Gravitational Wave Astronomy

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Lectures 1 & 2

Gravitational wave generation, propagation and detection

Lectures 3 – 5

The science enabled by gravitational wave observations

Gravitational waves from the coalescence of a binary black hole system (credit GSFC)

PENNSTATE 1855

Relevance of gravitational waves in astronomy

Gravitation powers the most energetic – and energetically efficient – astronomical phenomena

Type II supernovae powered by stellar core collapse

GRBs, quasars powered by accretion onto rapidly rotating black hole

Coalescing black hole binaries radiate at a rate of ~10^{57} erg/s over ~10^{-4} $(M/M_{\odot})$ s!

The central engines of gravitationally powered phenomena are electromagnetically obscured or invisible

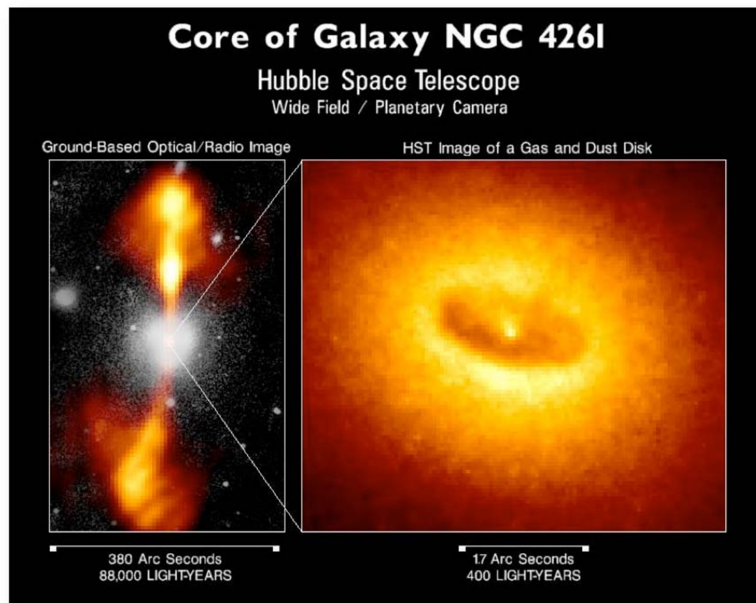
Energetic phenomena are associated with strong gravitational fields

$$1 \sim \left(\frac{v}{c}\right)^2 \sim \frac{GM}{c^2 L} \sim \phi_G$$

Energetic

Virial Thm

Gas, dust accumulate in potential & obscure engine powering eletromagnetic emission



Gravitational radiation carries the signature of the central engine's dynamics

Gravitational, electromagnetic wave analogy

Charge? Mass

Current? Momentum

Radiation? Time varying mass, current distributions

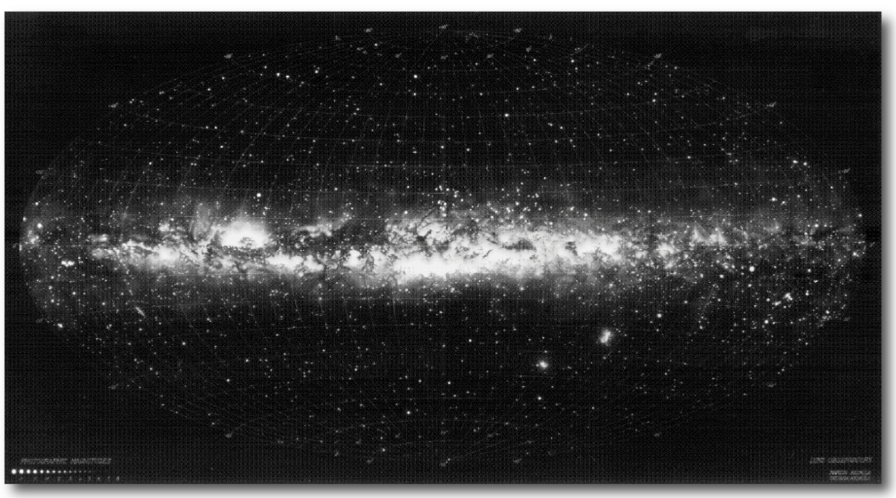
Gravitational radiation couples *very* weakly to matter

