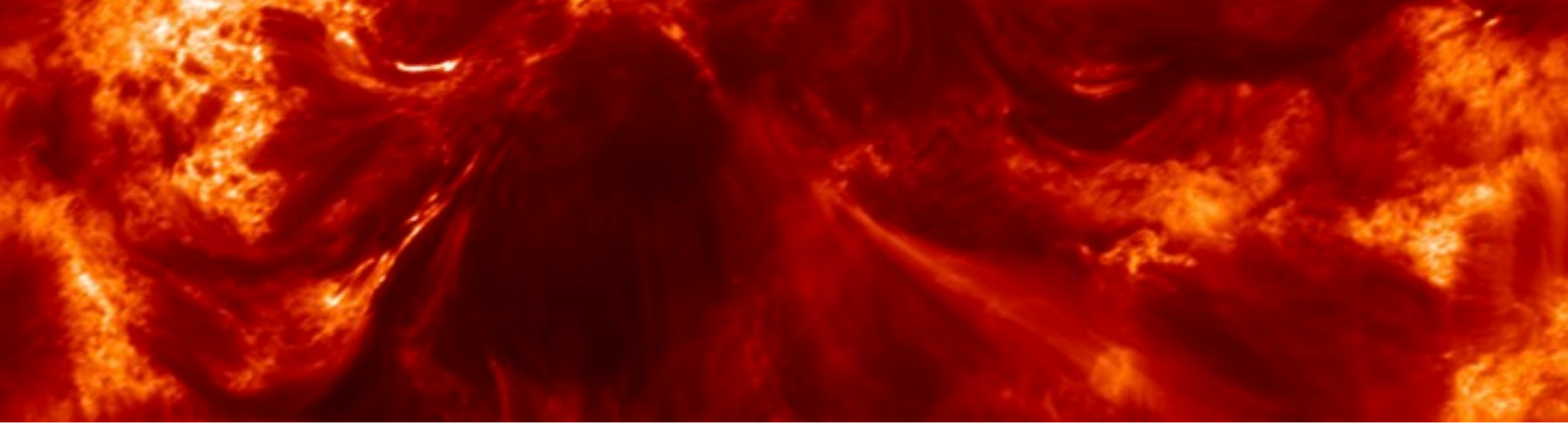
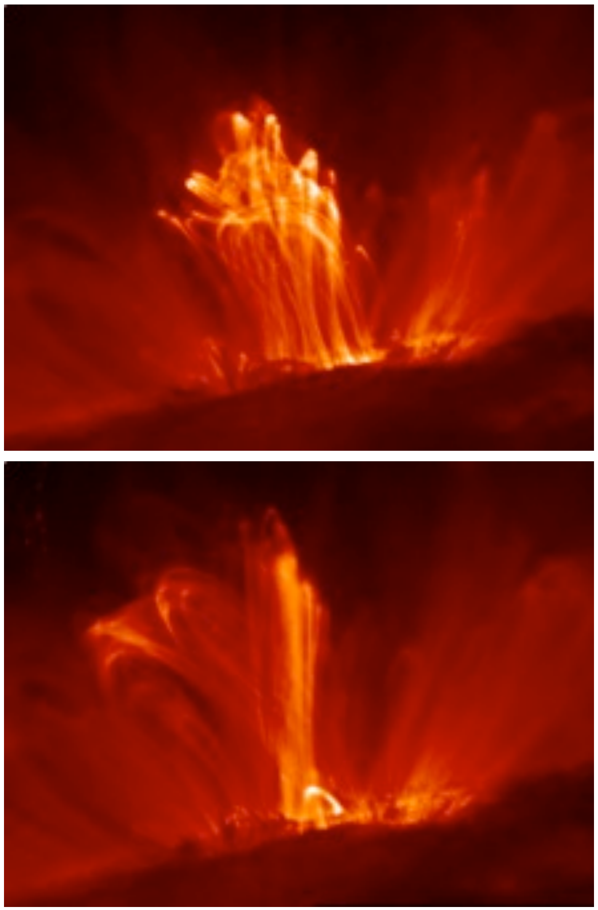


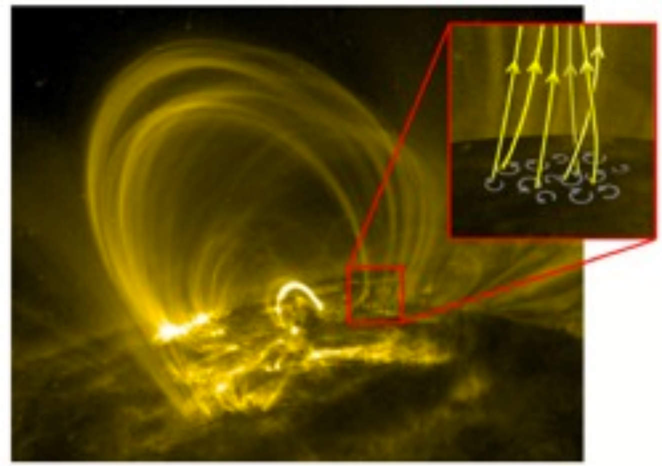
Braided magnetic loops in the solar corona: relaxation and heating

Antonia Wilmot-Smith, Gunnar Hornig, David Pontin
University of Dundee

Relaxation in the Solar Corona

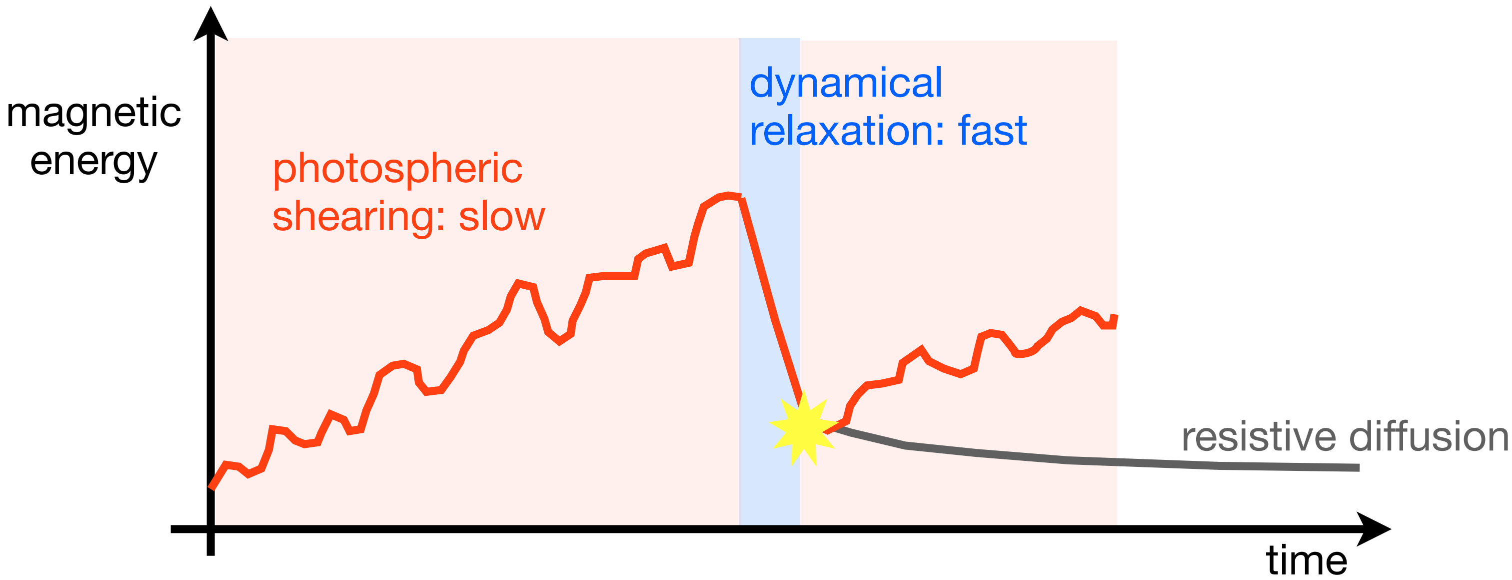


M1.1 class flare, AR 9166
14th September 2000.



- Magnetic energy & helicity injected slowly by footpoint motions.
- Energy released on Alfvénic timescale.

Key questions



- What structures will be produced by photospheric motions?
- What triggers a dynamical relaxation?
- What is the end result of a dynamical relaxation?

Modelling approach

- Ampere's Law

$$\nabla \times \mathbf{B} = \mu \mathbf{j}$$

- Faraday's Law

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

- Ohm's Law

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \frac{\mathbf{j}}{\sigma}$$

- Equation of Motion

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \mathbf{j} \times \mathbf{B} + \mathbf{F}$$

- Continuity Equation

$$\frac{D\rho}{Dt} \equiv \frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho = -\rho \nabla \cdot \mathbf{v}$$

- Solenoidal Constraint

$$\nabla \cdot \mathbf{B} = 0$$

+ appropriate energy equation.

Here: **B** magnetic field, **j** current density, **E** electric field, **v** plasma velocity, **ρ** plasma density, **μ** magnetic permeability, **σ** electric conductivity.

