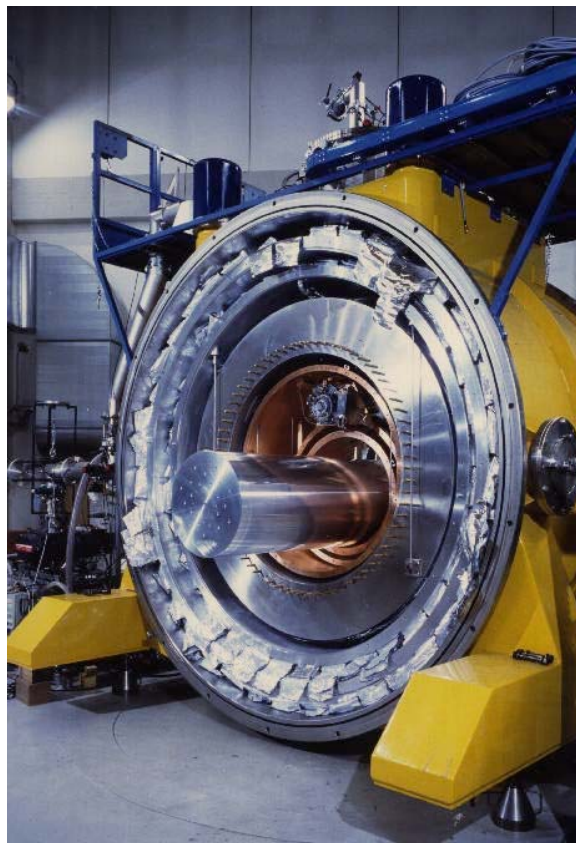
Gravitational Wave Astronomy: Propagation and Detection

Lee Samuel (Sam) Finn

Penn State University



PENNSTATE

Gravitational wave propagation

Gravitational waves have two polarizations states, propagate at *c*, propagate in free-space like EM waves

$$h_{jk}^{\mathrm{TT}} = \frac{2G}{c^2 r}\left[\ddot{I}_{jk}\right]^{\mathrm{TT}}_{\mathrm{ret}}$$

"Transverse-Traceless" gauge specialization. Project *transverse* to direction of wave propagation and remove trace

6	***I_{jk}* has six degrees of freedom**
-1	**Removing trace makes five**
-3	**Projecting transverse to direction of propagation removes three**
2	**Leaving 2 degrees of freedom**

Dispersion relation gives wave phase, group velocities, and geometrical optics propagation properties

$$\Box\bar{h}_{\mu\nu} = 0$$

$$\omega^2/c^2 \quad k^2 = 0$$

Same wave equation as EM, same free-space propagation properties

3

Gravitational waves are not reddened and suffer no extinction

Reddening, extinction complicate interpretation of electromagnetic intensities, spectra

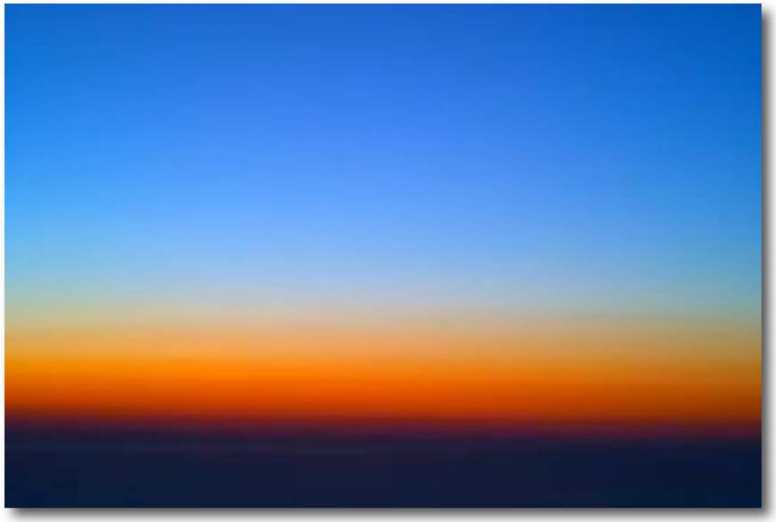
Reddening: relative change in red/blue intensity owing (principally) to Rayleigh & Raman scattering

Extinction: overall loss of intensity owing to scattering, absorption by gas, dust

Compare EM, gw reddening

Rayleigh: large λ scattering off neutral atom by dipole radiation; $\sigma_T \sim = [e^2/4\pi\varepsilon_0 m_e c^2]^2$

