **3 events** and Pix_B 1 or 2 events, Pix_A steals a count from Pix_B.

$$\{ \frac{p1^3}{3\,!} \cdot \exp(-p1) \} \cdot [\{ p2 \cdot \exp(-p2) \} + \{ \frac{p2^2}{2\,!} \cdot \exp(-p2) \}]$$

When Pix_A received 3 events and Pix_B 3 events, Pix_A steals a count from Pix_B with 50% chance.

$$0.5 \cdot \{ \frac{p1^3}{3\,!} \cdot \exp(-p1) \} \cdot \{ \frac{p2^3}{3\,!} \cdot \exp(-p2) \},$$

When Pix_A received **4 events** and Pix_B 1, 2 or 3 events, Pix_A steals a count from Pix_B.

$$\{ \frac{p1^4}{4\,!} \cdot \exp(-p1) \} \cdot [\{ p2 \cdot \exp(-p2) \} + \{ \frac{p2^2}{2\,!} \cdot \exp(-p2) \} + \{ \frac{p2^3}{3\,!} \cdot \exp(-p2) \}]$$

When Pix_A received 4 events and Pix_B 4 events, Pix_A steals a count from Pix_B with 50% chance.

$$0.5 \cdot \{ \frac{p1^4}{4\,!} \cdot \exp(-p1) \} \cdot \{ \frac{p2^4}{4\,!} \cdot \exp(-p2) \},$$

I assume "p1" (= n1/N) is reasonably small because most of star light falls in the central CCD pixel. If p1=0.4 (40 events/sec/CCD_pix), probability of 5 events arrival in a frame is only $6 \cdot 10^{-5}$. So, I ignore the cases of more than 5 events in Pix_A. It should be noted that the peak value of pulse height distribution for the events is 80-90LSBs and XMM's CCD camera employed 8bit ADC.. Both of Pix_B and Pix_A should have digitizing value of 255LSBs in the situation of 5 events arrival at Pix_A. The hardware of event detection allocates the event to Pix_B (following pixel) in the saturation. So, Pix_A has no chance to steal the count.

Summing up all possibilities listed above, the probability that Pix_A steals the count from Pix_B is

With 100% chance,
Pa =

$$+ \{ \frac{p1^2}{2\,!} \cdot \exp(-p1) \} \cdot [\{ p2 \cdot \exp(-p2) \}]$$

$$+ \left\{ \frac{p1^3}{3!} \cdot \exp(-p1) \right\} \cdot \left[\{p2 \cdot \exp(-p2)\} + \left\{ \frac{p2^2}{2!} \cdot \exp(-p2) \right\} \right]$$

$$+ \left\{ \frac{p1^4}{4!} \cdot \exp(-p1) \right\} \cdot \left[\{p2 \cdot \exp(-p2)\} + \left\{ \frac{p2^2}{2!} \cdot \exp(-p2) \right\} + \left\{ \frac{p2^3}{3!} \cdot \exp(-p2) \right] \right.$$

$$= \left[\ \{ p2 \cdot \exp(-p2) \} \right] \cdot \left[\exp(-p1) \cdot \left(\frac{p1^2}{2!} + \frac{p1^3}{3!} + \frac{p1^4}{4!} \right) \right]$$

$$+ \left[\left\{ \frac{p2^2}{2!} \cdot \exp(-p2) \right\} \right] \cdot \left[\exp(-p1) \cdot \left(\frac{p1^3}{3!} + \frac{p1^4}{4!} \right) \right]$$

$$+ \left[\left\{ \frac{p2^3}{3!} \cdot \exp(-p2) \right\} \right] \cdot \left[\exp(-p1) \cdot \left(\frac{p1^4}{4!} \right) \right] \tag{D.1}$$

With 50% chance,

$$Pa\_eq = \exp(-p2) \cdot \exp(-p1) \cdot \left(p2 \cdot p1 + \frac{p2^2}{2!} \cdot \frac{p1^2}{2!} + \frac{p2^3}{3!} \cdot \frac{p1^3}{3!} + \frac{p2^3}{4!} \cdot \frac{p1^3}{4!} \right) \tag{D.2}$$

Saturation effect for " Pix_B >= 5 events " with 0% chance,

$$Pb\_sat = \ 1 - \exp(-p2) \cdot \left(1 + p2 + \frac{p2^2}{2!} + \frac{p2^3}{3!} + \frac{p2^4}{4!} \right) \tag{D.3}$$

I will list up the probabilities that Pix_C steals count from Pix_B in a frame.
I assume "p3" (= n1/N) is reasonably small because most of star light falls in the central CCD pixel. So, I ignore the cases of more than 5 events in Pix_C. It should be noted that Both of Pix_B and Pix_C should have digitizing value of 255LSBs in the situation of 5 events arrival at Pix_C. The hardware of event detection allocates the event to Pix_C (following pixel) in the saturation. So, Pix_C should always steal the count when >5 events arrive

Summing up all possibilities, the probability that Pix_C steal the count from Pix_B is

With 100% chance,
Pc =

$$+\left\{\frac{p3^2}{2!}\cdot \exp(-p3)\right\}\cdot\left[\left\{p2\cdot \exp(-p2)\right\}\right]$$

$$+\left\{\frac{p3^3}{3!}\cdot \exp(-p3)\right\}\cdot\left[\left\{p2\cdot \exp(-p2)\right\}+\left\{\frac{p2^2}{2!}\cdot \exp(-p2)\right\}\right]$$

$$+\left\{\frac{p3^4}{4!}\cdot \exp(-p3)\right\}\cdot\left[\left\{p2\cdot \exp(-p2)\right\}+\left\{\frac{p2^2}{2!}\cdot \exp(-p2)\right\}+\left\{\frac{p2^3}{3!}\cdot \exp(-p2)\right]\right.$$

$$= \left[\ \left\{p2\cdot \exp(-p2)\right\}\right]\cdot\left[\exp(-p3)\cdot\left(\frac{p3^2}{2!}+\frac{p3^3}{3!}+\frac{p3^4}{4!}\right)\right]$$

$$+\left[\left\{\frac{p2^2}{2!}\cdot \exp(-p2)\right\}\right]\cdot\left[\exp(-p3)\cdot\left(\frac{p3^3}{3!}+\frac{p3^4}{4!}\right)\right]$$

$$+\left[\left\{\frac{p2^3}{3!}\cdot \exp(-p2)\right\}\right]\cdot\left[\exp(-p3)\cdot\left(\frac{p3^4}{4!}\right)\right] \quad \text{(D.4)}$$

With 50% chance,

$$Pc\_eq = \exp(-p2)\cdot \exp(-p3)\cdot\left(p2\cdot p3+\frac{p2^2}{2!}\cdot\frac{p3^2}{2!}+\frac{p2^3}{3!}\cdot\frac{p3^3}{3!}+\frac{p2^3}{4!}\cdot\frac{p3^3}{4!}\right) \quad \text{(D.5)}$$

Saturation effect for " Pix_C >= 5 events " with 100% chance,

$$Pc\_sat = 1-\exp(-p3)\cdot\left(1+p3+\frac{p3^2}{2!}+\frac{p3^3}{3!}+\frac{p3^4}{4!}\right) \quad \text{(D.6)}$$

There are overlapping in the combination for stealing counts.
When both of Pix_A and Pix_C received larger number of evens than Pix_B, Pix_B does not lose the count twice but only once.
I.e.,
The loss of count with 100% chance,
$1.0 \cdot (Pa+Pc-Pb\_sat + Pc\_sat - Pa\perp Pc - Pa \cdot Pc\_sat)$

where,

$$
\begin{aligned}
Pa\perp Pc = {} & [\exp(-p2)] \times \\
& \left[p2 \cdot \left\{\exp(-p1) \cdot \left(\frac{p1^2}{2!} + \frac{p1^3}{3!} + \frac{p1^4}{4!}\right)\right\} \cdot \left\{ \exp(-p3) \cdot \left(\frac{p3^2}{2!} + \frac{p3^3}{3!} + \frac{p3^4}{4!}\right)\right\} \right. \\
& + \frac{p2^2}{2!} \cdot \left\{ \exp(-p1) \cdot \left(\frac{p1^3}{3!} + \frac{p1^4}{4!} \right) \right\} \cdot \left\{ \exp(-p3) \cdot \left(\frac{p3^3}{3!} + \frac{p3^4}{4!} \right) \right\} \\
& \left. + \frac{p2^3}{3!} \cdot \left\{ \exp(-p1) \cdot \left(\frac{p1^4}{4!} \right) \right\} \cdot \left\{ \exp(-p3) \cdot \left(\frac{p3^4}{4!} \right) \right\} \right]
\end{aligned}
\tag{D.7}
$$

