Interaction of radiation with matter

- simple radiative transfer
- synchrotron self-absorption

- photoelectric absorption
- Compton and inverse Compton scattering *
- pair production

4C15/SS7 (2006/2007)

Radiation processes and their inverse processes

Photon emission processes have their corresponding absorption processes.

Emission processes	Absorption process
recombination	photo-ionization
e^-/e^+ annihilation	e^-/e^+ pair production
synchrotron emission	synchrotron self-absorption
inverse Compton scattering	Compton scattering

Radiative transfer - basic formulation

Radiative transfer equation

$$\frac{dI_\nu(\hat{\Omega})}{dz} = -k_\nu I_\nu(\hat{\Omega}) + j_\nu + \int\int d\nu' d\Omega \; \sigma(\nu,\nu';\hat{\Omega},\hat{\Omega}') I_{\nu'}(\hat{\Omega}')$$

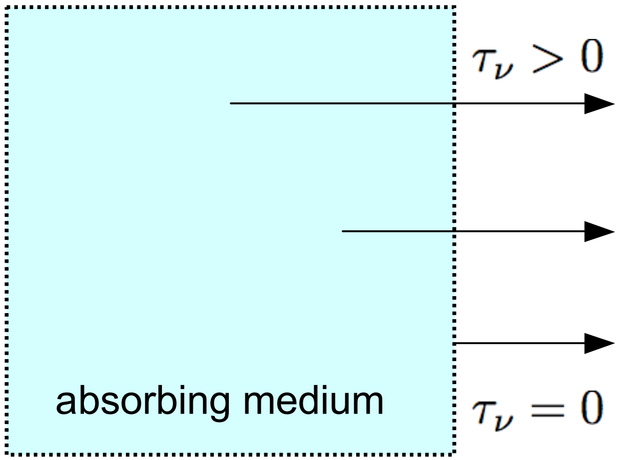
absorption emission scattering

Emission and absorption only

$$\frac{dI_\nu}{dz} = -k_\nu I_\nu + j_\nu$$

Optical depth (absorption)

$$d\tau_\nu = k_\nu dz$$

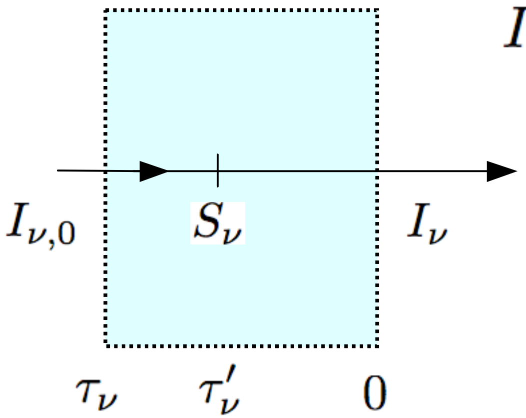


Radiative transfer - formal solution

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu \qquad\qquad S_\nu = \frac{j_\nu}{k_\nu}$$

source function

Formal solution to the radiative transfer equation

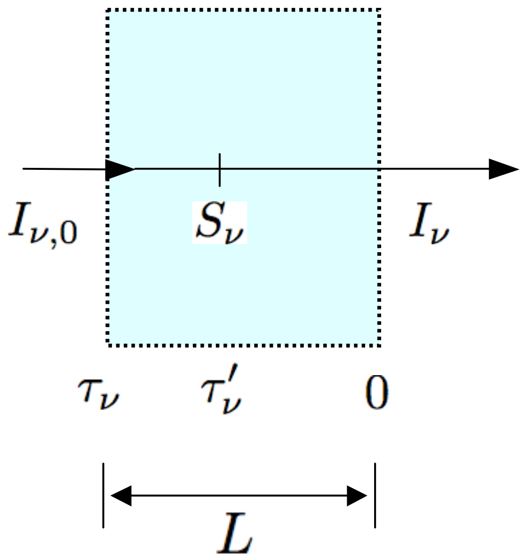


$$I_\nu = I_{\nu,0}\, e^{-\tau_\nu} + \int_0^{\tau_\nu} d\tau'_\nu\, S_\nu(\tau'_\nu)\, e^{-(\tau_\nu - \tau'_\nu)}$$

homogeneous medium

$$I_\nu = I_{\nu,0}\, e^{-\tau_\nu} + S_\nu \left(1 - e^{-\tau_\nu}\right)$$

Radiative transfer - opaque and transparent



transparent - small optical depth $\tau_\nu \ll 1$

$$e^{-\tau_\nu} = 1 - \tau_\nu + ...$$

$$\begin{aligned} I_\nu &= I_{\nu,0} + \tau_\nu S_\nu \\ &= I_{\nu,0} + j_\nu L \end{aligned}$$

opaque - large optical depth $\tau_\nu \gg 1$

$$e^{-\tau_\nu} \approx 0$$

$$I_\nu = S_\nu$$

Radiative transfer - thermal emission

When the emission and absorption are in local equilibrium, the emission and absorption coefficient are related by the Planck function.