Nature of an equilibrium state

- Consider the equation of motion:

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \mathbf{j} \times \mathbf{B} + \mathbf{F}$$

- Assume the equilibrium is quasi-static.
- Gas pressure is negligible in most of corona.
- Neglect gravity over typical scale heights. Neglect viscosity.
- Reduce to the force-free condition:

$$\mathbf{j} \times \mathbf{B} = \mathbf{0}$$

Force-free fields

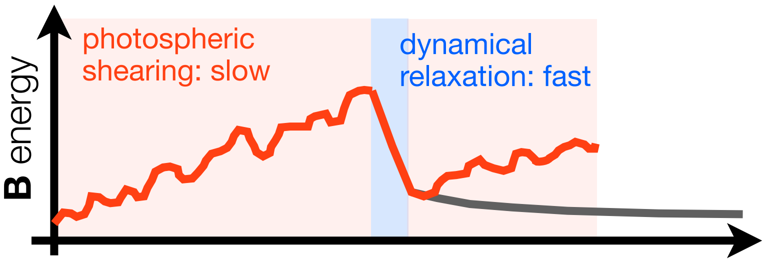
$$\mathbf{j} \times \mathbf{B} = \mathbf{0}, \quad \nabla \times \mathbf{B} = \mu\mathbf{j}, \quad \nabla \cdot \mathbf{B} = 0$$

- Simplest solution $\mathbf{j}=\mathbf{0}$ known as a potential field.
- More general case has $\mathbf{j} \parallel \mathbf{B}$: $\nabla \times \mathbf{B} = \alpha\mathbf{B}$.
- The force-free parameter α is not arbitrary but must be constant along magnetic field lines.

$$[\nabla \cdot (\nabla \times \mathbf{B} = \alpha\mathbf{B}) \Rightarrow \mathbf{0} = \nabla \cdot (\alpha\mathbf{B}) = \alpha\nabla \cdot \mathbf{B} + \mathbf{B} \cdot \nabla\alpha = \mathbf{B} \cdot \nabla\alpha]$$

- If α uniform in space ⇒ linear force-free field.
- If α varies between field lines ⇒ non-linear force-free field.

Which force-free field?



For an ideal dynamics main question is whether smooth force-free equilibria exist at all (Parker, 1979).

Yes: If the twist in the flux tube is 'not too large'.

e.g. Binneau (1972); van Ballegooijen (1985); Aly (1990); Craig & Sneyd (2005); Wilmot-Smith et al. (2009).

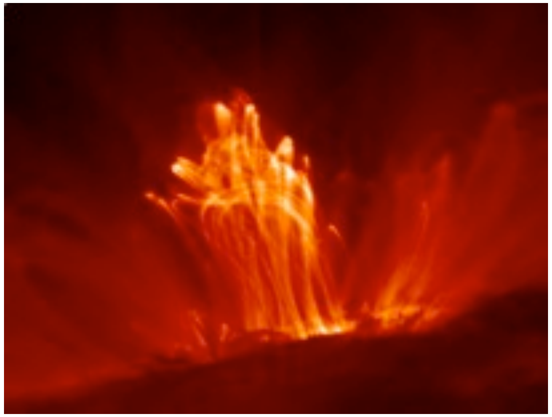
No: If the twist in the flux tube is 'large' (compare horizontal with vertical length scales).

e.g. Galsgaard & Nordlund (1996); Janse, Low & Parker (2007-2010); Bowness et al. (2011).

Here we consider non-ideal relaxations in which the magnetic field topology can change in localised regions due to magnetic reconnection events.

Which force-free field?

- Non-ideal relaxation is an energy minimisation process.



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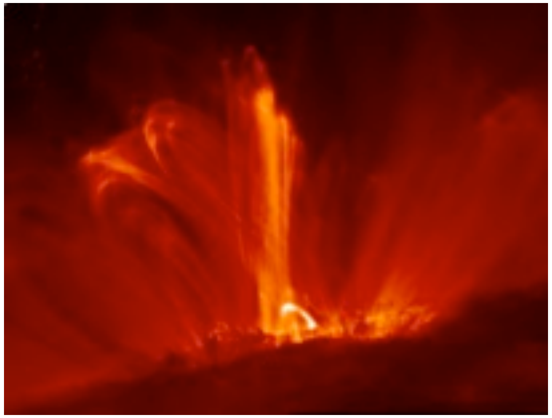

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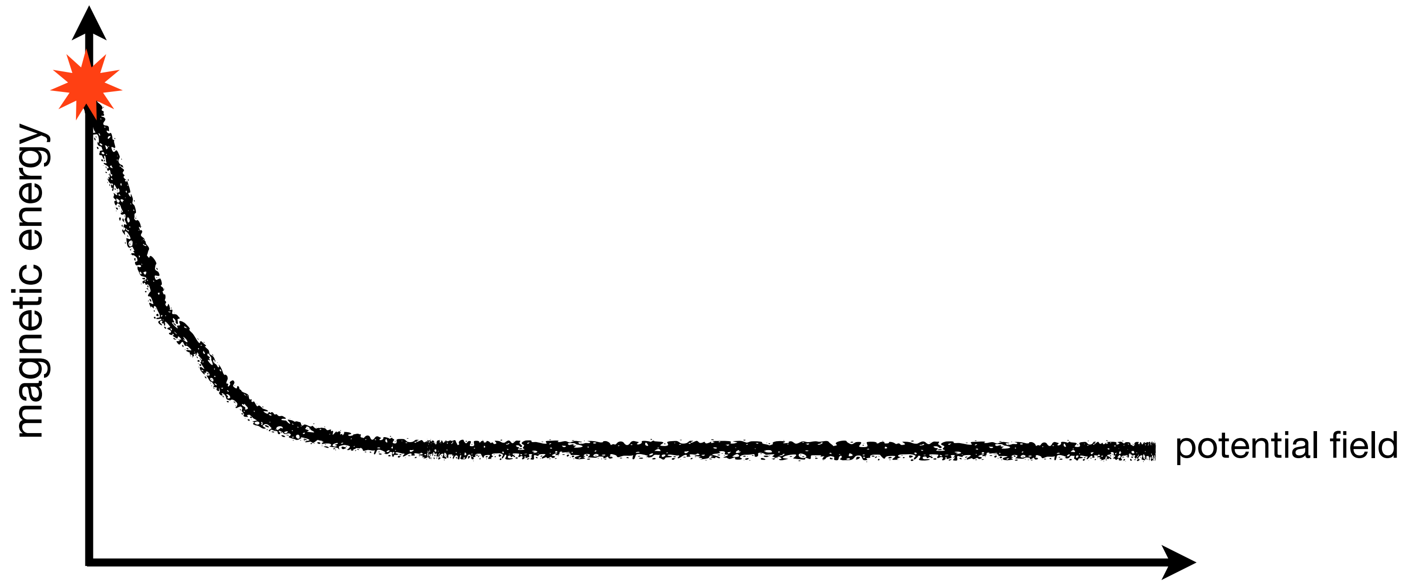
- Minimise $\int \frac{B^2}{2}\,dV$ subject to the same boundary conditions.

Which force-free field?

- Non-ideal relaxation is an energy minimisation process.



- Minimise $\int \frac{B^2}{2}\, dV$ subject to the same boundary conditions.

$\Rightarrow$ potential field

Which force-free field?

- Result from lab. plasmas (Taylor, 1974): conservation of total helicity

$$K_0 = \int_{V_0} \mathbf{A} \cdot \mathbf{B} \, dV$$

- Minimise $I = \int \frac{B^2}{2} \, dV - \lambda K_0$ via variational principle
- Resulting field is linear force-free, $\nabla \times \mathbf{B} = \alpha \mathbf{B}$, $\alpha \in \mathbb{R}$.
- Process known as Taylor relaxation.
- Parameter α determined by K_0.
- Adapted to the solar case via relative helicity.

Which force-free field?

- Non-ideal relaxation is an energy minimisation process.

- Minimise $\int \frac{B^2}{2} \, dV$ subject to the same boundary conditions and conservation of helicity $\Rightarrow$ linear force-free field.

Model: test a braided field

- Suppose photospheric motion occurs in random manner: braid has no net twist.
- Any sufficiently complex random braid will have a component of the pigtail type.
- Realistic aspect ratio, only 1% energy above that of the corresponding potential field.
- Field: uniform background + 6 isolated magnetic flux rings.