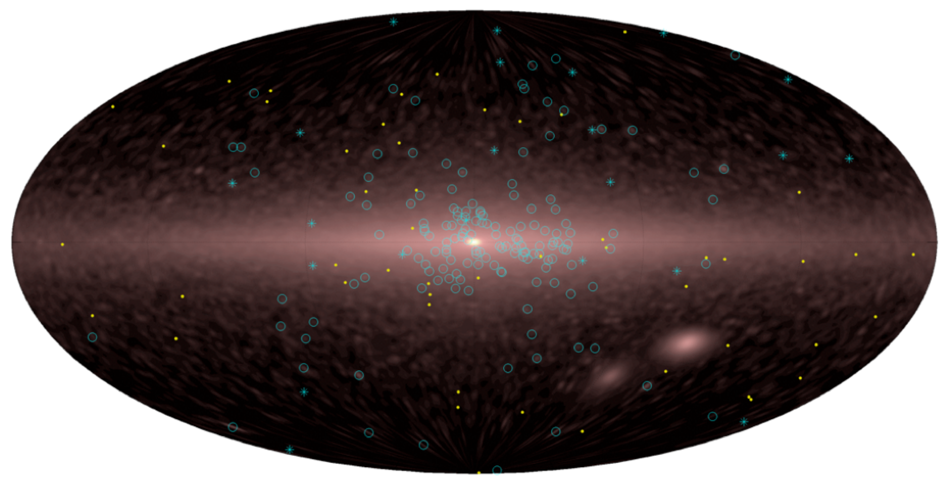
No (negligible) reddening

No (negligible) extinction

Undistorted view of central engine dynamics

No gravitational wave zone of avoidance!





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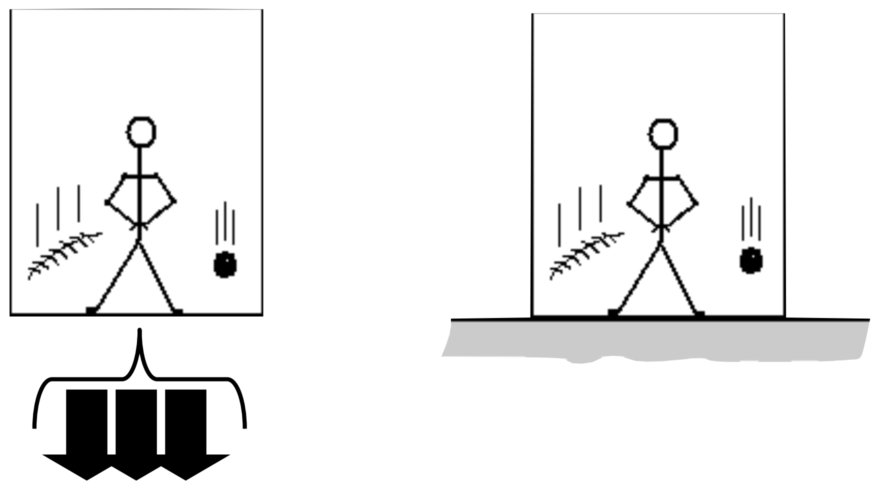
Understanding gravitational radiation: Gravitation, generally

Tidal forces are the real manifestation of gravitation



Equivalence Principle

Free test-body trajectories depend only on initial position, velocity

Can't distinguish gravity from coordinate system acceleration on basis of single trajectory

Gravity is what can't be explained by coordinate system acceleration

"Tidal force" acting on separation between nearby trajectories

$$\frac{\partial^2}{\partial t^2}\left(z_1^j - z_2^j\right) \qquad \left(\frac{\partial^2 \phi_G}{\partial x^j \partial x^k}\right)\left(z_1^k - z_2^k\right)$$

Gravity is tidal acceleration

Gravity is a space-time metric phenomena

Metric **describes separation between points in space-time...**

Space-time interval

Metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

Infinitesimal coordinate separation between space-time points

Change in rate of separation is tidal acceleration...

$$\frac{\partial^2}{\partial t^2}\left(z_1^j - z_2^j\right) \qquad \left(\frac{\partial^2 \phi_G}{\partial x^j \partial x^k}\right)\left(z_1^k - z_2^k\right)$$

Tidal acceleration is related to difference between Minkowski and space-time metrics

Minkowski metric

Metric perturbation

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\ddot{h}_{jk} \sim \frac{\partial^2 \phi_G}{\partial x^j \partial x^k}$$