When both of Pix_A and Pix_C received same number of evens as Pix_B, Pix_B does not lose the count twice but only once. The chance, however, increases from "1/2" to "2/3".
I.e.,
The loss of count with 50% (or 67%),
$1/2 \cdot (Pa\_eq + Pc\_eq) - 1/3 \cdot Pa\_eq\perp Pc\_eq$

where,

$$
\begin{aligned}
Pa\_eq\perp Pc\_eq = {} & \exp(-p1) \cdot \exp(-p2) \cdot \exp(-p3) \times \\
& \left(p1 \cdot p2 \cdot p3 + \frac{p1^2}{2!} \cdot \frac{p2^2}{2!} \cdot \frac{p3^2}{2!} + \frac{p1^3}{3!} \cdot \frac{p2^3}{3!} \cdot \frac{p3^3}{3!} + \frac{p1^3}{4!} \cdot \frac{p2^3}{4!} \cdot \frac{p3^3}{4!} \right)
\end{aligned}
\tag{D.8}
$$

Taking all of the above into account, the detection probability at Pix_B is

$$
\begin{aligned}
& [1 - \exp(-p2)] - 1.0 \cdot [Pa+Pc + \{1 - \exp(-p2)\} \cdot Pc\_sat - Pc \cdot Pb\_sat - Pa \cdot Pc\_sat - Pa\perp Pc] \\
& \quad - 1/2 \cdot (Pa\_eq + Pc\_eq) + 1/3 \cdot Pa\_eq\perp Pc\_eq
\end{aligned}
\tag{D.9}
$$

with real PDH but narrow event width

Table 8. Probabilities of stealing a count by Pix_C, when Pix_C < 255LSBs

Pix_B:		1-	2-	3-	4-events	Pix_B >255
Pix_C:	1 event	.505	.077	.005	.000	.002
	2 events	.791	.380	.070	.005	.128
	3 events	.325	.220	.056	.005	.659
	4 events	.040	.031	.009	.001	.957

There is a more complicated issue associated with pulse height distribution of the events. We assumed the event is allocated to Pix_C if Pix_C received 2 events while Pix_B 1 event. In the real case, the event stroke Pix_B might be brighter than the addition of the 2 events stroke Pix_C. For instance, the big event has the brightness of 200LSBs, while the 2 modest events 80LSBs and 90LSBs respectively. This probability is low but not zero(e.g. ~8%). Fig.14 shows pulse height distributions for single, double, triple and quadruple events. Significant portion of single event can be brighter than the addition of 2 events. The ratio is even more between double events and triple events. Such reversed allocation of the event is listed in Table 8 with the probabilities.
Same reversal can happen in opposite direction, i.e. when Pix_C received 1 event while Pix_B 2 events. These reversals are not the matter if Pix_B and Pix_C have same brightness in star image, since the reversal for the both directions cancels each other. Unfortunately, Pix_B is brighter than Pix_C in the star image, therefore this reversal effect shifts the count more toward Pix_C, though the quantity itself is very small as seen in the 4th -6th terms of the following Eq(D.10).
Introducing the parameters in Table 8, Eqs (D.4) and (D.5) are modified as follows,

With no longer100% chance,

$$Pc = [\ \{ p2 \cdot \exp(-p2) \}\ ] \cdot [\ \exp(-p3) \cdot (\ 0.50 \cdot p3 + 0.79 \frac{p3^2}{2\,!} + 0.33 \frac{p3^3}{3\,!} + 0.04 \frac{p3^4}{4\,!}\ )\ ]$$

$$+ [\ \{ \frac{p2^2}{2\,!} \cdot \exp(-p2) \}] \cdot [\ \exp(-p3) \cdot (\ 0.08 \cdot p3 + 0.38 \frac{p3^2}{2\,!} + 0.22 \frac{p3^3}{3\,!} + 0.03 \frac{p3^4}{4\,!})\ ]$$

$$+ [\ \{ \frac{p2^3}{3\,!} \cdot \exp(-p2) \}] \cdot [\ \exp(-p3) \cdot (\ 0.005 \cdot p3 + 0.07 \frac{p3^2}{2\,!} + 0.06 \frac{p3^3}{3\,!} + 0.01 \frac{p3^4}{4\,!}\ )\ ]$$

$$+ [\ \{ \frac{p2^4}{4\,!} \cdot \exp(-p2) \}] \cdot [\ \exp(-p3) \cdot (\ 0.000 \cdot p3 + 0.005 \frac{p3^2}{2\,!} + 0.005 \frac{p3^3}{3\,!} + 0.001 \frac{p3^4}{4\,!}\ )\ ]$$

$$\cong [\ \{\ p2 \cdot \exp(-p2)\ \}\ ] \cdot Pc\_1 + [\ \{ \frac{p2^2}{2\,!} \cdot \exp(-p2)\ \}\ ] \cdot Pc\_2$$

$$+ \left[\left\{ \frac{p2^3}{3!} \cdot \exp(-p2) \right\} \right] \cdot Pc\_3 \quad + \left[\left\{ \frac{p2^4}{4!} \cdot \exp(-p2) \right\} \right] \cdot Pc\_4 \tag{D.10}$$

The last column of Table 8 shows probability that Pix_C receives more than 255LSBs. With XMM's digital processing hardware, the event is automatically allocated to Pix_C when it gets 255LSBs whatever value of Pix_B. This is dominant mechanism to steal the count from Pix_B. Expressing this mechanism separately,

$$Pc\_sat = \quad 1 - \exp(-p3) \cdot \left(1 + p3 + \frac{p3^2}{2!} + \frac{p3^3}{3!} + \frac{p3^4}{4!}\right)$$

$$+ \exp(-p3) \cdot \left(0.02 \cdot p3 + 0.128 \cdot \frac{p3^2}{2!} + 0.659 \cdot \frac{p3^3}{3!} + 0.957 \cdot \frac{p3^4}{4!}\right) \tag{D.11}$$

Table 9. Probabilities of stealing a count by Pix_A, when Pix_B < 255LSBs

Pix_A:	1-	2-	3-	4-events	Pix_B >255
Pix_B: 1 event	.493	.921	.993	.998	.002
2 events	.081	.492	.802	.867	.128
3 events	.016	.121	.285	.336	.659
4 events	.003	.012	.034	.042	.957

Table 9 shows probability of stealing the count by Pix_A from Pix_B. It excludes the case when Pix_B receives >255LSBs, in which the event is automatically allocated to Pix_B. Since the digital saturation effect is the dominant mechanism in high count rate, less events are stolen by Pix_A. Introducing the parameters in Table 9, Eqs (D.1) and (D.2) are modified as follows,

With no longer100% chance,

$$Pa = \left[\quad \{ p2 \cdot \exp(-p2) \} \right] \cdot \left[\exp(-p1) \cdot \left(0.49 \cdot p3 + 0.92 \cdot \frac{p1^2}{2!} + 0.99 \cdot \frac{p1^3}{3!} + 1.00 \cdot \frac{p1^4}{4!} \right) \right]$$

$$+ \left[\left\{ \frac{p2^2}{2!} \cdot \exp(-p2) \right\} \right] \cdot \left[\exp(-p1) \cdot \left(0.08 \cdot p3 + 0.49 \cdot \frac{p1^2}{2!} + 0.80 \cdot \frac{p1^3}{3!} + 0.87 \cdot \frac{p1^4}{4!} \right) \right]$$

$$+ \left[\left\{ \frac{p2^3}{3!} \cdot \exp(-p2) \right\} \right] \cdot \left[\exp(-p1) \cdot \left(0.016 \cdot p3 + 0.12 \cdot \frac{p1^2}{2!} + 0.29 \cdot \frac{p1^3}{3!} + 0.34 \cdot \frac{p1^4}{4!} \right) \right]$$

$$+ \left[\left\{ \frac{p2^4}{4!} \cdot \exp(-p2) \right\} \right] \cdot \left[\exp(-p1) \cdot \left(0.003 \cdot p3 + 0.012 \frac{p1^2}{2!} + 0.034 \frac{p1^3}{3!} + 0.04 \frac{p1^4}{4!} \right) \right]$$

(D.12)

$$\cong \left[\left\{ p2 \cdot \exp(-p2) \right\} \right] \cdot Pa\_1 + \left[\left\{ \frac{p2^2}{2!} \cdot \exp(-p2) \right\} \right] \cdot Pa\_2$$

$$+ \left[\left\{ \frac{p2^3}{3!} \cdot \exp(-p2) \right\} \right] \cdot Pa\_3 + \left[\left\{ \frac{p2^4}{4!} \cdot \exp(-p2) \right\} \right] \cdot Pa\_4$$

(D.13)