Kirochoff's law: $j_\nu = k_\nu B_\nu(T)$

$$S_\nu = B_\nu(T)$$

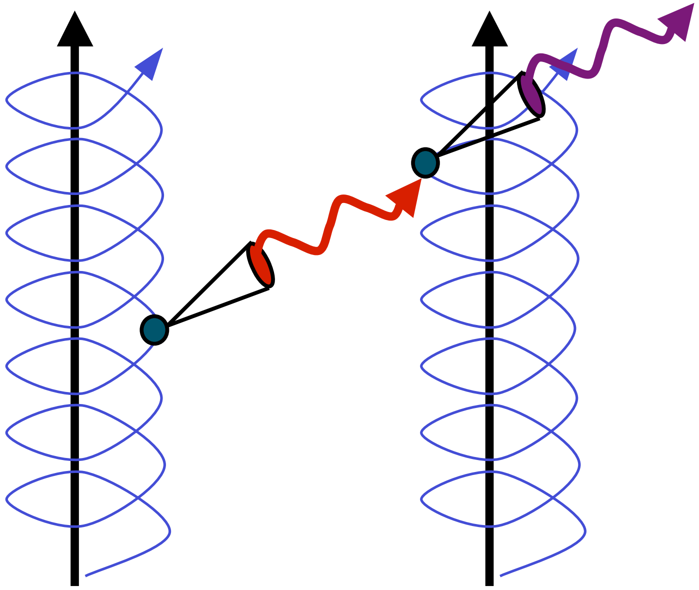
$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + B_\nu(T)$$

$$I_\nu = I_{\nu,0}\, e^{-\tau_\nu} + B_\nu(T)\left(1 - e^{-\tau_\nu}\right)$$

$$I_\nu = I_{\nu,0} + \tau_\nu B_\nu(T) \qquad \tau_\nu \ll 1$$

$$I_\nu = B_\nu(T) \qquad \tau_\nu \gg 1$$

Synchrotron self-absorption (I)



for power-law electron energy distribution

$$f(\gamma)d\gamma = C\gamma^{-p}d\gamma$$

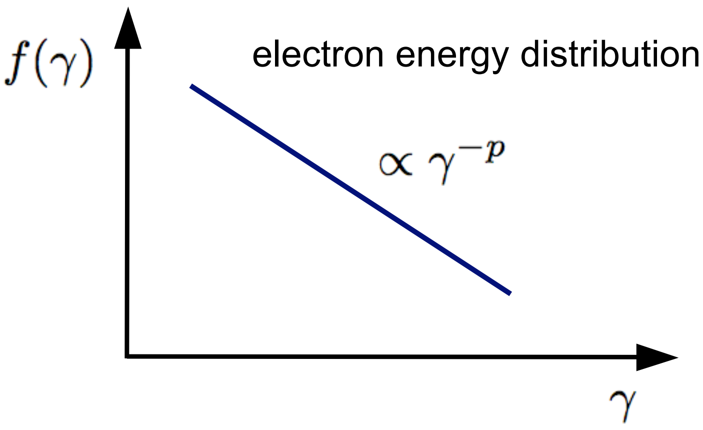
$$E = \gamma m_0 c^2$$

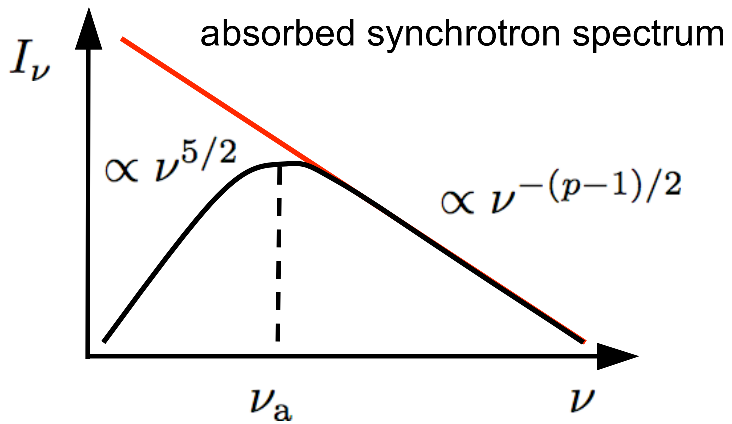
synchrotron absorption and emission coefficients:

$$k_\nu \propto \nu^{-(p+4)/2}$$

$$j_\nu \propto \nu^{-(p-1)/2}$$

Synchrotron self-absorption (II)