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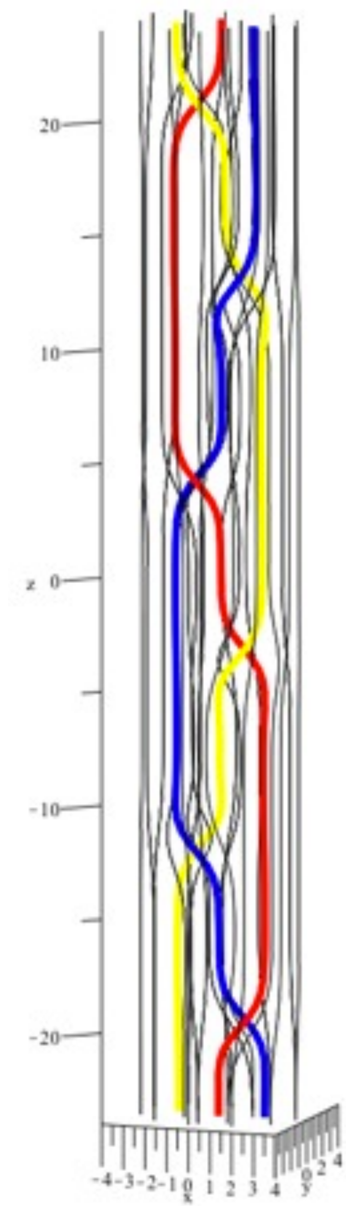

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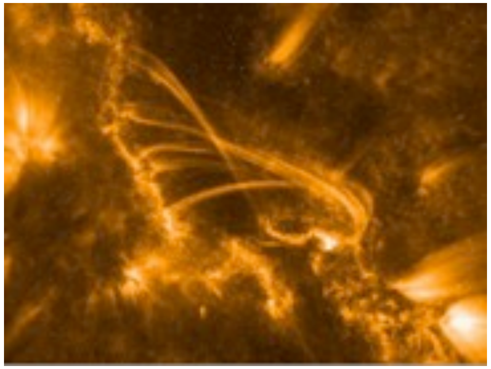

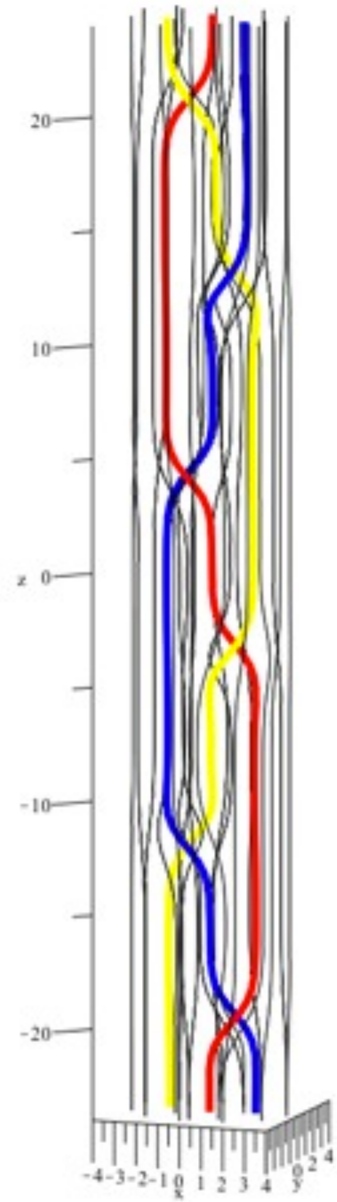

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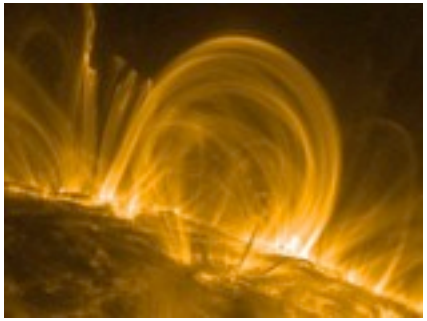
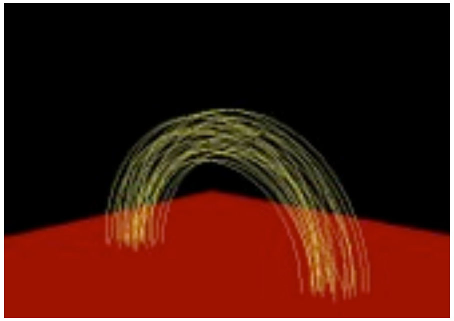
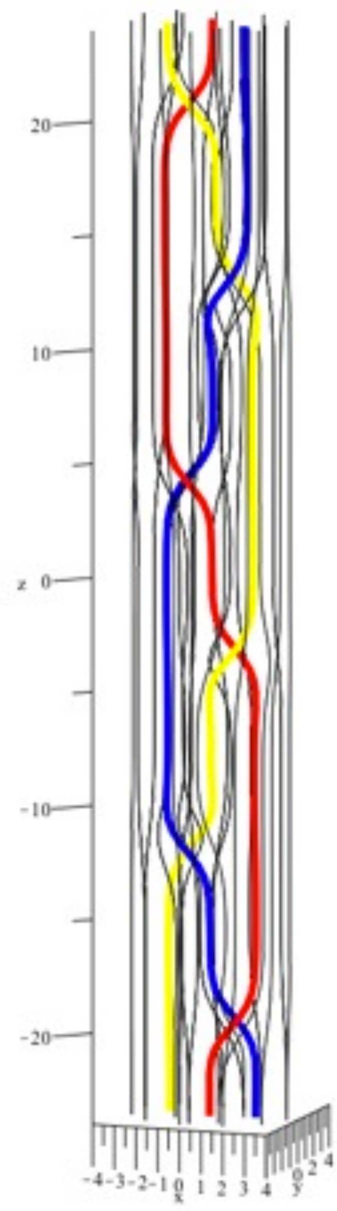
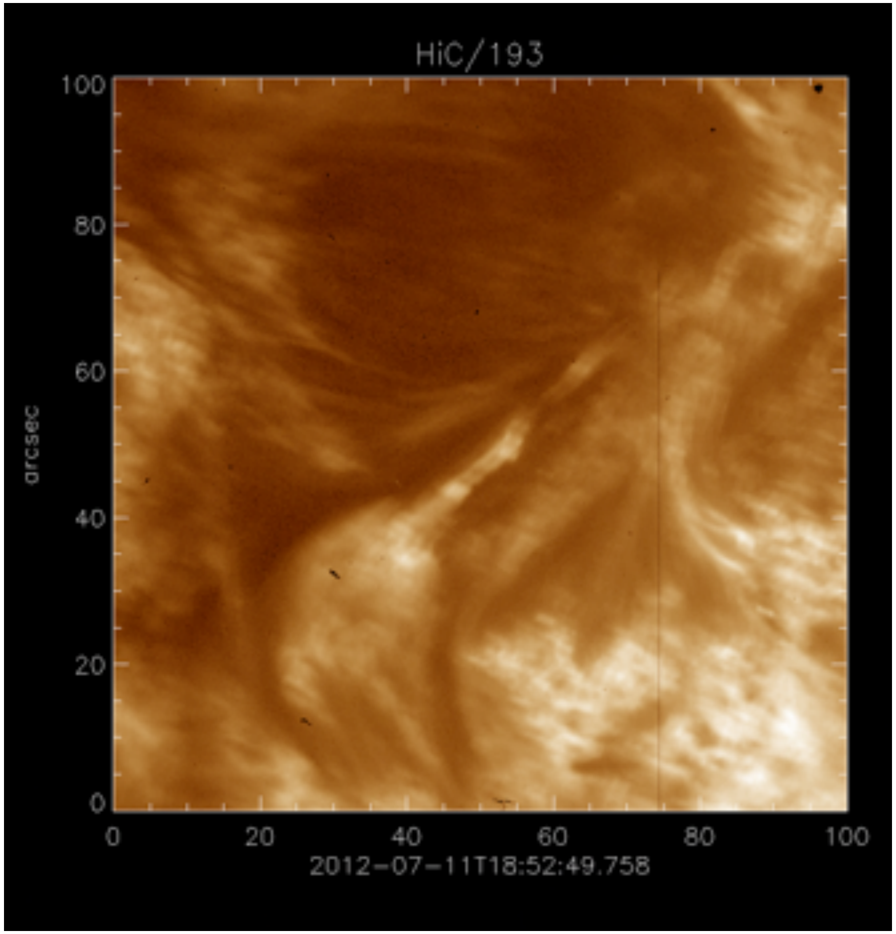
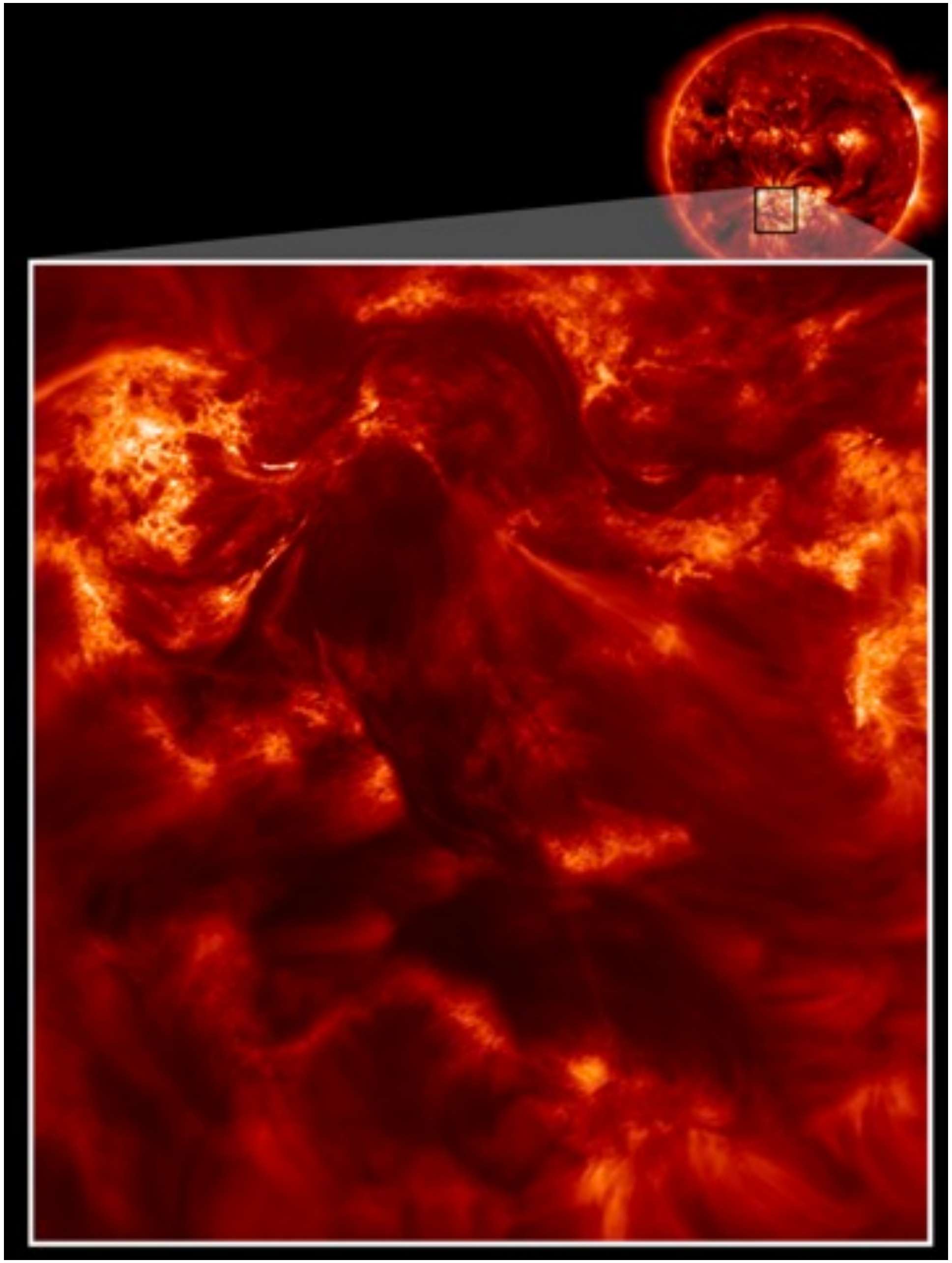