There are overlappings in the combination for stealing the count. When both of Pix_A and Pix_C are brighter than Pix_B, Pix_B does not lose the count twice. The overlapping probability is, however, not simply "Pa⊥Pc". For example, when all of the 3 Pixels receive single event, the probability is not 1/4 (="Pa⊥Pc") but 1/3. This is because the brightness of Pix_B is key factor for the occurrence. I.e. if Pix_B is fainter, chance of the overlapping is higher, while if Pix_B is brighter, chance of the overlapping is lower. We must calculate the individual cases, for instance Pix_A=2 events, Pix_B=1, Pix_C=2 events or Pix_A=3 events, Pix_B=2 events, Pix_C=2 events, and so on. This makes the equation too complicated, so I will use the correction factor of "1/3" as a makeshift approximation. It is not the case when Pix_C receives 255LSBs. Pix_B loses the count always independent of its brightness. If Pix_B is stolen by Pix_A as well, this is 100% over subtraction. Therefore, the overlapping by Pix_C=255LSB and brighter Pix_A is described simply by "Pa·Pc_sat".
Taking all of the above into account, the detection probability at Pix_B is

$$[1 - \exp(-p2)] - [\, Pa + Pc + \{1 - \exp(-p2)\} \cdot Pc\_sat - Pa \cdot Pc\_sat - Pa \perp Pc - 1/3\ (?)\ Pa \perp Pc \,]$$

where,

$$Pa \perp Pc = \left[\left\{ p2 \cdot \exp(-p2) \right\} \right] \cdot Pa\_1 \cdot Pc\_1 + \left[\left\{ \frac{p2^2}{2!} \cdot \exp(-p2) \right\} \right] \cdot Pa\_2 \cdot Pc\_2$$

$$+ \left[\left\{ \frac{p2^3}{3!} \cdot \exp(-p2) \right\} \right] \cdot Pa\_3 \cdot Pc\_3 + \left[\left\{ \frac{p2^4}{4!} \cdot \exp(-p2) \right\} \right] \cdot Pa\_4 \cdot Pc\_4$$

(D.14)

Section 4D. 2. Coincidence correction curve (1-dimension)
with real PDH and real event profile

At last, I will introduce all of key parameters. It is sad, but some of key parameters cannot be involved in equations explicitly.
The event width captured by CCD camera was assumed to be very small in the above. The real event profile, however, extends up to 1.1 CCD_pixels (FWHM). It is too complicated for an analytic equation to describe re-positioning of coincidence events, which must include gradient of PSF within a CCD pixel, pulse height distribution and event profile. Fig.16 shows an example of star image located at CCD pixel centre. Events falling in subpix-1 of Pix_C are not effective for re-position from Pix_B. If an event falling in the subpix-1 generates a brightness of 80 LSBs in Pix_C, it will give 68LSBs to Pix_B as well. While a event falling in the subpix-4 would generate 94 LSBs in Pix_C but would give only 37 LSBs to Pix_B. Therefore, events falling in the right half of Pix_C are more effective to steal the count from Pix_B. Since the centre of the PSF is located at Pix_B, majority of evens fall in the subpix_1 among Pix_C-subpixels. The terms involving "p3" (=n3/N) in Eq(D.10) must be reduced by significant factor. An event falling in subpix-8 of Pix_B is similarly ineffective for re-positioning from Pix_C. Majority of evens inside Pix_B are, however, located between subpix-3 to subpix-6, therefore the terms involving "p2" (=n2/N) are not necessarily to be reduced by a large factor.
The event falling in subpixel-8 of Pix_B has different role. It can add brightness to Pix_C. As an extreme example, 5 events in subpixel-8 of Pix_B would give 255 LSBs to Pix_C, hence Pix_B can lose the count without event in Pix_C itself. "Pc_sat" in Eq(D.11) will be more the dominant term with the real event profile.

In order to see dominant terms and minor terms, I will derive a linearity curve for one example;
a) PSF is FWHM=40um due to the photocathode gap effect
b) A star is located at the center of Pix_B
c) Pulse height distribution and event profile are from DEP_#8 intensifier

The FWHM=40um is the maximum width among all wavelengths (at ~3000A). The PSF was calculated in same manner as Section 3. I.e. The profile is basically (Lorentzan)^4, but is multiplied by "cos(r)" at the wing, so that it tends to zero beyond the energy limit (~50um). There are so many star positions, but I chose the centre for this example (see Fig. 16).
Since the event has spread, pulse height distribution changes along CCD Pixel. When events fall in the subpixel-2 of Pix_C, significant portion (~67%) of event energy goes to Pix_B (see Fig. 17). The brightness ratio is even closer when events fall in subpixle-1, while the ratio is larger when events fall in subpixels -3 -4 -5. Table 10 shows the peak value of PHDs for individual subpixels. If input light source is F-F, pulse height distribution at Pix_B is derived by adding the 8 PHDs with equal weight, then the peak=90 LSBs will be obtained. Now, we are looking at a star image, locating at the CCD pixel centre. Since subpixel-4 and -5 have the larger weights than subpixel-1 and -8, peak of the PHD for Pix_B is a little larger than 90LSBs. We can also make PHD at Pix_C against the leak energy of events. Peak of the PHD is 20 LSBs (see Fig. 18). Table 11. shows the peak values of PHDs when events fall one of subpixels of Pix_C. Since majority of star light fall in subpix_1, most of events spread the energy to Pix_B. In the consequence, peak of the PHD is 62 LSBs for Pix_B and the peak=84 LSBs for Pix_C (see Fig. 19).

This table must be derived by REAL measurements, later on

Table 10. Peak of pulse height distributions when events fall in subpixels of Pix_B

	Pix_A	Pix_B	Pix_C
Supixel-1	68	80	0
Supixel-2	58	87	7
Supixel-3	47	91	17
Supixel-4	37	94	27
Supixel-5	27	92	37
Supixel-6	17	88	47
Supixel-7	7	83	58
Supixel-8	0	80	68
Average for F-F input	----	90	----

This table must be derived by REAL measurements, later on

Table 11. Peak of pulse height distributions when events fall in subpixels of Pix_C

		Pix_B	Pix_C
Supixel-1		68	80
Supixel-2		58	87
Supixel-3		47	91
Supixel-4		37	94
Supixel-5		27	92
Supixel-6		17	88
Supixel-7		7	83
Supixel-8		0	80
Average for F-F input	----	----	90

Fig. 20 shows pulse height distributions for Pix_B when multiple events fall in Pix_B, while Fig. 21 for Pix_C when multiple events fall in Pix_C. Because of the localized illumination, PHD for Pix_B is a little brighter than that of Pix_C but not much difference. In both cases, CCD pixels are saturated if more than 4 events arrive in one frame. Fig. 22 shows pulse height distributions seen at Pix_B when multiple events fall in Pix_C, while Fig. 23 is seen at Pix_C when multiple events fall in Pix_B. The difference due to the localized illumination is more significant between these two PHDs. Pix_B is saturated if more than 6 events arrive in Pix_C without any event in Pix_B itself, while Pix_C needs more than 10 events at Pix_B to be saturated. The latter is, however, dominant mechanism to steal the count from Pix_B at high count rate. Even if Pix_C has a few events, it can be saturated with the help of Pix_B. Table 12 shows the probabilities of saturation at Pix_C when both of Pix_C and Pix_B received a few events. For instance, if two events arrive at Pix_C and 4 events at Pix_B, Pix_C is saturated with 80% probability. It is expected that far more events arrive at Pix_B. Therefore, it is rare chance for Pix_C to be brighter than Pix_B. But, Pix_C still can steal the count by this saturation mechanism when count rate is high.