Understanding gravitation: Gravitational wave generation

Weak gravitational waves in Minkowski space can be understood by analogy with electromagnetic waves

	Maxwell	**Einstein**
Source	Four-current density	Stress-Energy density
"Potential"	A_μ	$h_{\mu\nu}$
Force Law	Lorentz	Geodesic Equation with metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
Gauge group	U(1)	Poincare group (general coordinate transformations)
"Field Equations" in "Lorentz Gauge"	$\Box A_\mu = -4\pi J_\mu$ $\frac{\partial A^\mu}{\partial x^\mu} = 0$	$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^2} T_{\mu\nu}$ $\frac{\partial}{\partial x^\mu} \bar{h}^\mu_\nu = 0$ $(\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h$ $h = \eta^{\alpha\beta} h_{\alpha\beta})$

Electromagnetic waves are dipole at leading order

Solution of the wave equation...

$$\Box A^\mu = -4\pi J^\mu$$

$$A^\mu(t,\mathbf{x}) = -4\pi \int d^3x' \, \frac{J^\mu(t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}, \mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}$$

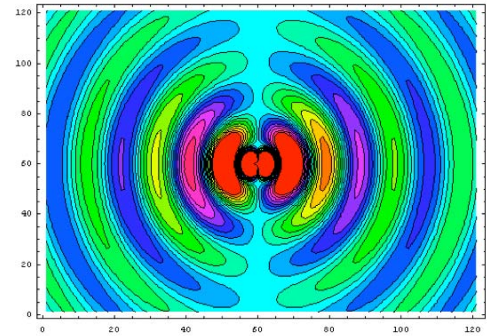
Expand |x–x'| in integrand for small |x'| and keep radiation (1/r) term...

$$\frac{1}{|\mathbf{x}|}\left[J^\mu(\mathbf{x}') + \frac{x'^j}{c}\dot{J}^\mu(\mathbf{x}') + \ldots\right]_{\rm ret}$$

Monopole contribution to radiation field vanishes

Charge conservation: total current in closed system sums to zero!

Dipole gives first non-vanishing contribution to radiation field



Gravitational waves are quadrupolar at leading order in velocity

Monopole radiation? None: mass is conserved

Dipole radiation? None: Dipole is center of mass, which – for a closed system – is unaccelerated

First non-vanishing contribution to radiation arises from quadrupole moment

$$h_{jk}^{\rm TT} = \frac{2G}{c^2 r}\left[\ddot{I\!\!\!-}_{jk}\right]_{\rm ret}^{\rm TT}$$

"Transverse-Traceless" gauge specialization. Project *transverse* to direction of wave propagation and remove trace

Trace-free quadrupole moment of matter distribution

$$I\!\!\!-_{jk} = \int d^3x \left[x^j x^k - \frac{x^2}{3}\delta_{jk}\right]\rho(\vec{x})$$

Gravitational waves are too weak to imagine a Hertz-type experiment

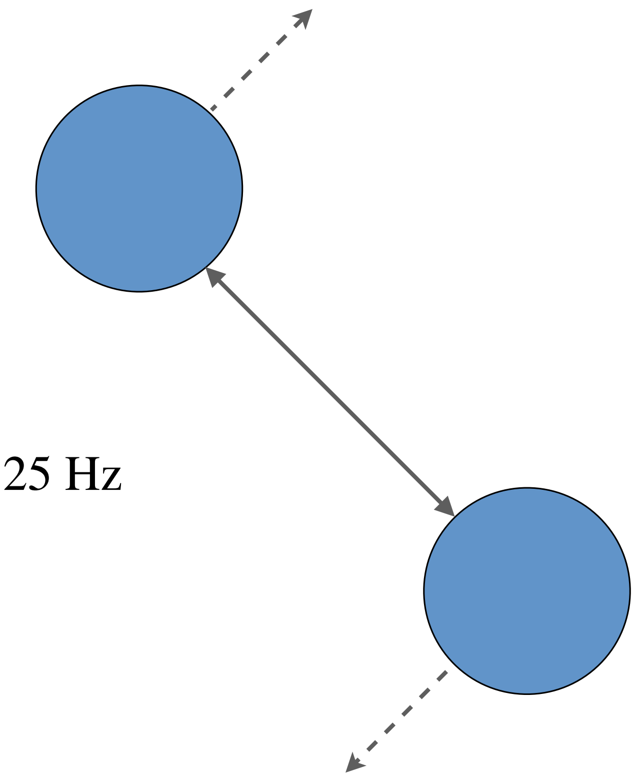
Spinning dumbbell

$$h \sim 10^{-39} \frac{1\,\mathrm{km}}{r} \frac{M}{1000\,\mathrm{kg}} \left(\frac{v}{300\,\mathrm{m/s}} \right)$$

