Good progress in the extrapolation of photospheric magnetograms

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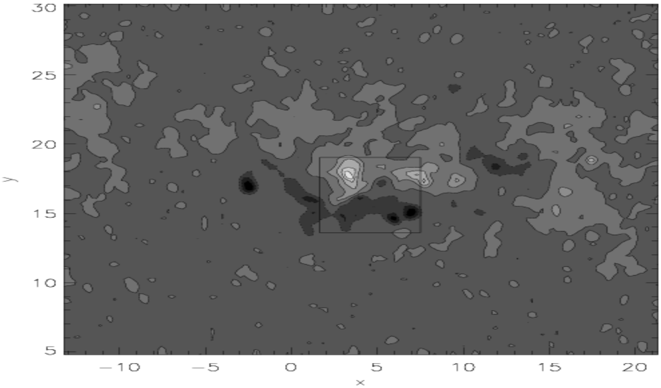
A solar example

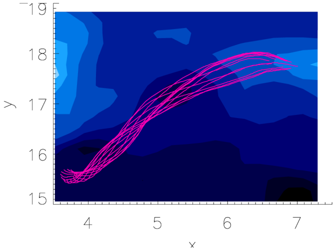
Bastille event: IVM 2000-07-14 16:33 $\subset$ MDI

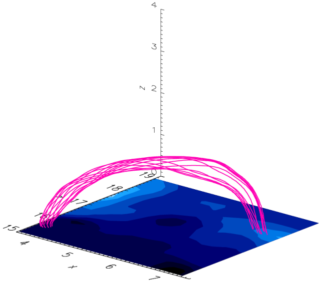


Extrapolated field: $\sigma_J = 0.14$ max $|\vec{\nabla} \cdot \vec{B}| = 1.5 \times 10^{-12}$

σ_J is the current-averaged sine angle between $\vec{B}$ and $\vec{J}$.

Post-flare loop in the Bastille event

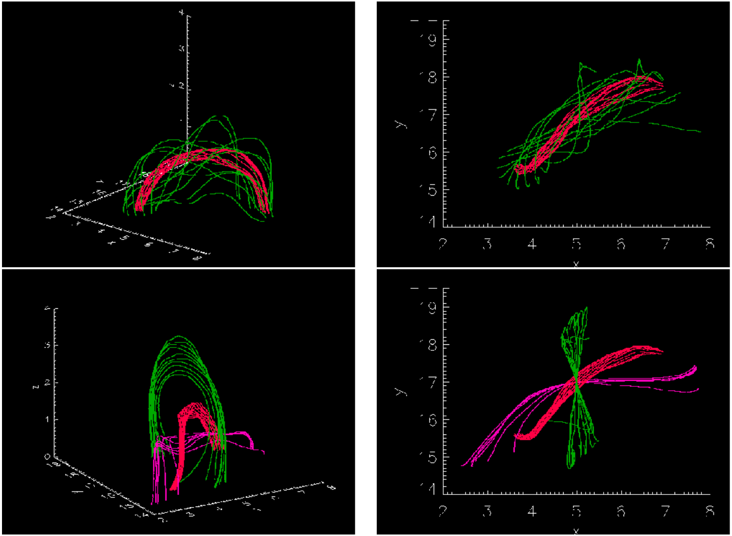




Flux rope with estimated twist of about 2π.

The loop in figure intersects the photosphere on a circle of diameter equal to the magnetogram resolution ($\Delta = 0.2$).

Post-flare loop in the Bastille event



Post-flare loop in the Bastille event

The extrapolation provides with the 3D magnetic field:

- Helicity, energy
- Topology, current sheets, stability analysis
- Does the flux rope exist prior the CME, and is it the only one?
  $\Longrightarrow$ extrapolations of time series (in progress)
- Energy and helicity budgets
- Full MHD simulation of extrapolated field: desktop CME (Tibor)?

All very interesting,

BUT

can we trust the results of the extrapolation code?