synchrotron source function

$$S_\nu = \frac{j_\nu}{k_\nu} \propto \nu^{5/2}$$

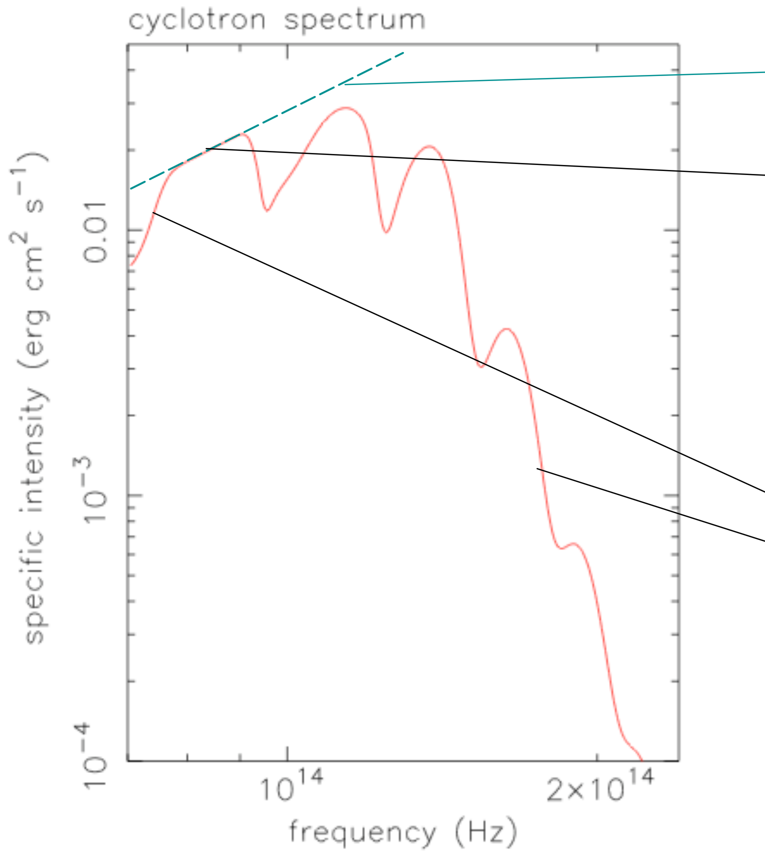
optically thick spectrum

$$I_\nu^{(\rm syn)} = S_\nu \propto \nu^{5/2}$$

optically thin spectrum

$$I_\nu^{(\rm syn)} = j_\nu^{(\rm syn)} L \propto \nu^{-(p-1)/2}$$

Thermal absorption of cyclotron radiation



Rayleigh-Jean spectrum

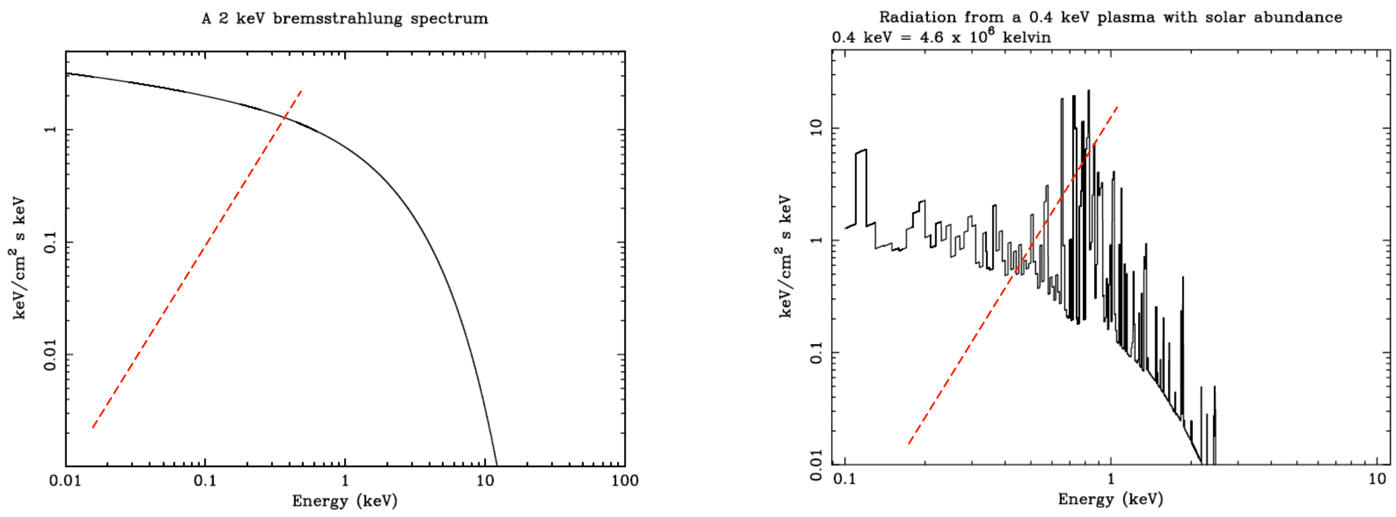
optically thick part

$$I_\nu^{(\mathrm{cyc})} = B_\nu(T) = I_\nu^{(\mathrm{RJ})}(T) \propto \nu^2$$

optically thin part