Binary neutron star system

$$h \sim 10^{-23} \frac{100\,\mathrm{Mpc}}{r} \boxed{\frac{M}{3\mathrm{M}_\odot} \quad \frac{150\,\mathrm{km}}{R}}$$

$f_{\mathrm{orb}} \sim 125$ Hz



Luminosity

$$\begin{aligned} L &= \frac{1}{32 G c \pi} \int d^2\Omega r^2 \dot{h}^{\mathrm{TT}}_{jk} \dot{h}^{\mathrm{TT}}_{jk} \\ &= \frac{1}{5} \frac{G}{c^5} \dddot{\mathcal{I}}_{jk} \dddot{\mathcal{I}}_{jk} \\ &\sim \frac{G}{c^5} \left(\frac{M v^2}{T} \right)^2 \sim \frac{\mathrm{K.E.}/T}{c^5/G} \frac{\mathrm{K.E.}}{T} \end{aligned}$$

$$\frac{c^5}{G} = 3.6 \times 10^{59} \frac{\mathrm{erg}}{\mathrm{s}} \quad (!)$$

Example: Gravitational waves from a binary star system

Two point masses, $m_1=m_2=M/2$, in a radius r circular orbit

Non-vanishing quadrupole moment components

$$
\begin{aligned}
I_{xx} &= \frac{Mr^2}{4}\cos^2\omega t \\
I_{xy} &= \frac{Mr^2}{4}\cos\omega t\sin\omega t \\
I_{yy} &= \frac{Mr^2}{4}\sin^2\omega t
\end{aligned}
$$

Remove trace

$$
\begin{aligned}
\bar{I}_{xx} &= \frac{Mr^2}{4}\left[\cos^2\omega t - \frac{1}{3}\right] \\
\bar{I}_{xy} &= \frac{Mr^2}{4}\cos\omega t\sin\omega t \\
\bar{I}_{yy} &= \frac{Mr^2}{4}\left[\sin^2\omega t - \frac{1}{3}\right] \\
\bar{I}_{zz} &= -\frac{Mr^2}{12}
\end{aligned}
$$

Second time-derivatives

$$
\begin{aligned}
\ddot{\bar{I}}_{xx} &= -\frac{M}{2}(\omega r)^2\cos 2\omega t \\
\ddot{\bar{I}}_{xy} &= -\frac{M}{2}(\omega r)^2\sin 2\omega t \\
\ddot{\bar{I}}_{yy} &= \frac{M}{2}(\omega r)^2\cos 2\omega t
\end{aligned}
$$

Project transverse, trace-free

For radiation propagating along z axis...

$$
\begin{aligned}
h_{xx} = -h_{yy} &= -\frac{GM(\omega r)^2}{c^2R}\cos 2\omega t \\
h_{xy} = h_{yx} &= -\frac{GM(\omega r)^2}{c^2R}\sin 2\omega t
\end{aligned}
$$

Luminosity

$$
L = \frac{2G}{5c^5}(\omega r)^4(\omega M)^2
$$

References and things to think about...

References

S. L. Shapiro, S. A. Teukolsky. 1983. *Black Holes, White Dwarfs and Neutron Stars* (Wiley: New York)

P. C. Peters, J. Mathews. 1963. *Gravitational radiation from point masses in a Keplerian orbit.* Phys. Rev. **131**:435

T. X. Thuan, J. P. Ostriker. 1974. *Gravitational radiation from stellar collapse.* Ap. J. Lett. **191**:L105 – L107.

C. Van Den Broeck. 2005. *The gravitational wave spectrum of non-axisymmetric, freely precessing neutron stars.* Class. Quant. Grav. **22**:1825 – 1839.

Things to try

What is the radiation from a rapidly rotating, non-axisymmetric, non-precessing neutron star?

What is the braking index of for a pulsar whose spin-down is dominated by gravitational waves?

Consider a circular orbit binary system, losing energy adiabatically to gravitational waves. How does the binary period change with time?

Next Lecture: Gravitational wave propagation and detection