Outline

1. The MF method as an extrapolation tool
   - Magneto-frictional relaxation
   - MF used as an extrapolation technique
2. Quality of extrapolations
   - Standard test: Low and Lou
   - Flux rope with return current
   - Flux rope without return current
   - Filament and arcade
3. Outlook
   - Mgm preprocessing

Extrapolation of photospheric measurements

- Find the magnetic field in a numerical box for given conditions at the photospheric boundary, assuming a perfectly force-free coronal field.

- Discard non-force-free effects close to the photosphere and errors in, or inconsistencies of, measured photospheric fields.

Outline

Frictional relaxation for MHD equilibrium solutions

The MF relaxation seeks for a solution of the time-independent MHD with prescribed boundary conditions introducing

- a time-like variable (ie using time-dependent MHD);
- a fictitious friction $\nu(x, y, z)$ in the momentum balance equation.

Starting from an approximate solution $\Longrightarrow$ drive the system toward equilibrium under the combined effect of dissipation and boundary conditions.

Force-free approximation

For a force-free field, the momentum balance equation,

$$\vec{v} = \frac{1}{\nu}\vec{J} \times \vec{B},$$

gives the velocity field that drives the system toward the force-free equilibrium in terms of the magnetic field. Hence, only

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B})$$

need be advanced in time $\Longrightarrow$ reduced numerical effort.

Outline

Initial and boundary conditions

What turns the MF relaxation into an extrapolation technique are

- Initial condition: from the normal component of the vector magnetogram the potential field is computed (Seehafer).
- Boundary conditions: the whole vector magnetogram is copied into the photospheric layer prior to relaxation, and then kept fixed.

The MF relaxation is then started, seeking the equilibrium solution that is force-free, has the magnetogram as (photospheric) boundary, and has small $\vec{\nabla} \cdot \vec{B}$ errors.

Boundary conditions

Two ghost layers of bc for $\vec{B}$ are prescribed by imposing

- Side and top boundaries (both ghost layers):

$$\text{normal component} : \vec{\nabla} \cdot \vec{B} = 0$$
$$\text{transverse components:} \ \vec{J} \times \vec{B} = 0$$

- Photospheric boundary
  - inner layer: all components: magnetogram
  - outer layer: normal component : $\vec{\nabla} \cdot \vec{B} = 0$
    transverse components: : 4th polynomial

The velocity field is windowed toward all but the photospheric boundary.

Flexibility: different bc can be applied to improve relaxation.

Performance tests

The extrapolation code was tested using four classes of nlfff

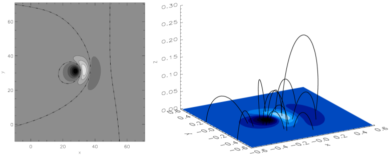
- Low and Lou
- Török and Kliem: model of coronal loop with return current
- Titov Demoulin: model of coronal loop with net current
- van Ballegooijen: filament and arcade

In all cases, the three magnetogram components at $z = 0$ are the only information used in the extrapolation.

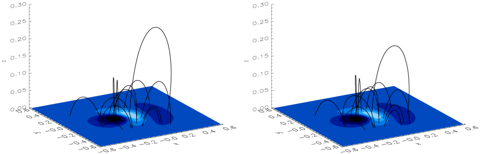
Outline

The Low and Lou test case

Semi-analytical solution to the nonlinear force-free equations[1].



Magnetogram (B_z) and initial (potential) field.



Input and extrapolated force-free solutions.

$$\sigma_J = < \sin \widehat{\vec{J}\,\vec{B}} >_{\vec{J}} = 0.015, \qquad \max|\vec{\nabla}\cdot\vec{B}| = 3\times 10^{-7}$$

Accurate reconstruction: force and divergence-free

[1] Low & Lou, ApJ **352** 343 (1990)

Comparison with other methods

Several metrics can be defined to compare input $\vec{b}$ and extrapolated $\vec{B}$ fields[2], e.g.

$$E'_M \equiv 1 - E_M; \qquad E_M \equiv \frac{1}{N}\sum_i \frac{|\vec{b}_i - \vec{B}_i|}{|\vec{B}_i|},$$

	C_{vec}	C_{CS}	E'_N	E'_M	ϵ	L_f	L_d	$\sigma_J \times 10^2$
MF	1.00	0.95	0.96	0.75	1.01	23.	10^{-06}	1.50
W	1.00	0.91	0.92	0.66	1.04	524.	205.	4.49

Figures of merit for the extrapolation of Case II, inner domain, with MF and Wiegelmann's (W) codes.