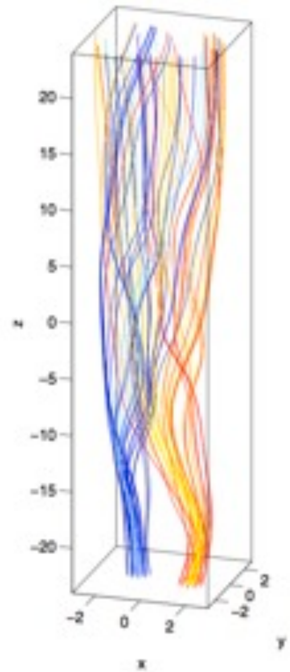
Hi-C 193Å, 0.2” resolution, C1.7 flare 11/07/12





Resistive MHD simulation

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E},$$
$$\mathbf{E} = -(\mathbf{v} \times \mathbf{B}) + \eta \mathbf{J},$$
$$\mathbf{J} = \nabla \times \mathbf{B},$$
$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}),$$
$$\frac{\partial}{\partial t}(\rho \mathbf{v}) = -\nabla \cdot (\rho \mathbf{v}\mathbf{v} + \underline{\underline{\tau}}) - \nabla P + \mathbf{J} \times \mathbf{B},$$
$$\frac{\partial e}{\partial t} = -\nabla \cdot (e\mathbf{v}) - P\nabla \cdot \mathbf{v} + Q_{visc} + Q_J,$$

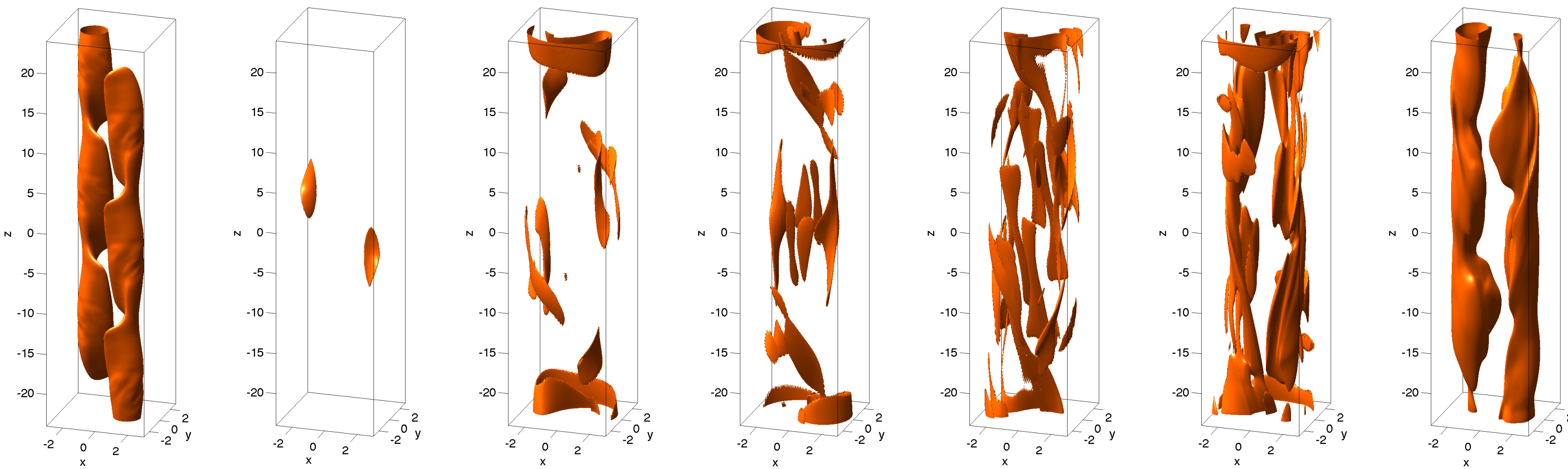


Full domain:
$[-6,6]^2 \times [-24,24]$

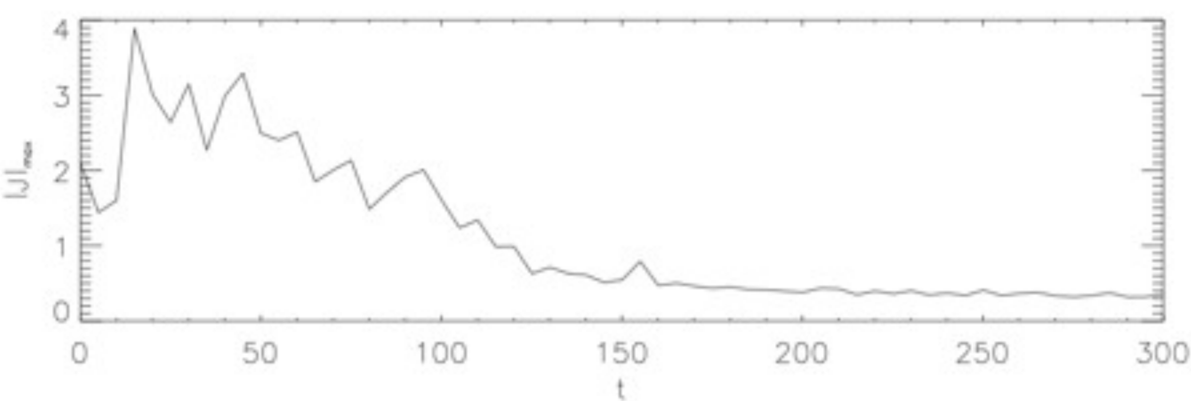
Braid in:
$[-3,3]^2 \times [-24,24]$

- Copenhagen *Stagger* code: rMHD finite difference scheme.
- Take uniform $\eta=10^{-3}$, 512^3 grid points, line-tied boundaries.
- Initial plasma beta ~ 0.1.
- Time: units of Alfvén time.

