

peacemoments - B-field Aligned Moments

velocity and heat flux vector

Given the GSE velocity $\mathbf{v}$ and the magnetic field $\mathbf{B}$, the projections of $\mathbf{v}$ parallel and perpendicular to $\mathbf{B}$ are given as:

$$\begin{aligned}\mathbf{v}_{\text{para}} &= (\mathbf{v} \cdot \mathbf{b}) \mathbf{b} \\ \mathbf{v}_{\text{perp}} &= \mathbf{v} - (\mathbf{v} \cdot \mathbf{b}) \mathbf{b}\end{aligned}$$

where $\mathbf{b}$ is the magnetic field normalized to 1, i.e. $\mathbf{b} = \mathbf{B} / |\mathbf{B}|$. These vectors are given in the output of the *peacemoments* software when the “B-field Aligned” output option is chosen.

Note that also:

$$\mathbf{v} = \mathbf{v}_{\text{para}} + \mathbf{v}_{\text{perp}}$$

and the magnitude of the GSE velocity is given by:

$$|\mathbf{v}| = \text{Sqrt}[\mathbf{v}_{\text{para}}^2 + \mathbf{v}_{\text{perp}}^2]$$

Note that $\mathbf{v}_{\text{para}}$ is not the same as $\mathbf{v}$ rotated to a coordinate system in which the z-axis is parallel to the magnetic field, which is given by:

$$\mathbf{v}_{\text{BFA}} = \mathbf{R}\mathbf{v}$$

where the rotation matrix $\mathbf{R}$ is given by:

$$\mathbf{R} = \begin{bmatrix} C & -b_x b_y / C & -b_x b_z / C \\ 0 & b_y / C & b_z / C \\ b_x & b_y & b_z \end{bmatrix} \quad (1)$$

where $C = \text{Sqrt}[b_y^2 + b_z^2]$. Here $\mathbf{b}$ is the normalized B-field vector.

The relations between $\mathbf{v}_{\text{BFA}}$, $\mathbf{v}_{\text{para}}$ and $\mathbf{v}_{\text{perp}}$ are:

$$v_{\text{para}}^2 = v_{\text{BFA}, z}^2$$

$$v_{\text{perp}}^2 = v_{\text{BFA}, x}^2 + v_{\text{BFA}, y}^2$$

pressure tensor and temperature

Firstly, the pressure tensor is rotated so that the z-axis is parallel to the magnetic field:

$$\mathbf{P}_{\text{BFA}} = \mathbf{R}^T \mathbf{P} \mathbf{R}$$

where the rotation matrix $\mathbf{R}$ is given in Eq. (1). Then

$$P_{\text{para}} = P_{\text{BFA}, zz}$$

$$P_{\text{perp}, 1} = P_{\text{BFA}, xx}$$

$$P_{\text{perp}, 2} = P_{\text{BFA}, yy}$$

$$P_{\text{perp}} = (P_{\text{BFA}, xx} + P_{\text{BFA}, yy})/2$$

The temperature is then given by

$$T_{\text{para}} = P_{\text{para}} / Nk$$

$$T_{\text{perp}, 1} = P_{\text{perp}, 1} / Nk$$

$$T_{\text{perp}, 2} = P_{\text{perp}, 2} / Nk$$

$$T_{\text{perp}} = P_{\text{perp}} / Nk$$

where N is the number density and k is Boltzmann's constant (1.3807×10^{-23}).

The relationship between T , T_{para} and T_{perp} is:

$$T = (2T_{\text{perp}} + T_{\text{para}}) / 3$$