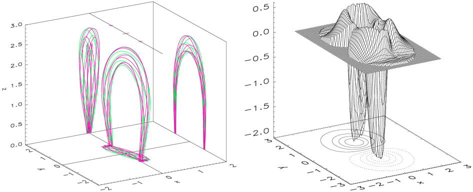
$\Longrightarrow$ In the LL case, the MF method gives the most accurate reconstruction among several methods (optimisation, Grad-Rubin, integral).

[2] Schrijver *et al*, Sol.Phys. **235** 161 (2006)

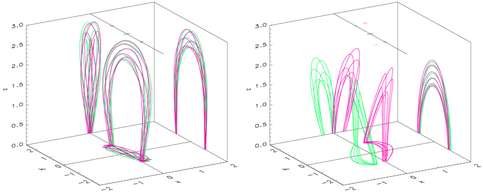
Outline

Nonlinear dipolar field: TK

Loop with return current and a twist that is about 80% of the instability threshold[3].



Field lines of the input field (left) and $\alpha(x, y, z = 0)$ (right)

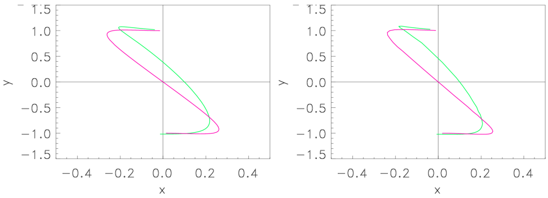


Extrapolated fields: nonlinear (left) and linear for the best fitting $\alpha = 0.5$ (right)

[3] Tórök and Kliem 2003 A&A 406,1043

Nonlinear dipolar field: TK

Twist is excellently reproduced



Projections of the central field line and a twisting line for the input (left) and extrapolated (right) fields

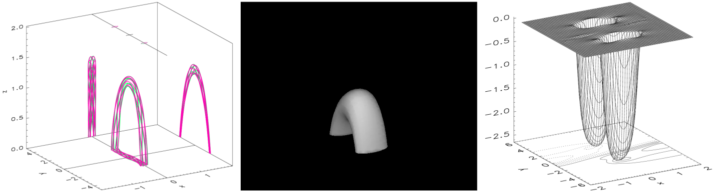
The nonlinear extrapolation is successful in reproducing both the global structure and the details of the twisted loop ($\sigma_J = 0.01$).

The linear extrapolation fails in both aspects: two flux tubes instead of one, and the best fitting value $\alpha = 0.5$ is much smaller than the actual maximum value of $|\alpha(z = 0)|$.

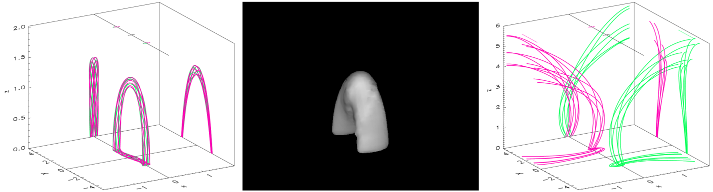
Outline

Nonlinear quadrupolar field: TD

Loop with no return current and a twist that is about 60% of the kink instability threshold[4].



Input field: Field lines (left), iso-surfaces of current density (centre), and $\alpha(x, y, z = 0)$ (right)

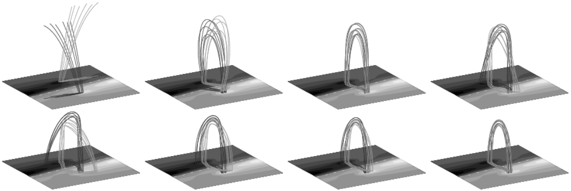


Extrapolated: nonlinear (left), isosurfaces of current density (centre), and linear for the best fitting $\alpha = -0.85$ (right)

[4] Kliem and Török, 2004, A&A 413 L23

Nonlinear quadrupolar field: TD

A very specific balance between the Lorentz self-force of the current-carrying flux rope and the force connected with the external potential field insures a stable equilibrium.