Table 12. Probability of Pix_C >255 LSBs ((real event profile))

Pix_B:	0-	1-	2-	3-	4-	5-	6-	7-	8-	9-	10-events		
Pix_C;													
0 event	.000	.000	.000	.002	.019	.082	.218	.416	.625	.794	.903	.967	.993
1 event	.001	.006	.039	.136	.310	.524	.719	.858	.938	.977	.992	1.00	1.00
2 events	.073	.211	.416	.629	.799	.907	.963	.987	.996	.999	1.00	1.00	1.00
3 events	.526	.724	.863	.941	.978	.993	.998	1.00	1.00	1.00	1.00	1.00	1.00
4 events	.911	.965	.988	.996	.999	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
5 events	.994	.998	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
6 events	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
7 events	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
8 events	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 13. Probability of Pix_C >255 LSBs ((real event profile))

Pix_A+C:	0-	1-	2-	3-	4-	5-	6-	7-	8-	9-	10-events
Pix_B;											
0 event	.000	.000	.005	.122	.521	.871	.983	.999	1.00	1.00	1.00
1 event	.001	.035	.275	.687	.931	.992	1.00	1.00	1.00	1.00	1.00
2 events	.116	.459	.815	.967	.997	1.00	1.00	1.00	1.00	1.00	1.00
3 events	.630	.898	.985	.999	1.00	1.00	1.00	1.00	1.00	1.00	1.00
4 events	.947	.993	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
5 events	.997	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
6 events	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
7 events	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

With >90% probability, Pix_C automatically steals the count due to the saturation when Pix_C receives more than 4 events, or Pix_B receives more than 10 events.

$$Pc\_4 = 1 - \exp(-p3) \cdot \left(1 + p3 + \frac{p3^2}{2\,!} + \frac{p3^3}{3\,!} \right) \tag{D.15}$$

$$Pb\_10 = 1 - \exp(-p2) \cdot \left(1 + p2 + \frac{p2^2}{2\,!} + \frac{p2^3}{3\,!} + \frac{p2^4}{4\,!} + \frac{p2^5}{5\,!} + \frac{p2^6}{6\,!} + \frac{p2^7}{7\,!} + \frac{p2^8}{8\,!} + \frac{p2^9}{9\,!} \right\} \tag{D.16}$$

Pc_4 plus Pb_10 contain overlapping, "Pc_4 x Pb_10". Subtracting the overlapping and adding remaining non-zero terms in Table 12, probability for stealing count by the saturation,

$$
\begin{aligned}
\mathrm{Pc\_sat} = {} & [1 - \exp(-p2)] \cdot \mathrm{Pc\_4} + \mathrm{Pb\_10} - \mathrm{Pc\_4} \cdot \mathrm{Pb\_10} \\
& + [\exp(-p2) \cdot \exp(-p3)] \times \\
& [\; \{ \; 0.07 \frac{p3^2}{2!} + 0.53 \frac{p3^3}{3!} \} \\
& + p2 \cdot \{ 0.21 \frac{p3^2}{2!} + 0.72 \frac{p3^3}{3!} \} \\
& + \frac{p2^2}{2!} \cdot \{ 0.04\, p3 + 0.42 \frac{p3^2}{2!} + 0.86 \frac{p3^3}{3!} \} \\
& + \frac{p2^3}{3!} \cdot \{ 0.14\, p3 + 0.63 \frac{p3^2}{2!} + 0.94 \frac{p3^3}{3!} \} \\
& + \frac{p2^4}{4!} \cdot \{ 0.02 + 0.31\, p3 + 0.80 \frac{p3^2}{2!} + 0.98 \frac{p3^3}{3!} \} \\
& + \frac{p2^5}{5!} \cdot \{ 0.08 + 0.52\, p3 + 0.91 \frac{p3^2}{2!} + 0.99 \frac{p3^3}{3!} \} \\
& + \frac{p2^6}{6!} \cdot \{ 0.22 + 0.72\, p3 + 0.96 \frac{p3^2}{2!} + 1.00 \frac{p3^3}{3!} \} \\
& + \frac{p2^7}{7!} \cdot \{ 0.42 + 0.86\, p3 + 0.99 \frac{p3^2}{2!} + 1.00 \frac{p3^3}{3!} \} \\
& + \frac{p2^8}{8!} \cdot \{ 0.63 + 0.94\, p3 + 1.00 \frac{p3^2}{2!} + 1.00 \frac{p3^3}{3!} \} \\
& + \frac{p2^9}{9!} \cdot \{ 0.79 + 0.98\, p3 + 1.00 \frac{p3^2}{2!} + 1.00 \frac{p3^3}{3!} \} \;]
\end{aligned}
\tag{D.17}
$$

Pix_B is saturated more often, because of the higher input rate and the larger leak of event energy from both of Pix_A and Pix_C. Table 13 shows the probabilities of Pix_B saturation. For instance, if one event arrives at each of Pix_A and Pix_C and 4 events at Pix_B, Pix_B is saturated with ~100% probability. It is more difficult for Pix_A to steal the count, because of the saturation of Pix_B.
With >90% probability, the saturation of Pix_B automatically prevents the steal by Pix_A when Pix_A plus Pix_C receive more than 6 events or Pix_B itself receives more than 4 events. The count does not go to Pix_B when Pix_C is saturated, but the saturation of Pix_B is narrowing the chance for Pix_A.

$$Pb\_4 = 1 - \exp(-p2)\cdot\{ 1 + p2 \frac{p2^2}{2!} + \frac{p2^3}{3!} \} \tag{D.18}$$

$$Pac\_6 = 1 - \exp(-p1-p3)\cdot\{ 1 + (p1+p3) + \frac{(p1+p3)^2}{2!} + \frac{(p1+p3)^3}{3!} + \frac{(p1+p3)^4}{4!} + \frac{(p1+p3)^5}{5!} \} \tag{D.19}$$

$$\begin{aligned}
Pb_sat = {} & Pb_4 + Pac_6 - Pb_4 \cdot Pac_6 \\
& + [\exp(-p1-p3) \cdot \exp(-p2)] \times \\
& [\quad \{ \quad 0.12 \frac{p2^2}{2!} + 0.63 \frac{p2^3}{3!} \} \\
& + (p1+p3) \cdot \{ 0.04\, p2 + 0.46 \frac{p2^2}{2!} + 0.90 \frac{p2^3}{3!} \} \\
& + \frac{(-p1-p3)^2}{2!} \cdot \{ 0.01 + 0.28\, p2 + 0.82 \frac{p2^2}{2!} + 0.99 \frac{p2^3}{3!} \} \\
& + \frac{(-p1-p3)^3}{3!} \cdot \{ 0.12 + 0.69\, p2 + 0.97 \frac{p2^2}{2!} + 1.00 \frac{p2^3}{3!} \} \\
& + \frac{(-p1-p3)^4}{4!} \cdot \{ 0.52 + 0.93\, p2 + 1.00 \frac{p2^2}{2!} + 1.00 \frac{p2^3}{3!} \} \\
& + \frac{(-p1-p3)^5}{5!} \cdot \{ 0.87 + 0.99\, p2 + 1.00 \frac{p2^2}{2!} + 1.00 \frac{p2^3}{3!} \} \ ]
\end{aligned} \tag{D.20}$$

Table 14a. Brightness competition Pix_C > Pix_B ((finite event width))
Non-saturation Pix_A = 0 event

Pix_B:	1-	2-	3-	4-	5-	6-	7-events
Pix_C;							
1 event	.041	.000	.000	.000	.000	.000	.000
2 events	.173	.002	.000	.000	.000	.000	.000
3 events	.118	.002	.000	.000	.000	.000	.000
4 events	.021	.000	.000	.000	.000	.000	.000
5 events	.001	.000	.000	.000	.000	.000	.000

Table 14b. Brightness competition Pix_C > Pix_B ((finite event width))
Non-saturation Pix_A = 1 event

Pix_B:	1-	2-	3-	4-	5-	6-	7-events
Pix_C;							
1 event	.0005	.000	.000	.000	.000	.000	.000
2 events	.0073	.002	.000	.000	.000	.000	.000
3 events	.0069	.002	.000	.000	.000	.000	.000
4 events	.0013	.000	.000	.000	.000	.000	.000
5 events	.0001	.000	.000	.000	.000	.000	.000

Table 15. Brightness competition Pix_A > Pix_B ((finite event width))
Non-saturation Pix_C = 0 event

Pix_B:	1-	2-	3-	4-	5-	6-	7-events
Pix_A;							
1 event	.040	.000	.000	.000	.000	.000	.000
2 events	.132	.001	.000	.000	.000	.000	.000
3 events	.059	.000	.000	.000	.000	.000	.000
4 events	.007	.000	.000	.000	.000	.000	.000
5 events	.000	.000	.000	.000	.000	.000	.000

Is there any chance for Pix_C to compete with Pix_B in terms of brightness in non-saturated regime ? Since incoming photon rate to Pix_B is 25 times as high as to Pix_C, Pix_B is almost always brighter. Pix_C has a small chance only at low count rate with the help of occasional big events.
Gradient of PSF within Pix_C makes the competition more difficult for Pix_C, since majority of events fall near the boundary of Pix_B. Fig.18 shows Pix_C receives 22% of event energy in average when the event falls in Pix_B, while Fig.19 shows Pix_C gives 73% of event energy in average when the event falls in Pix_C itself. These cross-talk values vary around the average depending on the event position within the CCD pixels. For the following calculation, I will

introduce a radical assumption that an event falls in Pix_C gives fixed ratio of 73% of event energy to Pix_B, and an event falls in Pix_B gives fixed ratio of 22% to Pix_C. Net brightness difference is only 27% for Pix_C and 78% for Pix_B. Comparing pulse height distributions in Figs. 20 and 21 as well as the net brightness differences, winning chance for Pix_C was calculated in various event combinations. The results are tabulated in Table 14a. They do not include the case for Pix_C > 255LSBs. If Pix_B receives more than 2 events, Pix_C has no chance to win. Pix_A also supports Pix_B for the competition between Pix_B and Pix_C, since 73% of energy is transferred to Pix_B. Table 14b shows winning chance for Pix_C when Pix_A receives one events. Pix_C has not chance more than 1%. Therefore, Pix_A must receive zero event. The same calculation for Pix_A vs. Pix_B is tabulated in Table 15. Since the saturation of Pix_B happens more easily, the probabilities are slightly lower. The maximum probability happens at 2 events for Pix_A vs. 1 event for Pix_B. Again, Pix_C must be zero event.