Non-ideal relaxation



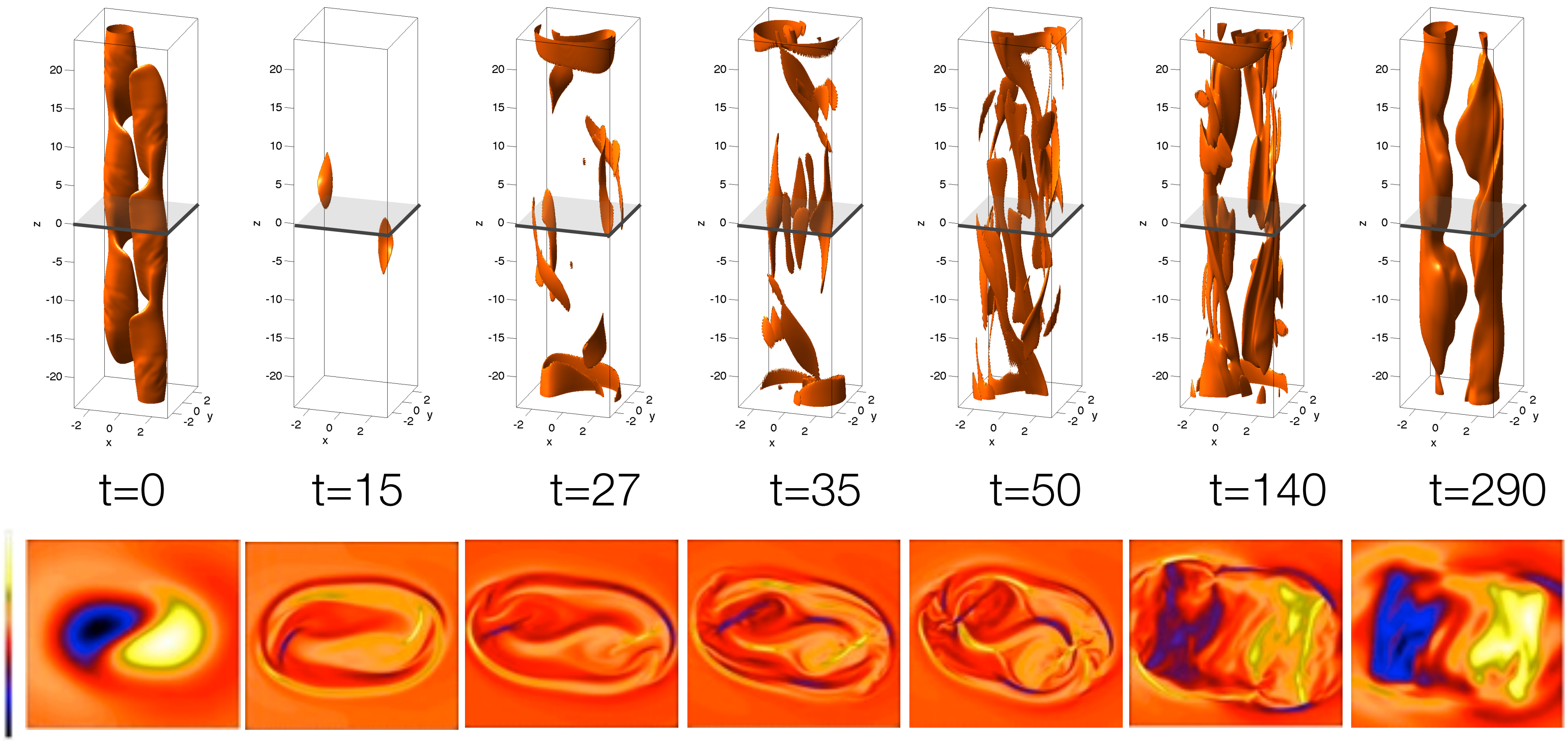
t=0 t=15 t=27 t=35 t=50 t=140 t=290



↑ Isosurfaces of current $|\mathbf{J}| = |\mathbf{J}|_{max}/2$.

← Maximum $|\mathbf{J}|$ in domain with time.

Non-ideal relaxation

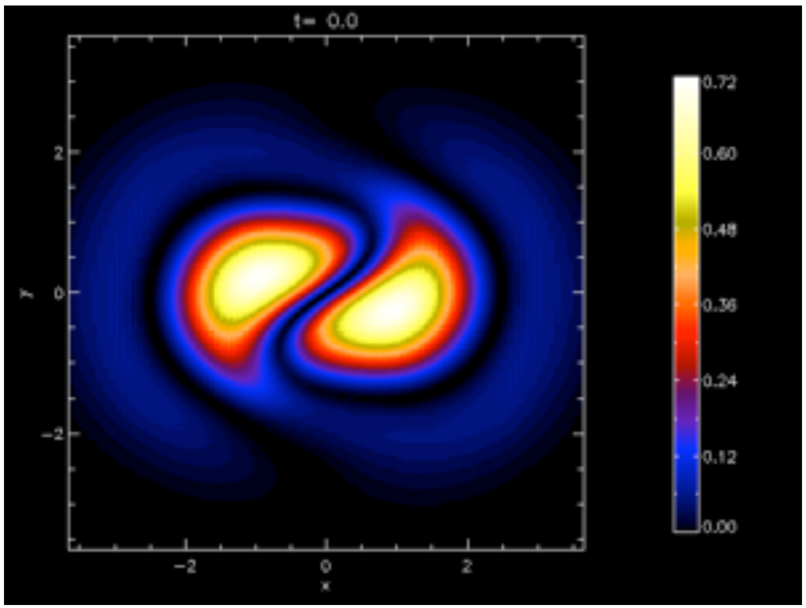


J_z, central plane

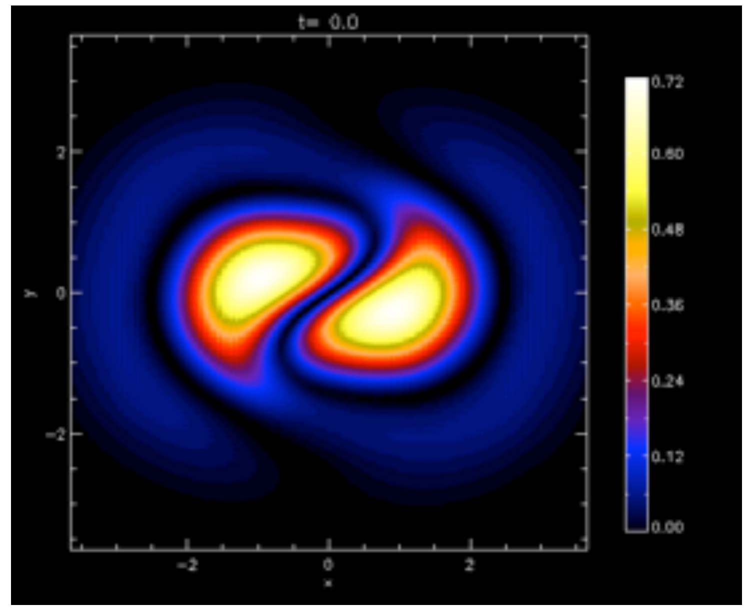
Magnetic Reynolds number comparison

|**J**| at z=0

$\eta=10^{-3}$



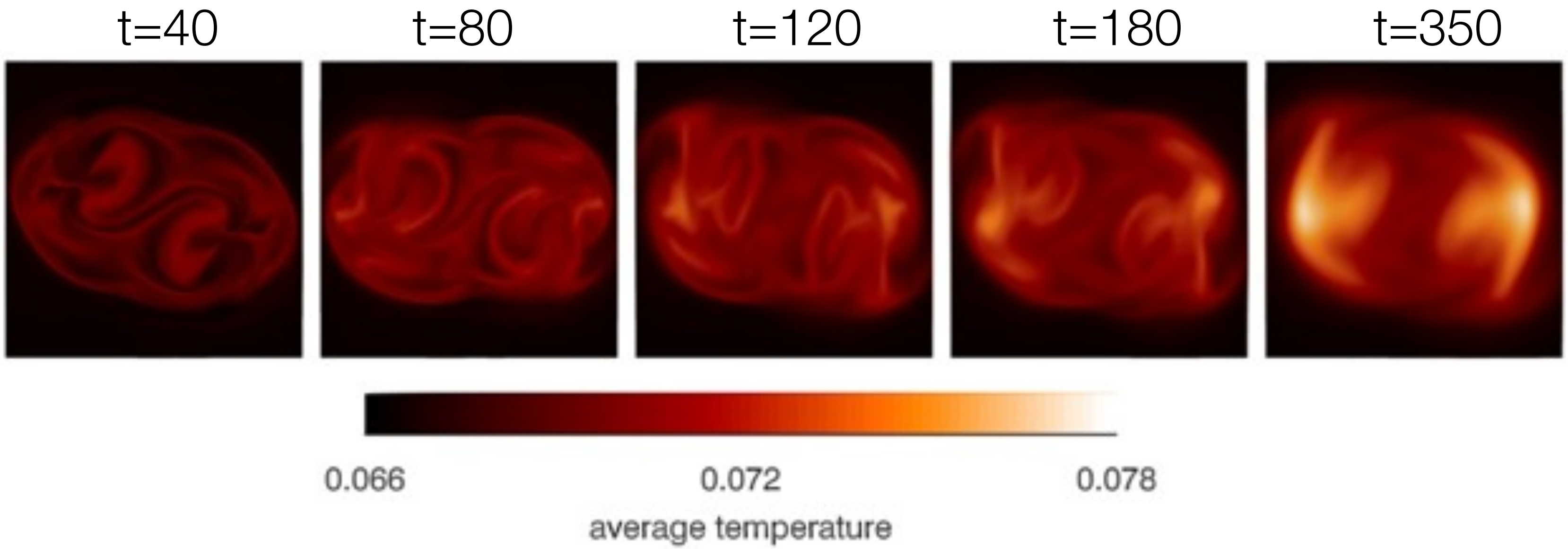
$\eta=2\text{x}10^{-4}$



- Relaxation time increases with magnetic Reynolds number.
- Greater current sheet fragmentation, more & faster rec^{n}.