$$I_\nu^{(\mathrm{cyc})} = j_\nu^{(\mathrm{cyc})} L$$

Internal thermal absorption

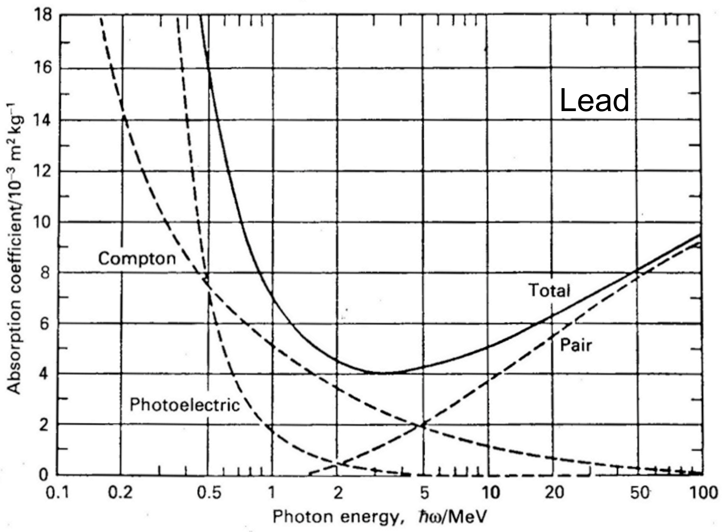


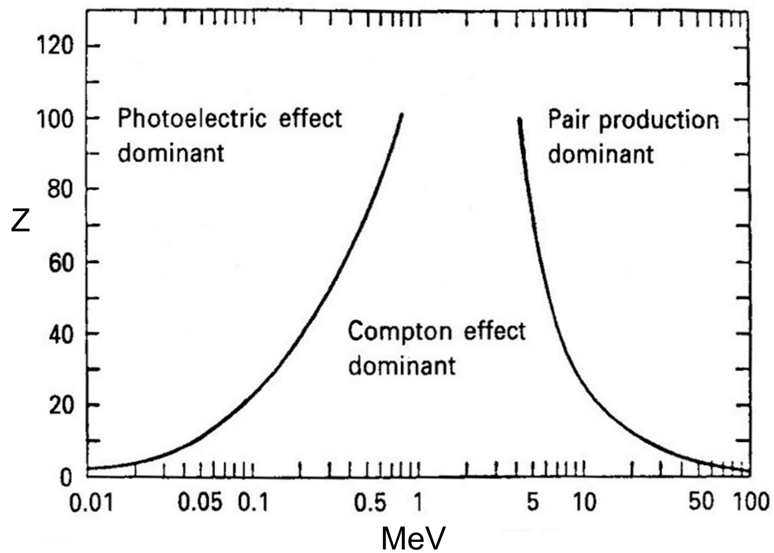
some features:

- Rayleigh-Jean (black body) spectrum at low frequency
- peak frequency at which the optical depth is about unity

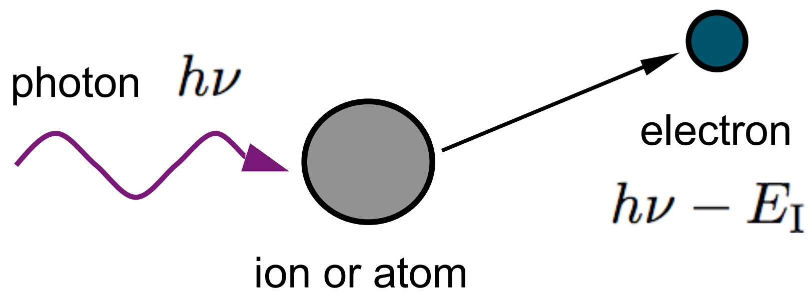
Some common photon processes in high-energy astrophysics

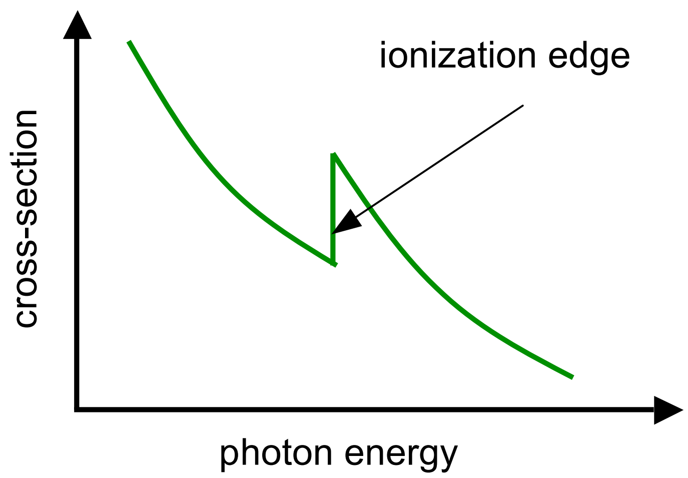
- photoelectric effect
- Compton scattering
- pair production