$$Pc = [\exp(-p1)] \cdot [\{p2 \cdot \exp(-p2)\}] \times [\exp(-p3) \cdot \{0.04\, p3 + 0.17 \frac{p3^2}{2!} + 0.12 \cdot \frac{p3^3}{3!} + 0.02 \cdot \frac{p3^4}{4!}\}] \qquad \text{(D.21)}$$

$$Pa = [\exp(-p3)] \cdot [\{p2 \cdot \exp(-p2)\}] \times [\exp(-p1) \cdot \{0.04\, p1 + 0.13 \frac{p1^2}{2!} + 0.06 \cdot \frac{p1^3}{3!} + 0.00 \cdot \frac{p1^4}{4!}\}] \qquad \text{(D.22)}$$

Table 16a. Brightness competition Pix_C > Pix_B, when PSF at Pix_C is flat
Non-saturation Pix_A = 0 event

Pix_B:	1-	2-	3-	4-	5-	6-	7-events
Pix_C;							
1 event	.501	.076	.005	.000	.000	.000	.000
2 events	.724	.269	.032	.001	.000	.000	.000
3 events	.250	.100	.013	.001	.000	.000	.000
4 events	.026	.009	.001	.000	.000	.000	.000
5 events	.001	.000	.000	.000	.000	.000	.000

Table 16b. Brightness competition Pix_C > Pix_B, when PSF at Pix_C is flat
Non-saturation Pix_A = 1 event

Pix_B:	1-	2-	3-	4-	5-	6-	7-events
Pix_C;							
1 event	.1420	.011	.000	.000	.000	.000	.000
2 events	.4076	.064	.003	.000	.000	.000	.000
3 events	.1465	.026	.001	.000	.000	.000	.000
4 events	.0135	.002	.000	.000	.000	.000	.000
5 events	.0004	.000	.000	.000	.000	.000	.000

We saw significant role of the gradient of PSF within Pix_C in the above. I assumed the light came from the star only, neglecting sky background. The main reason for the reduction of sampling window to 1 CCD pixel is to minimize the disturbance from background or nearby stars. Therefore, it may be more usual that light to Pix_A and Pix_C is mainly from other sources. In such case, the gradients within Pix_C and Pix_A are small. I artificially flattened the PSF within Pix_C and Pix_A and carried out same calculations. With this new PSF, an event falls in Pix_C gives only 22% of energy to Pix_B, and an event falls in Pix_B gives the same energy (22%) to Pix_C. PHD for Pix_C when events fall in Pix_C becomes brighter but only by a few %, therefore the probability of Pix_C saturation does not increase much. The net brightness difference jumps up to 78% from 27%, therefore the winning chance of Pix_C by brightness competition increases significantly. The results are tabulated in Tables 16a and 16b. In this time, we cannot ignore the case when Pix_A received one event.

$$
\begin{aligned}
Pc = {} & [\exp(-p1)] \cdot [\exp(-p2)] \cdot [\exp(-p3)] \times \\
& \Big[\{p2\} \cdot \Big\{ 0.50\,p3 + 0.72 \frac{p3^2}{2\,!} + 0.25 \cdot \frac{p3^3}{3\,!} + 0.03 \cdot \frac{p3^4}{4\,!} \Big\} \\
& + \Big\{ \frac{p2^2}{2\,!} \Big\} \cdot \Big\{ 0.08\,p3 + 0.27 \frac{p3^2}{2\,!} + 0.10 \cdot \frac{p3^3}{3\,!} + 0.01 \cdot \frac{p3^4}{4\,!} \Big\} \\
& + \Big\{ \frac{p2^3}{3\,!} \Big\} \cdot \Big\{ 0.01\,p3 + 0.03 \frac{p3^2}{2\,!} + 0.01 \cdot \frac{p3^3}{3\,!} + 0.00 \cdot \frac{p3^4}{4\,!} \Big\} \Big] \\
& + [p1 \cdot \exp(-p1)] \cdot [\exp(-p2)] \cdot [\exp(-p3)] \times \\
& \Big[\{p2\} \cdot \Big\{ 0.14\,p3 + 0.41 \frac{p3^2}{2\,!} + 0.15 \cdot \frac{p3^3}{3\,!} + 0.01 \cdot \frac{p3^4}{4\,!} \Big\} \\
& + \Big\{ \frac{p2^2}{2\,!} \Big\} \cdot \Big\{ 0.01\,p3 + 0.06 \frac{p3^2}{2\,!} + 0.03 \cdot \frac{p3^3}{3\,!} + 0.00 \cdot \frac{p3^4}{4\,!} \Big\}
\end{aligned}
\qquad (D.23)
$$

Probabilities of a)more than 4 events in Pix_C, b)more than 10 events in Pix_B, c)saturation of Pix_C by modest events in Pix_B and Pix_C, d)Pix_C is brighter than Pix_B with PSF by star and e)Pix_C is brighter than Pix_B with flat PSF, were calculated against input rate between 0-1000% of frame rate. I assumed sky background of 3 counts/CCD_pix/frame, which corresponds to 200kc/sec/full area with 256x256 CCD readout format. Fig. 24 shows the expected count rate seen at Pix_B. The 0-dimension equation looks good approximation up to ~150% of frame rate. Fig. 25 shows individual components. The probability of (a)>4 events at Pix_C is lower than 0.1% at all input rate. The probability of (e)brighter Pix_C with PSF by star is also lower than 0.1%. Flat PSF at Pix_C would not happen by sky background at high star count rate, therefore the digital saturation of Pix_C is the only mechanism to steal the count from Pix_B. We can ignore the role of Pix_A.

Section 4E. Coincidence correction curve (2-dimension)
with real PDH and real event width

Finally, equation applicable to the practical situation will be provided here. We consider the case, in which star image is spread among 3x3 CCD pixel array, but we sample the counts from the central CCD pixel only. Introducing the parameters described in Table 2,
We assume that "n11" events arrived at Pix_AA during "N" frames,
"n12" events at Pix_AB, "n13" events at Pix_AC,
"n21" events at Pix_BA, "n22" events at Pix_BB, "n23" events at Pix_BC,
"n31" events at Pix_CA, "n32" events at Pix_CB, "n33" events at Pix_CC.

Table 17. Photo-events configuration and counts by system

```
x  x  o
x  O  o    (1)
x  o  o
                 o: one of pixels is brighter than Pix_BB
                 x: fainter than Pix_BB

o  o  sX
o  O  sX  (2)
o  sX sX
                 sX: Not saturation

nc nc  s
nc O   s    (3)
nc s   s
(Digital saturation)  s: one of pixels got 255LSBs
```

Focusing on a particular frame, we consider whether the count is stolen from Pix_BB by surrounding 8 CCD pixels or not. I will derive a linearity curve for the same example as Section 4D.2, i.e.;
a) PSF is FWHM=40um due to the photocathode gap effect
b) A star is located at the center of Pix_BB
c) Pulse height distribution and event profile are from DEP_#8 intensifier

First of all, we consider the cases(1) and (2) of Table 17. Since most of photo-events fall in the central pixel, Pix_BB, and PSF has steep gradients at the surrounding 8 CCD pixels, there is not more than 0.1% probability for Pix_B to lose event by brightness competition as investigated in the previous section. The probability for Pix_BC to be brighter than Pix_BB is basically same as Eq (D.21). Since closest neighbours, Pix_AB, Pix_BA and Pix_CB, transfer significant portion of event energy to Pix_BB, they have to have zero events. Since the distances of Pix_AA, Pix_AC, Pix_CC and Pix_CA to the centre of PSF is longer by factor 1/SQRT(2), interactions to Pix_BB are far smaller hence their contributions can be ignored.

$$Pbc = [\exp(-p12-p21-p32)] \cdot [\{p22 \cdot \exp(-p22)\}] \times$$

$$[\exp(-p23) \cdot \{0.04\, p23 + 0.17 \cdot \frac{p23^2}{2!} + 0.12 \cdot \frac{p23^3}{3!} + 0.02 \cdot \frac{p23^4}{4!}\}] \quad (E.1)$$

The probability for Pix_CB is almost same as for Pix_BC, depending on the orientation of ellipse of event profile. If major axis is along X-direction, it is a little larger than that for Pix_BC. The probability for Pix_BA is basically same as Eq(D.22). The probability for Pix_AB is almost same as for Pix_BA, depending on the orientation of ellipse of event profile. If major axis is along X-direction, it is a little larger than that for Pix_BA. Because the longer distance from PSF centre, probabilities for Pix_AA, Pix_AC, Pix_CC and Pix_CA are significantly smaller than that for Pix_BC.