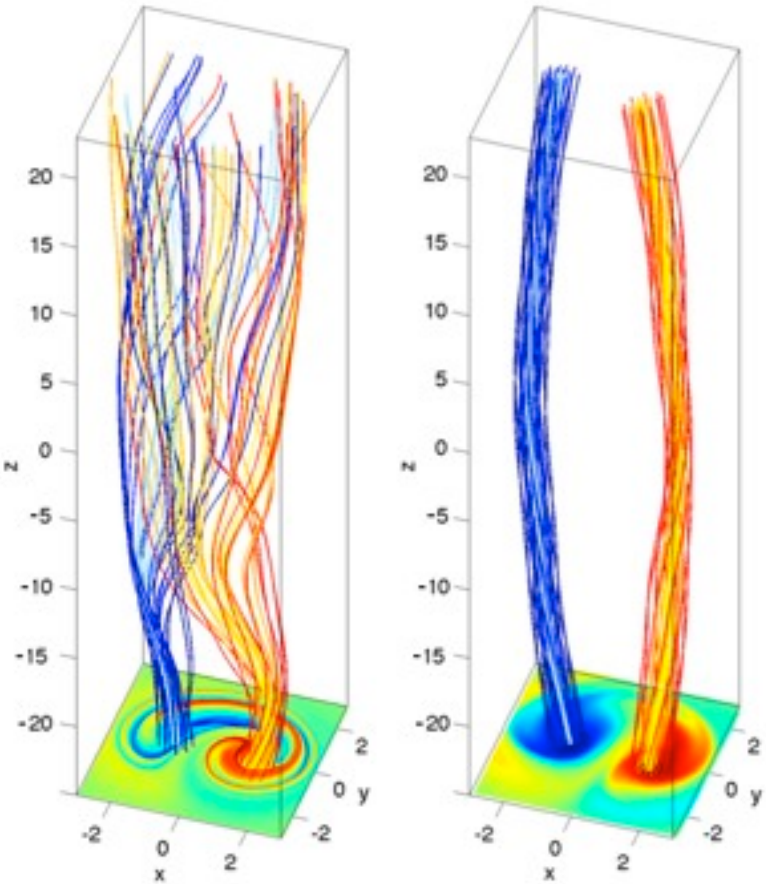
Relaxation: energy release

- 66.2% of free magnetic energy released in the relaxation.
- Homogeneous heating of the loop.



Temperature averaged along magnetic field lines and shown in the z=0 plane.

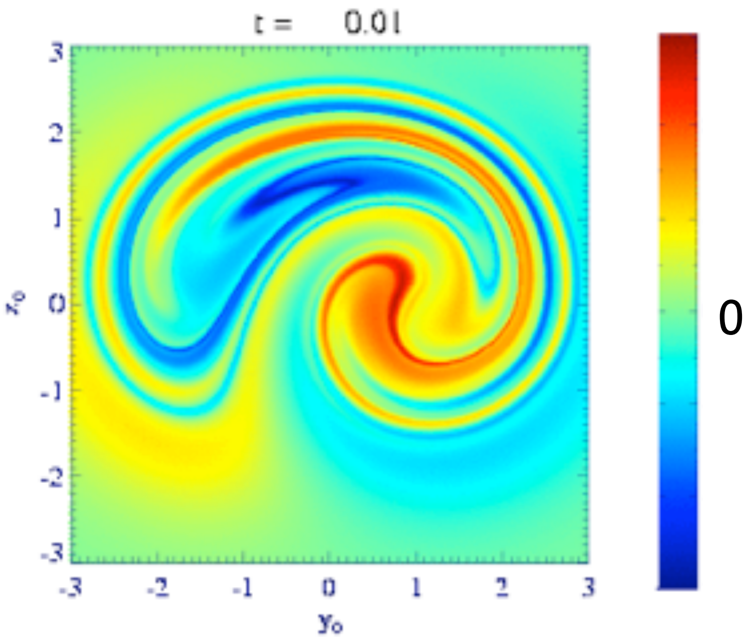
Relaxation: end state



Initial State   Final State

- Trace field lines from two sets of circles on lower boundary.
- Lower boundary shows force-free parameter α ($\nabla\times\mathbf{B} = \alpha\mathbf{B}$).
- Field evolves into two unlinked flux tubes of opposite helicity.
- Final state non-linear fff.
- Overall helicity remains zero.

Relaxation: end state



Force-free parameter
$\alpha^*=\mathbf{j}\cdot\mathbf{B}/\mathbf{B}\cdot\mathbf{B}$ on lower boundary

- Trace field lines from two sets of circles on lower boundary.
- Lower boundary shows force-free parameter α ($\nabla\times\mathbf{B} = \alpha\mathbf{B}$).
- Field evolves into two unlinked flux tubes of opposite helicity.
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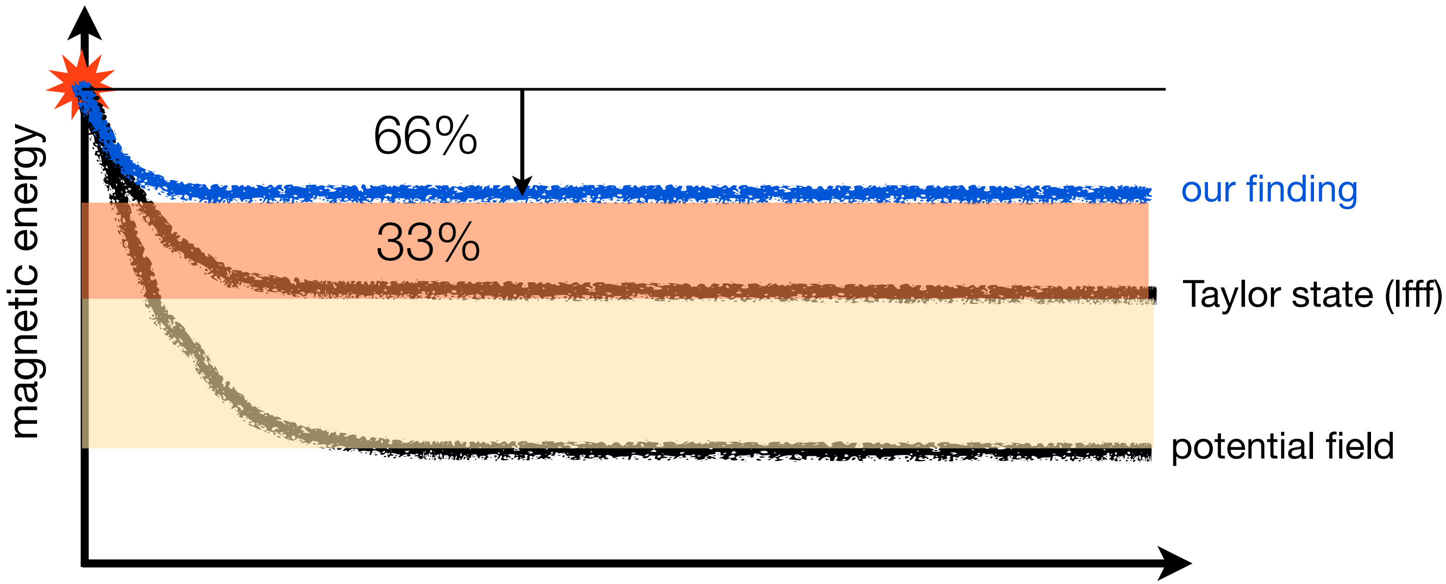
Which force-free field?

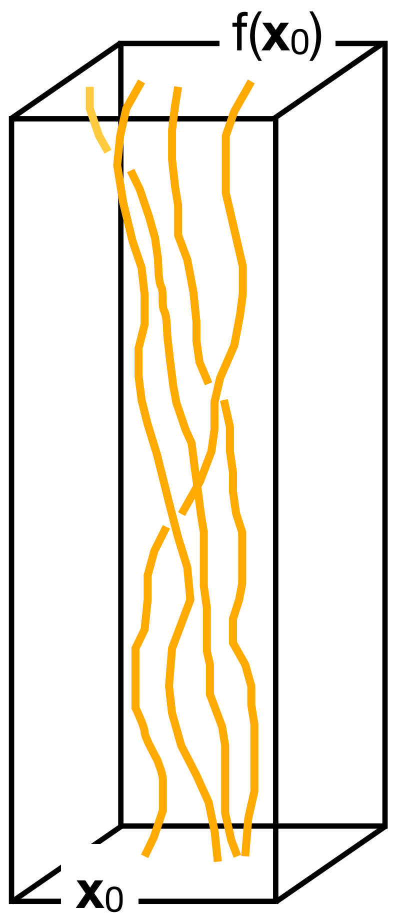
- Non-ideal relaxation is an energy minimisation process.



- Minimise $\int \frac{B^2}{2}\, dV$ subject to the same boundary conditions and conservation of helicity $\Rightarrow$ linear force-free field.

Suggests additional constraint acts!

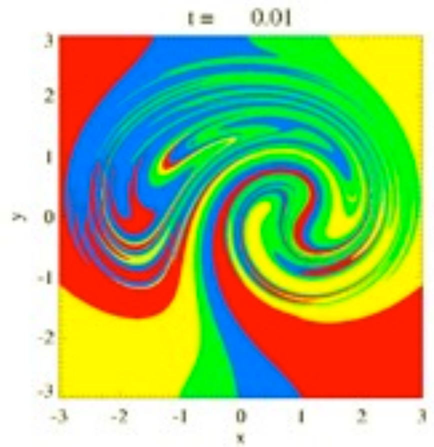
Topological Degree

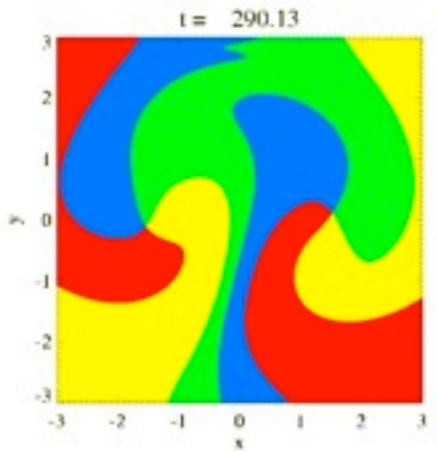


- Field line mapping $f(x_0,y_0)$ from lower to upper boundary includes number of fixed points.
- Generic fixed points can be elliptic (+1) or hyperbolic (-1)
- Sum of all fixed points gives index of field.
- Index of field = index of boundary.
- Index conserved during relaxation.
- Taylor state can't be reached if index incompatible with initial state.