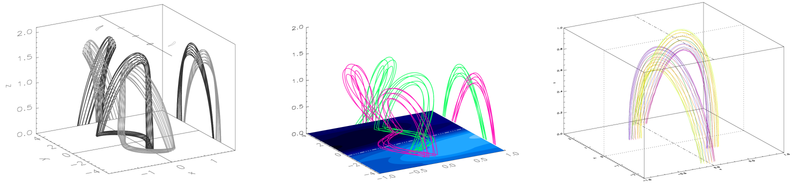
From left to right and top to bottom: evolution at $(0, 5, 10, 15, 45, 90, 110, 230) \times 10^3$ iterations

The nonlinear extrapolation successfully reproduces the complexity of the original field in all its aspects ($\sigma_J = 0.02$).

In this case, the linear extrapolation fails even more dramatically than in the dipolar case.

TD puzzle: Imperfect reconstruction

: let's raise the upper boundary a bit . . .
Method failure or peculiar equilibrium?



Extrapolation of TD using MF, Optimisation[5], and full MHD codes

Testing of boundary conditions (Tibor): flexibility is essential.

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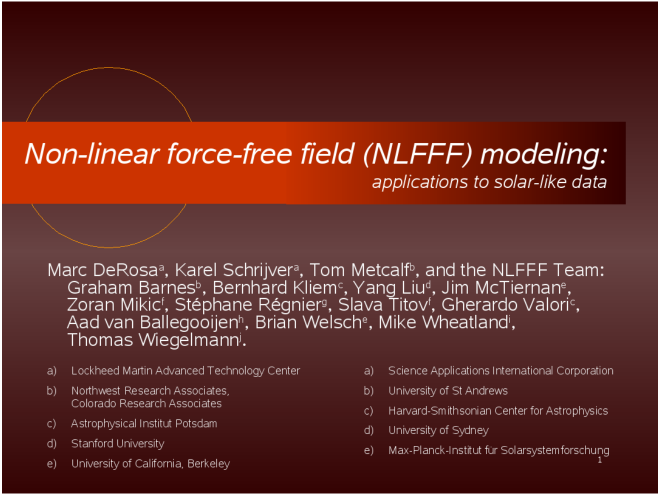
[5] Wiegelmann *et al*, 2006 A&A, 453, 737-74