It was turned out that digital saturation is the dominant mechanism to steal the count from the central CCD pixel as seen in the previous section (see case(3) in Table 17). The probability for Pix_BC to be saturated is basically same as Eq(D.21). It turned out probability of Pix_BC to have more than 4 events is smaller than 0.1% at all count rates, but I leave the term Pbc_4 here. Neighbouring pixels Pix_AC and Pix_CC can transfer energies to Pix_BC, but they needs to be more than 10 events to create Pix_BC saturation by themselves own. The probability is almost zero, hence ignored.

$$Pbc\_4 = 1 - \exp(-p23) \cdot (1 + p23 + \frac{p23^2}{2!} + \frac{p23^3}{3!}) \quad (E.2)$$

$$Pbb\_10 = 1 - \exp(-p22) \times$$

$$
\left(1 + p22 + \frac{p22^2}{2!} + \frac{p22^3}{3!} + \frac{p22^4}{4!} + \frac{p22^5}{5!} + \frac{p22^6}{6!} + \frac{p22^7}{7!} + \frac{p22^8}{8!} + \frac{p22^9}{9!} \right\} \tag{E.3}
$$

Pbc_mod = [exp(-p22) · exp(-p23)] x

$$
\begin{aligned}
\Big[\quad & \Big\{ \quad 0.07 \frac{p23^2}{2!} + 0.53 \frac{p23^3}{3!} \Big\} \\
+ \; & p22 \cdot \Big\{ \quad 0.21 \frac{p23^2}{2!} + 0.72 \frac{p23^3}{3!} \Big\} \\
+ \; & \frac{p22^2}{2!} \cdot \Big\{ \quad 0.04\, p23 + 0.42 \frac{p23^2}{2!} + 0.86 \frac{p23^3}{3!} \Big\} \\
+ \; & \frac{p22^3}{3!} \cdot \Big\{ \quad 0.14\, p23 + 0.63 \frac{p23^2}{2!} + 0.94 \frac{p23^3}{3!} \Big\} \\
+ \; & \frac{p22^4}{4!} \cdot \Big\{ 0.02 + 0.31\, p23 + 0.80 \frac{p23^2}{2!} + 0.98 \frac{p23^3}{3!} \Big\} \\
+ \; & \frac{p22^5}{5!} \cdot \Big\{ 0.08 + 0.52\, p23 + 0.91 \frac{p23^2}{2!} + 0.99 \frac{p23^3}{3!} \Big\} \\
+ \; & \frac{p22^6}{6!} \cdot \Big\{ 0.22 + 0.72\, p23 + 0.96 \frac{p23^2}{2!} + 1.00 \frac{p23^3}{3!} \Big\} \\
+ \; & \frac{p22^7}{7!} \cdot \Big\{ 0.42 + 0.86\, p23 + 0.99 \frac{p23^2}{2!} + 1.00 \frac{p23^3}{3!} \Big\} \\
+ \; & \frac{p22^8}{8!} \cdot \Big\{ 0.63 + 0.94\, p23 + 1.00 \frac{p23^2}{2!} + 1.00 \frac{p23^3}{3!} \Big\} \\
+ \; & \frac{p22^9}{9!} \cdot \Big\{ 0.79 + 0.98\, p23 + 1.00 \frac{p23^2}{2!} + 1.00 \frac{p23^3}{3!} \Big\} \Big]
\end{aligned} \tag{E.4}
$$

$$Pbc\_sat = [1 - \exp(-p22)] \cdot Pbc\_4 + Pbb\_10 - Pbc\_4 \cdot Pbb\_10 + Pbc\_mod \tag{E.5}$$

The probability for Pix_CB is almost same as for Pix_BC, depending on the orientation of ellipse of event profile. If major axis is along X-direction, it is a little smaller than that for Pix_BC, since the transfer from Pix_BB reduces. Assuming the event profile round,

$$Pcb\_4 = 1 - \exp(-p32) \cdot \left(1 + p32 + \frac{p23\verb|^|2}{2\,!} + \frac{p23\verb|^|3}{3\,!} \right) \tag{E.6}$$

$$Pcb\_mod = [\exp(-p22) \cdot \exp(-p32)] \ \mathrm{x}$$

$$\begin{aligned}
[\quad & \{ \quad 0.07 \frac{p32\verb|^|2}{2\,!} + 0.53 \frac{p32\verb|^|3}{3\,!} \} \\
& + p22 \cdot \{ \quad 0.21 \frac{p32\verb|^|2}{2\,!} + 0.72 \frac{p32\verb|^|3}{3\,!} \} \\
& + \frac{p22\verb|^|2}{2\,!} \cdot \{ \quad 0.04\, p32 + 0.42 \frac{p32\verb|^|2}{2\,!} + 0.86 \frac{p32\verb|^|3}{3\,!} \} \\
& + \frac{p22\verb|^|3}{3\,!} \cdot \{ \quad 0.14\, p32 + 0.63 \frac{p32\verb|^|2}{2\,!} + 0.94 \frac{p32\verb|^|3}{3\,!} \} \\
& + \frac{p22\verb|^|4}{4\,!} \cdot \{ 0.02 + 0.31\, p32 + 0.80 \frac{p32\verb|^|2}{2\,!} + 0.98 \frac{p32\verb|^|3}{3\,!} \} \\
& + \frac{p22\verb|^|5}{5\,!} \cdot \{ 0.08 + 0.52\, p32 + 0.91 \frac{p32\verb|^|2}{2\,!} + 0.99 \frac{p32\verb|^|3}{3\,!} \} \\
& + \frac{p22\verb|^|6}{6\,!} \cdot \{ 0.22 + 0.72\, p32 + 0.96 \frac{p32\verb|^|2}{2\,!} + 1.00 \frac{p32\verb|^|3}{3\,!} \} \\
& + \frac{p22\verb|^|7}{7\,!} \cdot \{ 0.42 + 0.86\, p32 + 0.99 \frac{p32\verb|^|2}{2\,!} + 1.00 \frac{p32\verb|^|3}{3\,!} \}
\end{aligned}$$

$$+\frac{p22^8}{8!}\cdot\{0.63+0.94\,p32+1.00\,\frac{p32^2}{2!}+1.00\,\frac{p32^3}{3!}\}$$

$$+\frac{p22^9}{9!}\cdot\{0.79+0.98\,p32+1.00\,\frac{p32^2}{2!}+1.00\,\frac{p32^3}{3!}\}\,]$$

(E.7)

$$Pcb\_sat = [1 - \exp(-p22)\,]\cdot Pcb\_4 + Pbb\_10 - Pcb\_4\cdot Pbb\_10 + Pcb\_mod$$

(E.8)

There is overlapping if both of Pix_BC and Pix_CB get saturation. Adding all and subtracting overlapping, the expected count at Pix_BB is

$$[1 - \exp(-p22)\,]\cdot(1.0 - Pbc\_4 - Pcb\_4) - Pbb\_10 - (Pbc\_4 + Pcb\_4)\cdot Pbb\_10$$
$$-(Pbc\_mod + Pcb\_mod) + Pbc\_mod \perp Pcb\_mod$$

where,

$$Pbc\_mod \perp Pcb\_mod = [\,\exp(-p22)\cdot\exp(-p23)\cdot\exp(-p32)\,]\ x$$

$$[\quad\{\quad 0.07\,\frac{p23^2}{2!}+0.53\,\frac{p23^3}{3!}\}$$
$$x\ \{\quad 0.07\,\frac{p32^2}{2!}+0.53\,\frac{p32^3}{3!}\}$$

$$+\ p22\cdot\{\quad 0.21\,\frac{p23^2}{2!}+0.72\,\frac{p23^3}{3!}\}$$
$$x\ \{\quad 0.21\,\frac{p32^2}{2!}+0.72\,\frac{p32^3}{3!}\}$$

$$+\frac{p22^2}{2!}\cdot\{\quad 0.04\,p23+0.42\,\frac{p23^2}{2!}+0.86\,\frac{p23^3}{3!}\}$$
$$x\ \{\quad 0.04\,p32+0.42\,\frac{p32^2}{2!}+0.86\,\frac{p32^3}{3!}\}$$

$$+\frac{p22^3}{3!}\cdot\{\quad 0.14\,p23+0.63\,\frac{p23^2}{2!}+0.94\,\frac{p23^3}{3!}\}$$

$$\times \left\{ \quad 0.14\, p32 + 0.63 \frac{p32^2}{2!} + 0.94 \frac{p32^3}{3!} \right\}$$