Topological Degree

Initial State: index +2





Final State: index +2

- Field line mapping $f(x_0,y_0)$ from lower to upper boundary includes number of fixed points.
- Generic fixed points can be elliptic (+1) or hyperbolic (-1).
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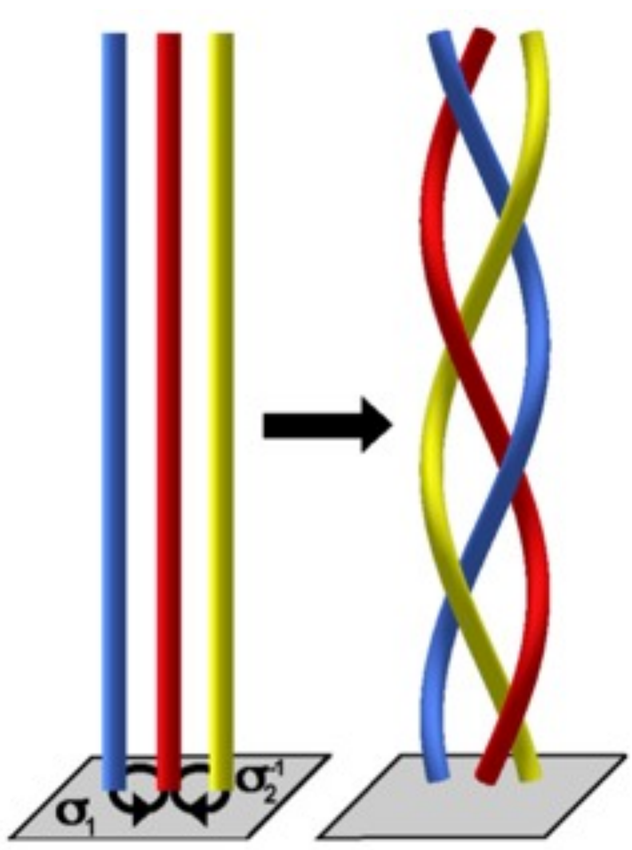
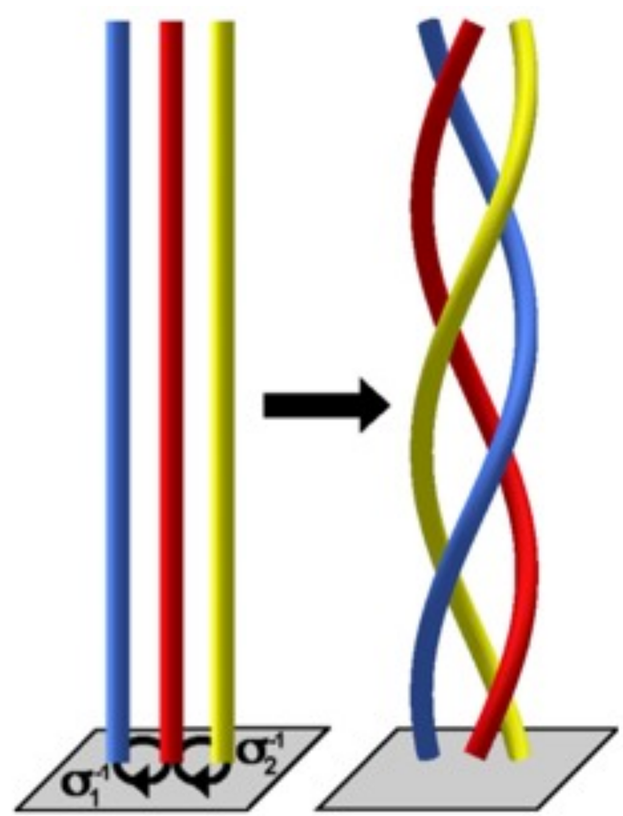
Complex vs. Coherent Braiding

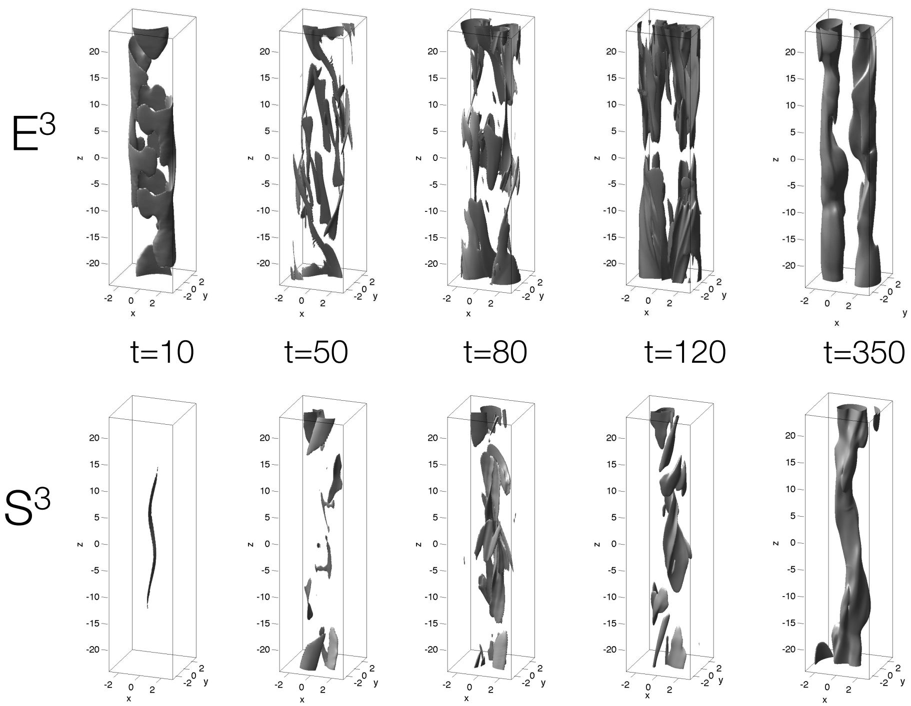




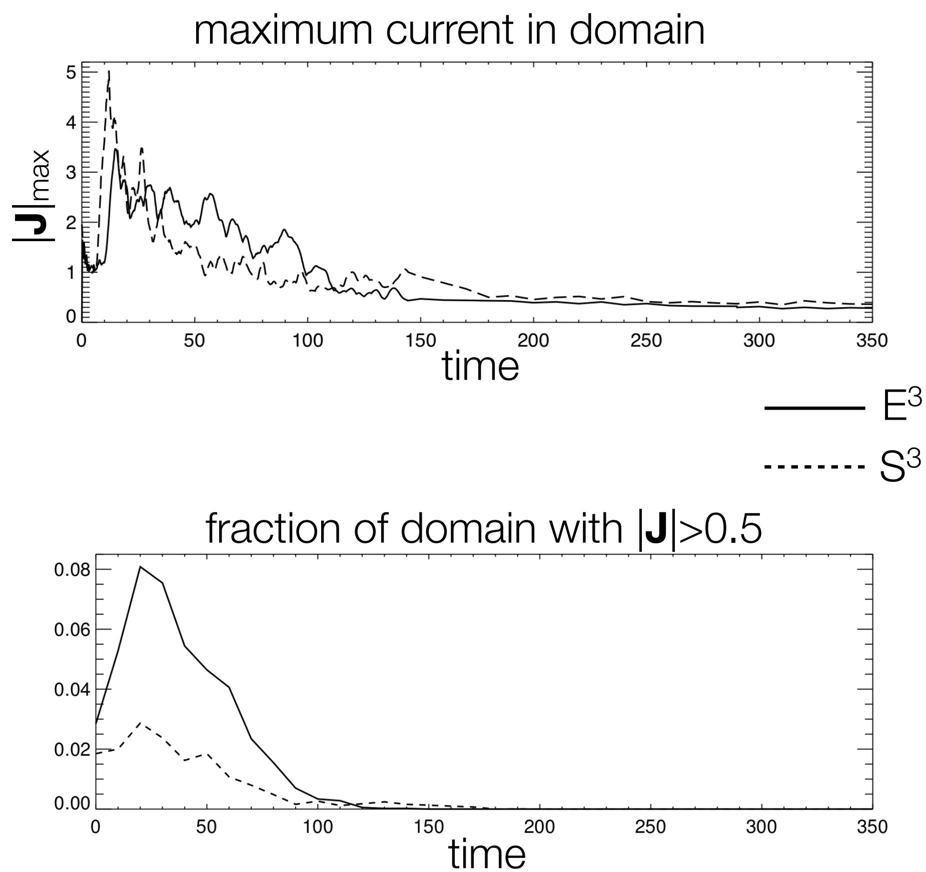
E^3 – oppositely directed rotational stirring motions