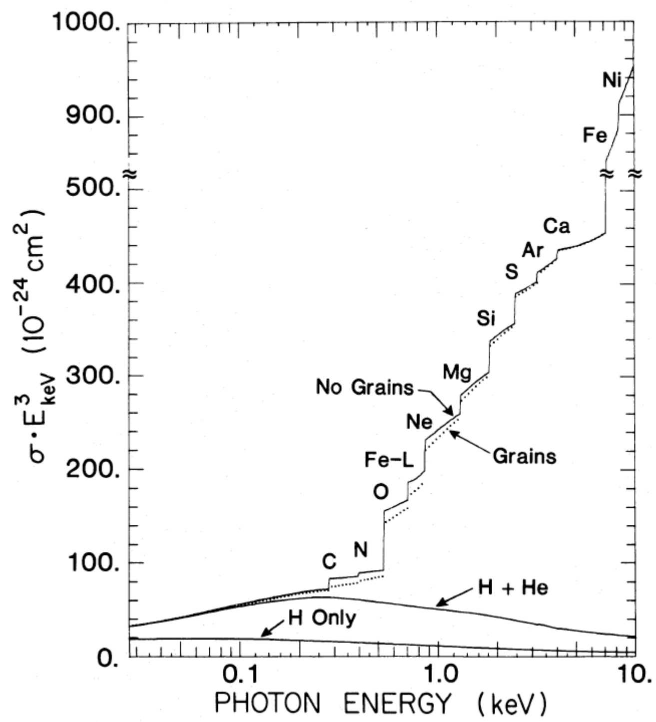




Photoioniztion







Photoelectric absorption cross-section

For $E_{\mathrm{I}} < h\nu < m_{\mathrm{e}}c^2$,

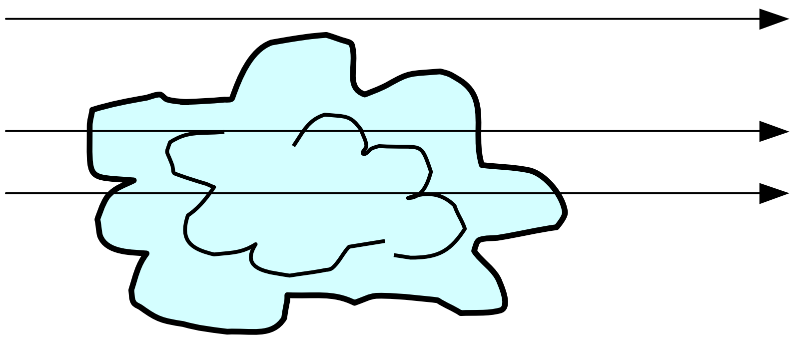
the photoelectric absorption cross-section for photons is given by

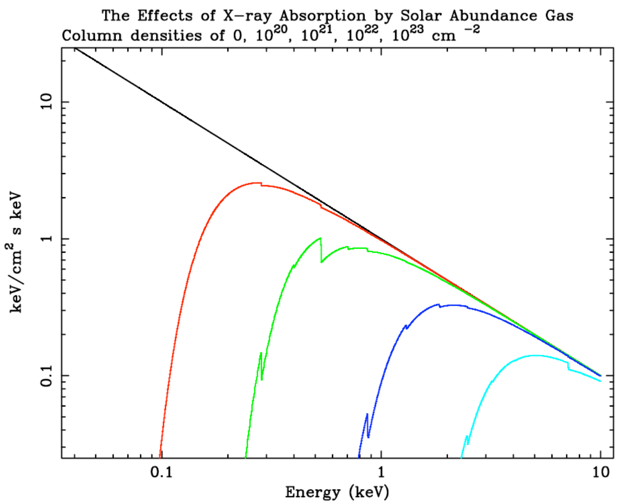
$$\sigma_{\mathrm{K}} \approx 2\sqrt{2}\, \sigma_{\mathrm{T}} \alpha^4 Z^5 \left(\frac{m_{\mathrm{e}}c^2}{h\nu} \right)^{7/2} ,$$

where E_{I} is the electron binding energy, α is the fine-structure constant, and σ_{T} is the Thomson cross-section.