$$+ \frac{p22^4}{4!} \cdot \left\{ 0.02 + 0.31\, p23 + 0.80 \frac{p23^2}{2!} + 0.98 \frac{p23^3}{3!} \right\}$$
$$\times \left\{ 0.02 + 0.31\, p32 + 0.80 \frac{p32^2}{2!} + 0.98 \frac{p32^3}{3!} \right\}$$

$$+ \frac{p22^5}{5!} \cdot \left\{ 0.08 + 0.52\, p23 + 0.91 \frac{p23^2}{2!} + 0.99 \frac{p23^3}{3!} \right\}$$
$$\times \left\{ 0.08 + 0.52\, p32 + 0.91 \frac{p32^2}{2!} + 0.99 \frac{p32^3}{3!} \right\}$$

$$+ \frac{p22^6}{6!} \cdot \left\{ 0.22 + 0.72\, p23 + 0.96 \frac{p23^2}{2!} + 1.00 \frac{p23^3}{3!} \right\}$$
$$\times \left\{ 0.22 + 0.72\, p32 + 0.96 \frac{p32^2}{2!} + 1.00 \frac{p32^3}{3!} \right\}$$

$$+ \frac{p22^7}{7!} \cdot \left\{ 0.42 + 0.86\, p23 + 0.99 \frac{p23^2}{2!} + 1.00 \frac{p23^3}{3!} \right\}$$
$$\times \left\{ 0.42 + 0.86\, p32 + 0.99 \frac{p32^2}{2!} + 1.00 \frac{p32^3}{3!} \right\}$$

$$+ \frac{p22^8}{8!} \cdot \left\{ 0.63 + 0.94\, p23 + 1.00 \frac{p23^2}{2!} + 1.00 \frac{p23^3}{3!} \right\}$$
$$\times \left\{ 0.63 + 0.94\, p32 + 1.00 \frac{p32^2}{2!} + 1.00 \frac{p32^3}{3!} \right\}$$

$$+ \frac{p22^9}{9!} \cdot \left\{ 0.79 + 0.98\, p23 + 1.00 \frac{p23^2}{2!} + 1.00 \frac{p23^3}{3!} \right\}$$
$$\times \left\{ 0.79 + 0.98\, p32 + 1.00 \frac{p32^2}{2!} + 1.00 \frac{p32^3}{3!} \right\} \;] \tag{E.9}$$

Since the distances of Pix_AC and Pix_CC are longer from PSF centre, probabilities of Pix_AA, Pix_AC and Pix_CC saturation are significantly smaller, hence ignored.
Probabilities of a)more than 10 events in Pix_BB, b)saturation of Pix_BC by modest events in Pix_B and Pix_C, and c)Pix_BC is brighter than Pix_BB with flat PSF, were calculated against input rate between 0-1000% of frame rate. I assumed sky background of 3 counts/CCD_pix/frame, which corresponds to 200kc/sec/full area with 256x256 CCD readout format. Fig. 26 shows the expected count rate seen at Pix_BB. The 0-dimension equation looks good approximation up to ~ 120% of frame rate. Fig. 25 shows individual components. The probability of (c)brighter Pix_CC with PSF by flat PSF is lower than that of 1-dim model.

5. Discussion

This section will be completed later on ~ May

Appendix. Point spread function of a proxy focused image intensifier

The XMM/Swift 's intensifier employs proximity focusing to achieve low distortion image in a compact and rugged structure. The resolution is highly depends on the photocathode gap "**h**", and the voltage applied to the gap "**Vc**" (see Fig. Ap1). When a photon with the energy of "**hν**" hits the photocathode, a photo-electron is emitted to the photocathode gap space. The electron displaces the position by "**ρ**" during the travel of the gap, depending on its transverse component of initial velocity. Since the electron is pull down by a constant acceleration of $(e \cdot Vc / m_e \cdot h)$, its trajectory becomes parabola, i.e. same as free fall under a constant gravity. Assuming "**t**" is time for traveling the photocathode gap,

$$h = 1/2\,(e \cdot Vc / m_e \cdot h) \cdot t^2 + (v_0 \cdot \cos\theta) \cdot t \qquad \text{(Ap.1)}$$

$$\rho = (v_0 \cdot \sin\theta) \cdot t \qquad \text{(Ap.2)}$$

Solving Eq (Ax.1),

$$t = \frac{-(v_0 \cdot \cos\theta) + \sqrt{(v_0 \cdot \cos\theta)^2 + 2\,(e \cdot Vc / m_e \cdot h) \cdot h}}{(e \cdot Vc / m_e \cdot h)}$$

Substituting to Eq (Ax.2),

$$\rho = h \cdot \frac{m_e}{eVc}\, v_0 \sin\theta \left[\sqrt{\frac{2\,eVc}{m_e} + (v_0 \cdot \cos\theta)^2} - (v_0 \cdot \cos\theta) \right]$$

$$= 2h \cdot \sqrt{\frac{1/2\, m_e v_0^2}{eVc}}\, \sin\theta \left[\sqrt{1 + \frac{1/2\, m_e v_0^2}{eVc} (\cos\theta)^2} - \sqrt{\frac{1/2\, m_e v_0^2}{eVc} (\cos\theta)^2} \right]$$

$$= 2h \cdot \sqrt{\frac{(K.E.)}{eVc}}\, \sin\theta \left[\sqrt{1 + \frac{(K.E.)}{eVc} (\cos\theta)^2} - \sqrt{\frac{(K.E.)}{eVc} (\cos\theta)^2} \right] \qquad \text{(Ap.3)}$$

For example, if applying 400V to the photocathode gap for 2480A input photon, $eVc = 400$ eV, and $h\nu(2480A) = 5$eV.

Since photo-electron loses energy corresponding to the surface potential of 1eV when escaping to the vacuum space,
$(K.E.) = h\nu - 1eV = 4eV$.

i.e. $$\frac{(K.E.)}{eVc} \sim 0.01 \quad .$$

Assuming $\frac{(K.E.)}{eVc} << 1$,

$$\rho = 2h \cdot \sqrt{\frac{(K.E.)}{eVc}} \sin\theta \left[1 - \left(\sqrt{\frac{(K.E.)}{eVc}} - \frac{1}{2} \frac{(K.E.)}{eVc} \cos\theta \right) \cos\theta \right.$$

$$\left. - \frac{1}{8} \left\{ \frac{(K.E.)}{eVc} (\cos\theta)^2 \right\}^2 + \frac{1}{16} \left\{ \frac{(K.E.)}{eVc} (\cos\theta)^2 \right\}^3 + \ldots\ldots \right]$$

Ignoring the accuracy of 0.001,

$$\rho \cong 2h \cdot \sqrt{\frac{(K.E.)}{eVc}} \sin\theta \left[1 - \left(\sqrt{\frac{(K.E.)}{eVc}} - \frac{1}{2} \frac{(K.E.)}{eVc} \cos\theta \right) \cos\theta \right] \qquad \text{(Ap.4)}$$

This shows that the maximum displacement can happen at θ=90 degrees.
I.e.,

$$\rho < 2h \cdot \sqrt{\frac{(K.E.)}{eVc}} \qquad \text{(Ap.5)}$$

Assuming the photocathode gap is 250µm and photocathode voltage 400V, the displacement of the 2480A photon must be smaller than 50µm. Hence, the photon spread due to the photocathode gap must be inside the circle of D=100µm (= 1.35 CCD pixel).

Eqs (Ax.3) and (Ax.4) give the displacement of a single photoelectron with a given escape velocity and angle. Assuming the angle distribution of the photo-electron is **f(θ)** per solid angle and surface density of its projection on the MCP top surface is **g(ρ)** as shown in Fig. Ap1. The angle **θ** and displacement **ρ** are related as

$$f(\theta) \sin\theta \cdot d\theta \, d\phi = g(\rho) \ \rho \cdot d\rho \, d\phi \qquad \text{(Ap.6)}$$

Ignoring 2nd and 3rd terms of Eq (Ax.4), i.e. in the accuracy of 10%,

$$\rho \sim 2h \cdot \sqrt{\frac{(K.E.)}{eVc}} \sin\theta$$

If escape velocity of photo-electron has not dependence on escape angle,

$$d\rho \sim 2h \cdot \sqrt{\frac{(K.E.)}{eVc}} \cos\theta \cdot d\theta$$

Substituting these into Eq (Ax.6),

$$g(\rho) = \frac{f(\theta)}{\cos\theta} \cdot \frac{1}{(2h)^2 \cdot [\, (K.E.) / eVc \,]}$$

where,

$$0 < \rho < 2h \cdot \sqrt{\frac{(K.E.)}{eVc}}$$

It is widely believed that the angle distribution of photo-electron emission is *Lambertion*. Then,

$$g(\rho) = \frac{1}{(2h)^2 \cdot [\, (K.E.) / eVc \,]} \qquad \text{(Ap.7)}$$

There is no explicit dependence on ρ the above equation. If the (K.E.) is really constant, the PSF will be a flat circle with the diameter of 100μm the 2480A photons. In the real image, PSF is much narrower (D~ 40μm) and concentrated. This is probably due to slower escape velocity at grazing angle or the angle distribution is not as bad as Lambertion.