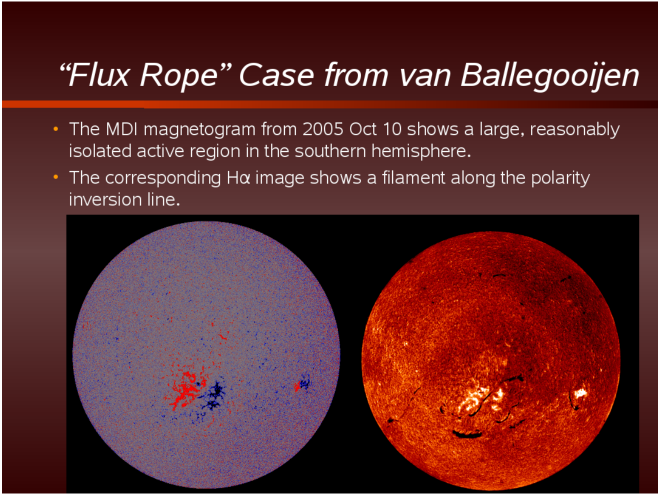
Outline

Aad van Ballegooijen's field

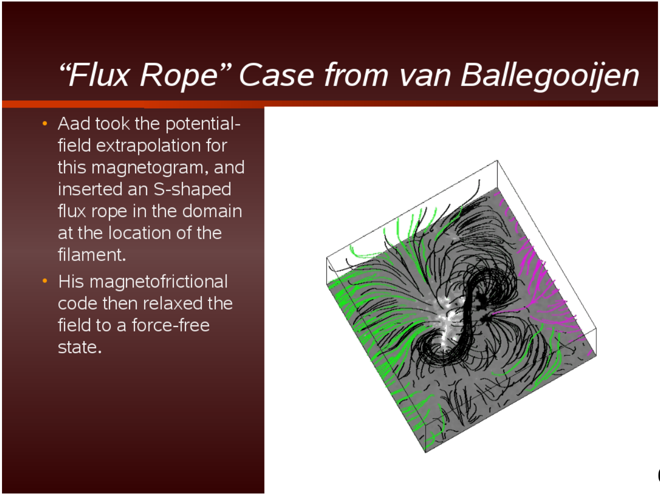


Presented by Marc deRosa at SPD 2006.

Aad van Ballegooijen's field



Aad van Ballegooijen's field

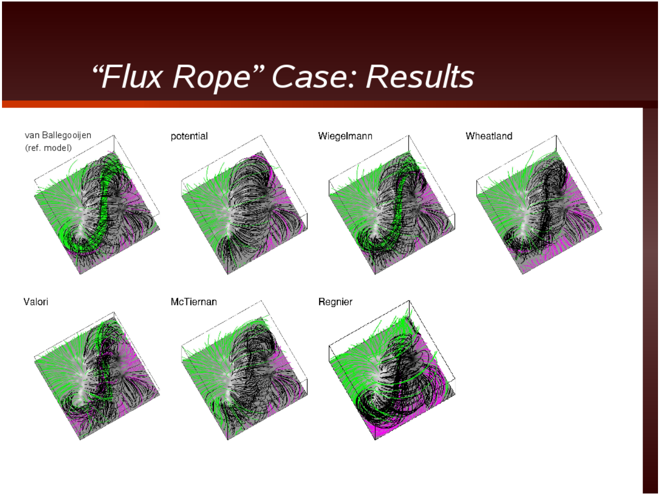


[6]

[6] van Ballegooijen, 2004 ApJ 612,519-529

Aad van Ballegooijen's field

Preliminary results



Aad van Ballegooijen's field

Ours: $\sigma_J = 0.1$ mgm force-free compatible, but small scales are present

Quantitative Comparison Metrics

	CS	1-NVE	CWcos	E/Epot
reference model	1.00	1.00	0.98	1.36
Wiegelmann	0.99	0.93	0.96	1.34
Wheatland	0.93	0.77	0.97	1.21
Valori	0.92	0.75	0.92	1.29
McTiernan	0.89	0.71	0.84	1.05
Régnier*	0.90	0.59	0.69	1.34
potential solution	0.90	0.65	(undef.)	1.00

*Régnier's calculation was performed at lower resolution.

CS = Cauchy-Schwarz

$$\mathrm{CS} \equiv \frac{1}{N}\sum_i \frac{|\vec{v}_i \cdot \vec{w}_i|}{|\vec{v}_i||\vec{w}_i|},$$

NVE = Normalized Vector Error

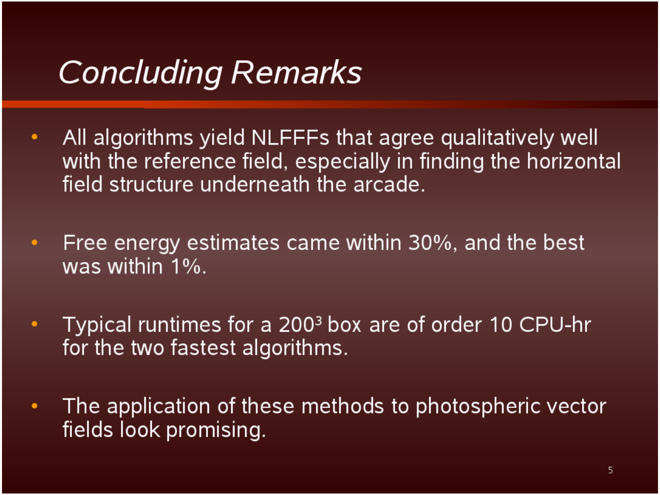
$$\mathrm{NVE} \equiv \frac{\sum_i |\vec{w}_i - \vec{v}_i|}{\sum_i |\vec{v}_i|}.$$