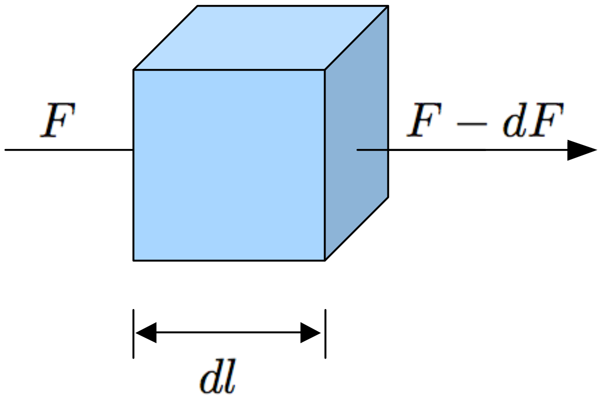
Note that it depends on Z^5 and on $(h\nu)^{-7/2}$.

Photoelectric absorption of soft X-rays by interstellar media (ISM)





Attenuation by ISM photoelectric absorption (I)



Consider a volume element with a thickness dl and with an element Z which has a number density of n_z .

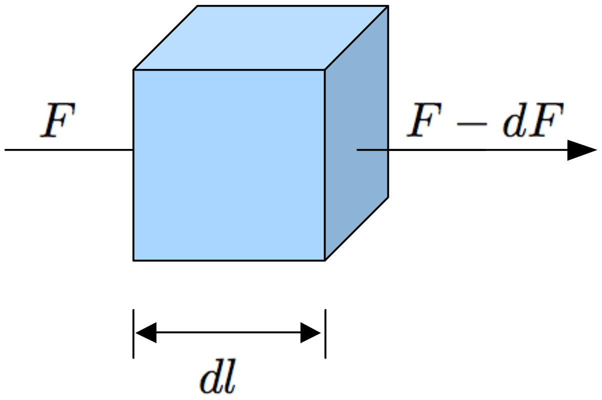
Suppose that the cross-section of the element is σ_z .

The fraction of the volume element blocked by the presence of the element Z is $n_z \sigma_z dl$.

The fraction of the flux F lost in the volume element is therefore

$$\frac{dF}{F} = -n_z \sigma_z(E) dl \ .$$

Attenuation by ISM photoelectric absorption (II)



Integrating over the light path in the ISM yields

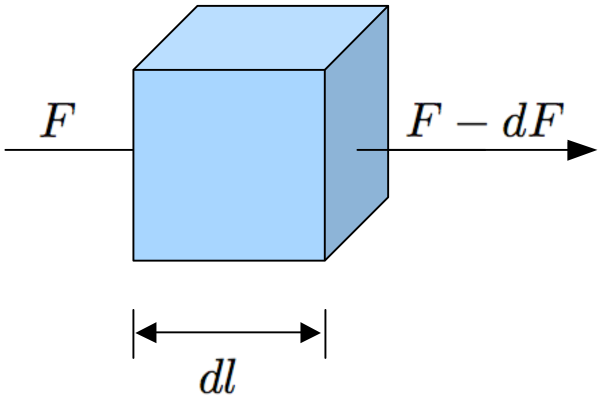
$$\int \frac{dF}{F} = -\sigma_z(E) \int dl\ n_z$$

$$\Rightarrow F = F_0 \exp\left(-\sigma_z(E) \int dl\ n_z\right)$$

Including all elements in the line-of-sight,

$$F = F_0 \exp\left(-\sum_z \left[\sigma_z(E) \int dl\ n_z\right]\right)$$

Attenuation by ISM photoelectric absorption (III)