////////////////////////// Determining Photo-electron distribution //////////////////////////////

Following Eberhardt (1977), Eq (Ax.4) is described with other parameters, (K.E.)// and (K.E.)⊥ .

$$\rho \cong 2h \cdot \sqrt{\frac{(K.E.)\perp}{eVc}} \left[\, 1 - \sqrt{\frac{(K.E.)//}{eVc}} + \frac{1}{2} \cdot \frac{(K.E.)//}{eVc} \,\right] \qquad \text{(Ap.8)}$$

$$d\rho \cong h \cdot \sqrt{\frac{eVc}{(K.E.)\perp}} \left[\, 1 - \sqrt{\frac{(K.E.)//}{eVc}} + \frac{1}{2} \cdot \frac{(K.E.)//}{eVc} \,\right] \frac{d\,(K.E.)\perp}{eVc}$$

$$+ h \cdot \sqrt{\frac{(K.E.)\perp}{eVc}} \left[\, - \sqrt{\frac{eVc}{(K.E.)//}} + 1 \,\right] \frac{d\,(K.E.)//}{eVc} \qquad \text{(Ap.9)}$$

Assuming the kinetic energy distribution of the photo-electron is **f ((K.E.)//, (K.E.)⊥)** per unit energy and surface density of its projection on the MCP top surface is **g(ρ)** or integrated around 2π is **G(ρ)**. The kinetic energies and displacement **ρ** are related as

$$f\,((K.E.)_{//}, (K.E.)_{\perp}) \cdot d\,(K.E.)_{\perp} \cdot d\,(K.E.)_{//} = G(\rho)\, d\rho \qquad \text{(Ap.10)}$$

If the kinetic energy distribution is known for the specific photocathode at a wavelength, PSF of spot image **g(ρ)** is predicted from Eq (Ax.10).

Ignoring the 3rd tem of Eq (Ax.7), i.e. with the accuracy of 1%,

$$d\rho \cong h \cdot \left[\sqrt{\frac{eVc}{(K.E.)_{\perp}}}\ \frac{d\,(K.E.)_{\perp}}{eVc} - \frac{1}{\tan\theta}\ \frac{d\,(K.E.)_{\perp}}{eVc} - \tan\theta\ \frac{d\,(K.E.)_{//}}{eVc} \right] \qquad \text{(Ap.11)}$$

This is still not simple enough to derive photo-electron distribution from observed PSF, so ignoring the 2nd tem as well, i.e. with the accuracy of 10%,

$$d\rho \cong h \cdot \left[\sqrt{\frac{eVc}{(K.E.)_{\perp}}}\ \frac{d\,(K.E.)_{\perp}}{eVc} \right] \qquad \text{(Ap.12)}$$

Substituting into Eq (Ax.10),

$$F(\ (K.E.)_{\perp}\ ) = G(\rho)\ d\rho$$

$$F(\ (K.E.)_{\perp}\ ) = G(\rho) \cdot \sqrt{\frac{eVc}{(K.E.)_{\perp}}}\ \frac{h}{eVc} \qquad \text{(Ap.13)}$$

Now, we can derive the photo-electron distribution. It, however, should be noted that Eq(Ax.13) does not derive (K.E.)// distribution, hence other approach necessary to estimate accurate PSF in all conditions.

If very small voltage applied to the photocathode gap,

From Eq (Ax.1),

$$h = 1/2\ (e \cdot Vc / m_e \cdot h) \cdot t^2 + (v_0 \cdot \cos\theta) \cdot t \qquad \text{(Ap.14)}$$

$$t \cong \frac{(v_0 \cdot \cos\theta)}{2\ (e \cdot Vc / m_e \cdot h)} \left[\frac{1}{2}\ \frac{4\ (e \cdot Vc / m_e \cdot h)\ h}{(v_0 \cdot \cos\theta)^2} - \frac{1}{8} \left\{ \frac{4\ (e \cdot Vc / m_e \cdot h)\ h}{(v_0 \cdot \cos\theta)^2} \right\}^2 \right.$$

$$+\frac{1}{16}\left\{\frac{4(e\cdot Vc/m_e\cdot h)h}{(v_0\cdot\cos\theta)^2}\right\}^3-\frac{5}{2^7}\left\{\frac{4(e\cdot Vc/m_e\cdot h)h}{(v_0\cdot\cos\theta)^2}\right\}^4\ ]$$

$$=\frac{h}{(v_0\cdot\cos\theta)}-\frac{(e\cdot Vc/m_e)\cdot h}{(v_0\cdot\cos\theta)^3}+\frac{2(e\cdot Vc/m_e)^2\cdot h}{(v_0\cdot\cos\theta)^5}-\frac{5(e\cdot Vc/m_e)^3\cdot h}{(v_0\cdot\cos\theta)^7}$$

$$=\frac{h}{(v_0\cdot\cos\theta)}\left[1-\frac{(e\cdot Vc/m_e)}{(v_0\cdot\cos\theta)^2}+\frac{2(e\cdot Vc/m_e)^2}{(v_0\cdot\cos\theta)^4}-\frac{5(e\cdot Vc/m_e)^3}{(v_0\cdot\cos\theta)^6}\right]$$

$$=\frac{h}{(v_0\cdot\cos\theta)}\left[1-\frac{1}{2}\frac{e\cdot Vc}{(K.E.)_{//}}+\frac{1}{2}\frac{(e\cdot Vc)^2}{(K.E.)_{//}^2}-\frac{5}{8}\frac{(e\cdot Vc)^3}{(K.E.)_{//}^3}\right]$$

Substituting to Eq (Ax.2),

$$\rho\cong h\cdot\tan\theta\left[1-\frac{1}{2}\frac{e\cdot Vc}{(K.E.)_{//}}+\frac{1}{2}\frac{(e\cdot Vc)^2}{(K.E.)_{//}^2}-\frac{5}{8}\frac{(e\cdot Vc)^3}{(K.E.)_{//}^3}\right]\qquad\text{(Ap.15)}$$

Angle distribution of photo-electron can be derived straightforward by applying zero volt to photocathode gap. The kinetic energy distribution is derived by applying Vc corresponding 0-0.5 of (K.E.)//. The problem is relatively large energy gap of MCPs, which requires <250nm. It is ideal to deposit photocathode material on top of input surface of 1st MCP, so that MCP has sensitivity even for low energy electrons.

As stated at Eq(Ax.7), escape velocity of photo-electron at grazing angle is not as fast as at normal angle. S20 photocathode was deposited semi-transparent thickness for XMM-intensifiers, so that a photo-electron generated at any depth can reach bottom surface of photocathode. Since absorption coefficient of the photocathode is large for a VUV photon, the VUV photon is converted to an electron at the top surface of the photocathode. If the photocathode is thick (thicker than mean free pass), the electron cannot reach bottom surface of the photocathode. If thinner than mean free pass, the electron can reach the bottom surface after several collisions at the expense of losing kinetic energy. Since the photoelectron experiences collisions, it also loses information on direction. Therefore, the angle distribution should be *Lambertion.*
Since absorption coefficient for optical red photons is lower, most of red photons pass through the photocathode. Only a lucky red photon is converted to an electron but at the anywhere in the photocathode layer. A photo-electron generated at the lower layer of the photocathode reach the bottom with few collisions, therefore it still has significant kinetic energy and perhaps some information on direction.

We have evidence that resolution of XMM-intensifier gets worse at the shorter wavelength. It reaches maximum at 300nm, but star recovering at further short wavelengths. Same phenomenon was reported on CsTe photocathode of MAMA detector used for HST.
These can be explained by the thickness of photocathode and colour dependence of conversion

layer of photo-electron.

There is an interesting question whether momentum of input photon is transferred to emitted photo-electron at $\lambda > 3000$A ? This is particularly important for fast F focusing optics or oblique incidence optics. If initial velocity of photo-electron has peak along incident photon direction in stead of normal to the surface, narrowing the input beam in the optics helps resolution in UV wavelengths. If input light is not normal to photocathode, the projection on the MCP surface can change with photon energy and photocathode voltage. This affects astrometry.
The relation of momentum between input photon and emitted photoelectron can be tested by narrow light beam with oblique incidence at low photocathode voltages with the help of Eq(Ax.7) If position of image at the output of the intensifier moves with photocathode voltage, there is a correlation.

Ref.

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