S^3 – same sign rotational stirring motions

Complex vs. Coherent Braiding



Isosurfaces at 50% $|\mathbf{J}|_{max}$



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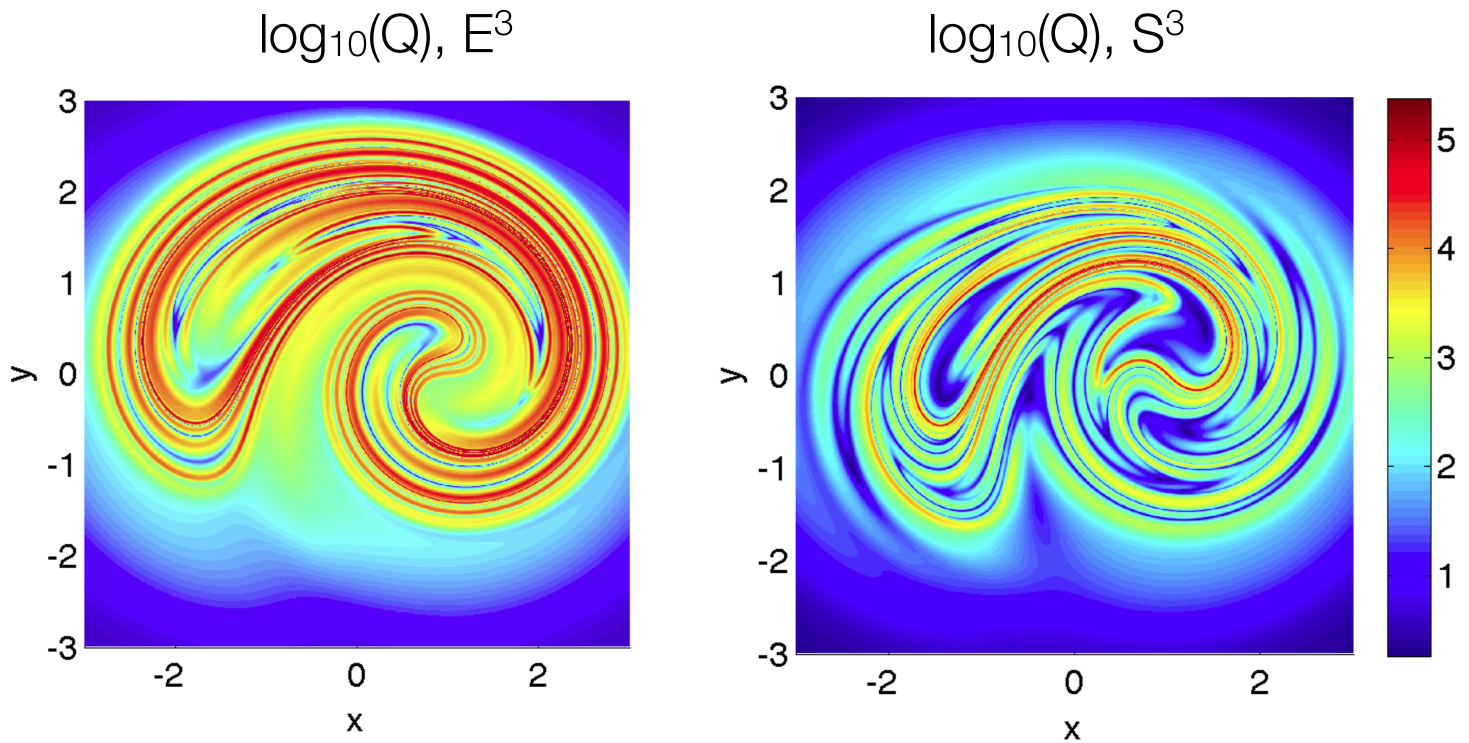
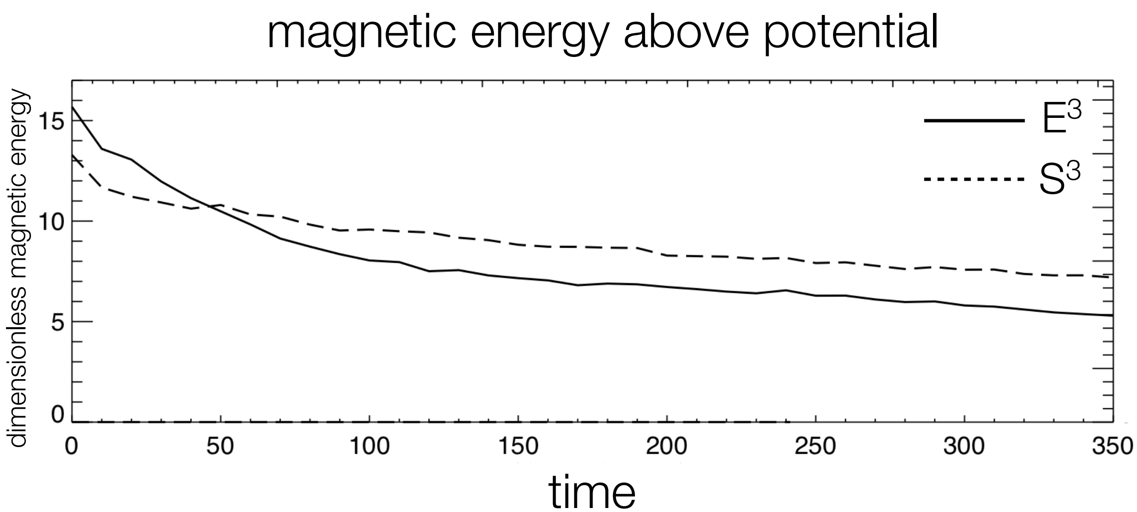


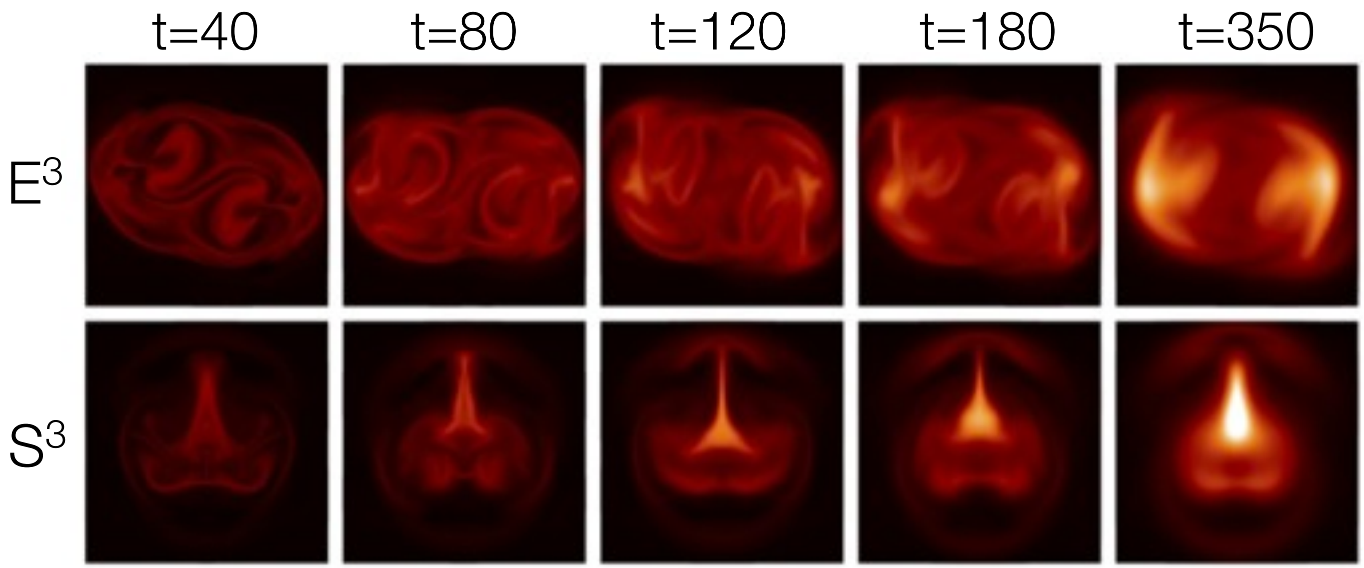
- Topological entropy: $T(E^3) \approx 3.3$, $T(S^3) \approx 2.3$
- Estimate with algorithm of Thiffeault (Chaos 20, 017516, 2010).

Complex vs. Coherent Braiding



- 66.2% free energy released for E^3, 45.8% for S^3.
- Higher degree of complexity in initial state of E^3 allows for a more efficient relaxation.
- Complex braiding leads to homogeneous heating, coherent braiding to local heating.

Complex vs. Coherent Braiding



Temperature averaged along field lines, z=0

- 66.2% free energy released for E^3, 45.8% for S^3.
- Higher degree of complexity in initial state of E^3 allows for a more efficient relaxation.
- Complex braiding leads to homogeneous heating, coherent braiding to local heating.

Conclusions

- Sufficiently complex braids are incompatible with a stationary equilibrium. Instability can occur even when free magnetic energy is low (~1% in our example).
- Constraint above conservation of total helicity acts in non-ideal field relaxations, limiting energy release.
- Complex photospheric motions lead to homogeneous loop heating while coherent motions to localised heating.