Suppose that the flux attenuation can be expressed as

$$F = F_0 \exp\left(-\sigma_{\rm eff}(E) N_{\rm H}\right)$$

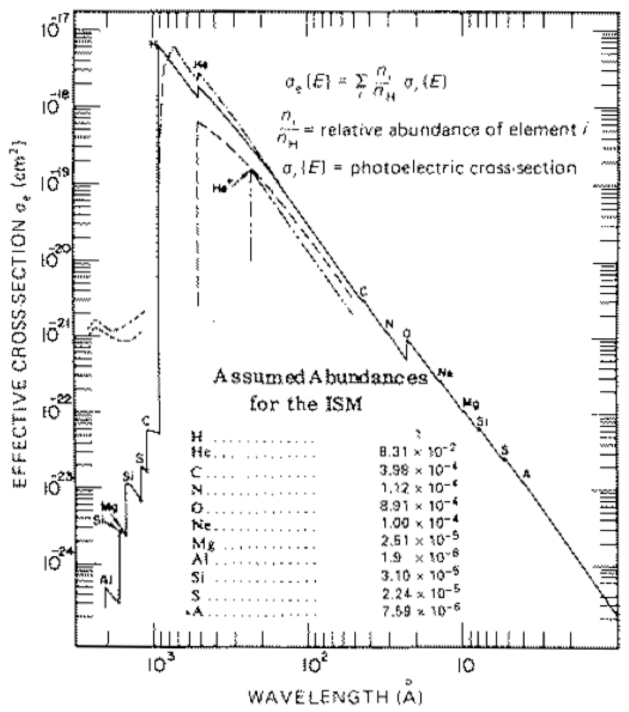
$$N_{\rm H} = \int dl\ n_{\rm H}$$

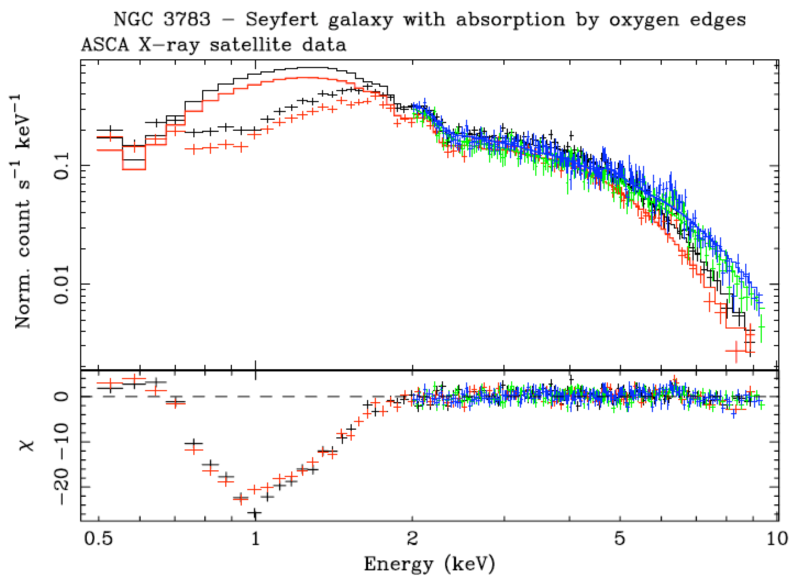
line-of-sight hydrogen column density

$$\sigma_{\rm eff}(E) = \sum_{\rm z} \left[\sigma_{\rm z}(E) \frac{n_{\rm z}}{n_{\rm H}}\right]$$

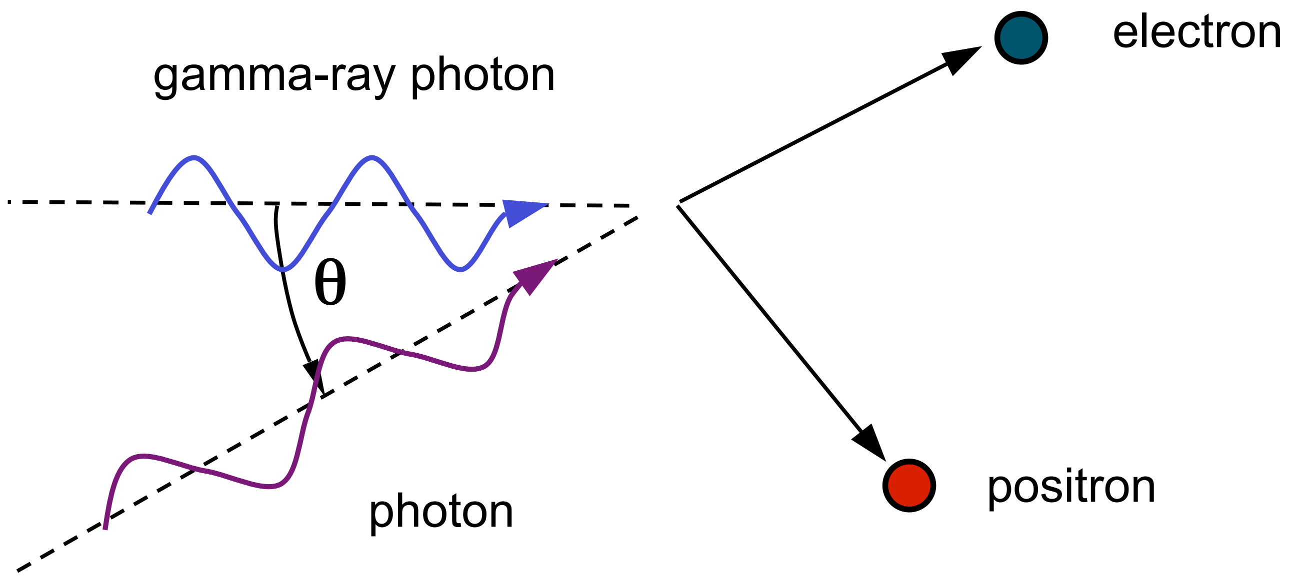
effective cross-section weighted over the abundance of elements with respect to hydrogen

Effective cross-section and absorption features





Electron-positron pair production (I)



Two photons, one of which must have energy $E > m_e c^2$, collide and create an electron-positron (e^+/e^-) pair. The process is effectively a form of gamma-ray absorption

Electron-positron pair production (II)