$$\mathrm{CWcos} = \frac{\sum_i |\vec{J}_i|\sigma_i}{\sum_i |\vec{J}_i|}$$

$$\sigma_i = \frac{|\vec{J}_i \cdot \vec{B}_i|}{|\vec{J}_i||\vec{B}_i|}$$

CWcos = current-weighted cosine between ***J*** and ***B***

Aad van Ballegooijen’s field



Summary

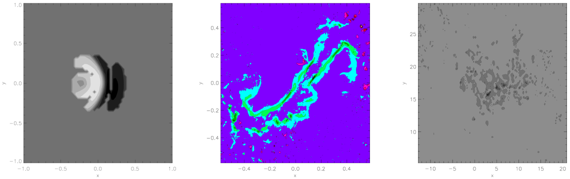
- As long as magnetograms are ff-consistent and scales well resolved, the MF method reproduces the field with high accuracy
- The method and its implementation are very flexible in allowing for
  - non force-free effects
  - tailoring of non-photospheric boundary conditions
  - different discretisation
  - treatment of large dataset using stretched grids
- Reconstruction quality seems to be influenced by the presence of small length scales in the mgm, but not by flux balance, extension of nonlinearities, fl connectivity

Outline

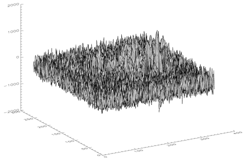
Length scales and force-freeness

Small length scales in the mgm $\Longrightarrow$ worse relaxation (and reconstruction)

α contour plots for the Low and Lou, van Ballegoijen, and Bastille cases



$\sigma_J =$ 0.01   0.1   >0.14



Transverse component in Huairou 2000-07-14-0119

In a measured magnetogram

- small scales
- error in 180 deg ambiguity removal
- noise
- finite β

Preprocessing

Magnetograms can be preprocessed prior to extrapolation in order to

- reduce noise and smooth small scales
  $\Longrightarrow$ facilitate extrapolation
- remove the non force-free part
  $\Longrightarrow$ study influence of the (thin) non-force-free layer

Preprocessing as an investigation tool.

Preprocessing equations

It is based on a minimisation process that changes the field within measurement errors.

From the force-free condition, $\vec{\nabla} \cdot (\vec{B}\vec{B} - 1/2B^2)$, applied to the mgm[7]

$$L_{force} = \int_{mgm} d\vec{x} \left[(B_x B_z)^2 + (B_y B_z)^2 + (B_z^2 - B_x^2 - B_y^2) \right] = 0$$

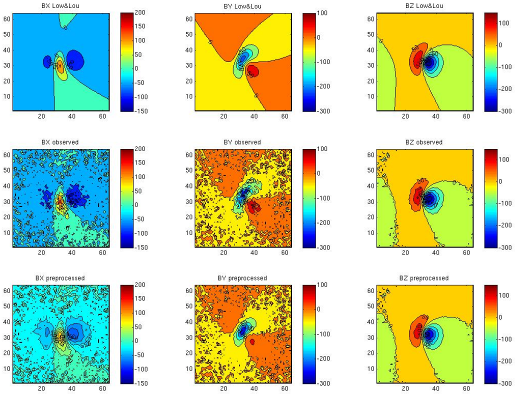
and analogous expression for the torque.

Simulated annealing is used to find the magnetic field that minimises L.

Similarly, a smoothing operator can be devised, but so far results are unsatisfactory.

[7] Molodenskii Sov. Astr., 1969, 12, 585-588

Preprocessing example: Low and Lou



The difference in L between "observed" and preprocessed magnetograms is 3 to 5 orders of magnitude.

$\Longrightarrow$ Easy to reduce to force-free, less to smooth without flattening excessively the field profiles.

What we need

- Improve discretisation
- Which techniques to remove noise?
- And small scales?