CLUSTER II – Rotation from S/C to GSE coordinates

based on an email by Steve Schwartz; expanded by Andrew Lahiff

1. Rotate by SUNAZ degrees about the XB axis to a primed coordinate system in which the sun at the spin start lies in the x'-y' plane. SUNAZ is given by:

   SUNAZ = 26.2 - SPOS*360/(16*1024)

where 26.2 is the angle (in degrees) between the sun sensor and the YB axis, and

SPOS = 176 (before 00:05:00 August 17th, 2003)
SPOS = 1200 (after 00:05:00 August 17th, 2003)

2. We now will construct the rotation matrix – it is made from the vectors ex', ey' and ez'.
   AUX contains the spin axis latitude (λ) and longitude (ϕ). Note that

   latitude $\lambda = \phi$ (here ϕ_{GSE} is the azimuthal angle)
   longitude $\phi = 90 - \theta_{GSE}$ (here θ_{GSE} is the polar angle)

Now, ex' is the spin axis, the components of which in GSE are known:

$(\cos\lambda \cos\phi, \cos\lambda \sin\phi, \sin\lambda)$

This was obtained using the usual relation between Cartesian and spherical coordinates, and:

$\cos(90 - A) = \sin A$
$\sin(90 - A) = \cos A$

3. ez' is perpendicular to ex_{GSE}, since the sun is the x'-y' plane. Therefore, ez' in GSE coordinates is

   (0, e32, e33)

since (1, 0, 0) . (e31, e32, e33) = 0. At this stage we do not know the values of e32 and e33.

4. ez' is perpendicular to ex', therefore:

$$(\cos\lambda\cos\phi\,,\ \cos\lambda\sin\phi\,,\ \sin\lambda)\ .\ (0,\ e32,\ e33) = 0$$

This gives a relation between e32, e33, λ and ϕ , i.e.

$$e33 = -\ (\cos\lambda\sin\phi\ /\ \sin\lambda)\ e32$$

The vector ez' is then given by:

$$(0, 1, -\cos\lambda\sin\phi\ /\ \sin\lambda)\ e32$$

This needs to be normalized to 1. This gives:

$$e32 = \sin\lambda\ /\ \mathrm{Sqrt}[\cos^2\lambda\sin^2\phi + \sin^2\lambda] = \beta\sin\lambda$$

Therefore ez' is given by

$$(0,\ \sin\lambda\,,\ -\cos\lambda\sin\phi)\ \beta$$

5. Next, ey' is perpendicular to both ez' and ex', hence:

$$ey' = ez'\ x\ ex'$$

This gives:

$$ey'(0) = \beta\ (\sin^2\lambda + \cos^2\lambda\sin^2\phi)$$
$$ey'(1) = -\ \beta\cos^2\lambda\ \cos\phi\sin\phi$$
$$ey'(2) = -\ \beta\cos\lambda\ \sin\lambda\cos\phi$$

6. The rotation matrix is then given by:

$$R = x'\ ex' + y'\ ey' + z'\ ez'$$