4-momenta of the 2 photons

$$k^\mu = k_1^\mu + k_2^\mu$$

$$k^\mu k_\mu = \left(k_1^\mu + k_2^\mu\right)\left(k_{1\mu} + k_{2\mu}\right)$$

$$= k_1^\mu k_{1\mu} + k_2^\mu k_{2\mu} + 2k_1^\mu k_{2\mu}$$

$$k_1^\mu k_{1\mu} = k_2^\mu k_{2\mu} = 0$$

$$k_1^\mu k_{2\mu} = \frac{(\hbar\omega_1)(\hbar\omega_2)}{c^2}\left[1 - \hat{\Omega}_1 \cdot \hat{\Omega}_2\right]$$

$$= \frac{(\hbar\omega_1)(\hbar\omega_2)}{c^2}\left[1 - \cos\theta\right]$$

$$k^\mu k_\mu = \frac{2(\hbar\omega_1)(\hbar\omega_2)}{c^2}\left[1 - \cos\theta\right]$$

Electron-positron pair production (III)

4-momenta of the electron-positron pair

$$p^\mu = p_1^\mu + p_2^\mu$$

$$p^\mu p_\mu = \left(p_1^\mu + p_2^\mu\right)\left(p_{1\mu} + p_{2\mu}\right)$$
$$= p_1^\mu p_{1\mu} + p_2^\mu p_{2\mu} + 2p_1^\mu p_{2\mu}$$

For minimum photon energies, it requires that electron-positron pair does not have linear momentum.

$$\vec{\beta}_1 = \vec{\beta}_2 = 0$$
$$\gamma_1 = \gamma_2 = 1$$
$$p_1^\mu p_{1\mu} = p_2^\mu p_{2\mu} = p_1^\mu p_{2\mu} = (m_\mathrm{e} c)^2$$

$$p^\mu p_\mu = 4\,(m_\mathrm{e} c)^2$$

Electron-positron pair production (IV)

Conservation of energy-momentum implies

$$p^\mu = k^\mu$$

Hence, we have

$$p^\mu p_\mu = k^\mu k_\mu$$

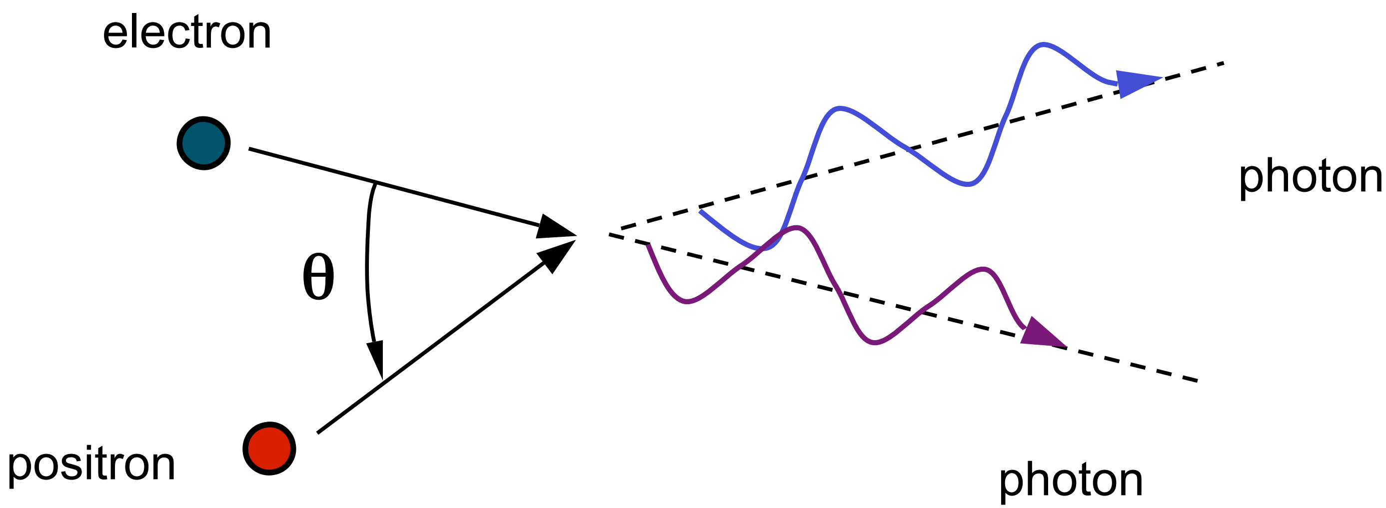
It follows that

$$(\hbar\omega_1) = \frac{2(m_{\mathrm{e}}c^2)^2}{(\hbar\omega_2)\left[1 - \cos\theta\right]}$$

$$\min{(\hbar\omega_1)} = \frac{(m_{\mathrm{e}}c^2)^2}{(\hbar\omega_2)}$$

The minimum energy of one of the photons must therefore be larger than the electron/positron rest mass.

Electron-positron pair annihilation



It is a reverse process of electron-positron pair production.