Gravitational analog: scattering by induced neutral atom *quadrupole*; $\sigma_G \sim [G m_e/c^2]^2$



$\sigma_G/\sigma_T \sim 7\text{x}10^{-85}$

Gravitational wave detection

In TT coordinates (gauge), free objects at coordinate rest remain at coordinate rest

Free objects move along geodesics

Geodesic equation

$$\frac{d^2x^\mu}{d\tau^2} = -\Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}$$

Connection coefficients

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2}g^{\mu\nu}\left[g_{\nu\alpha,\beta} + g_{\nu\beta,\alpha} - g_{\alpha\beta,\nu}\right]$$

Initial 4-velocity of object at rest

$$\frac{dx^\mu}{d\tau} = (1,0,0,0)$$

TT gauge metric for weak waves propagating in flat space

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

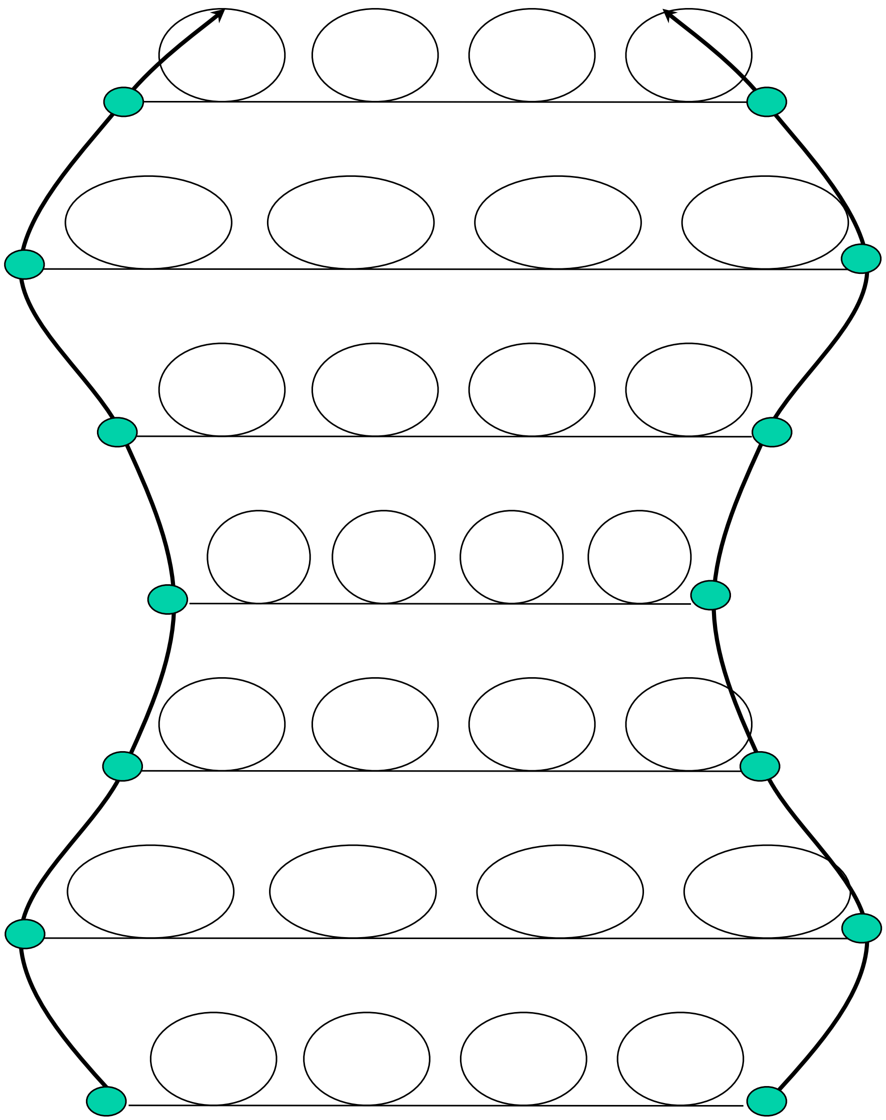
Relevant connection coefficients

$$\Gamma^\mu_{tt} = \frac{1}{2}\eta^{\mu\nu}\left[h_{\nu t,t} + h_{\nu t,t} - h_{tt,\nu}\right]$$

Coordinate acceleration vanishes!

$$\frac{d^2x^\mu}{d^2\tau} = 0$$

Gravitational waves can be detected by monitoring acoustic modes in elastic bodies

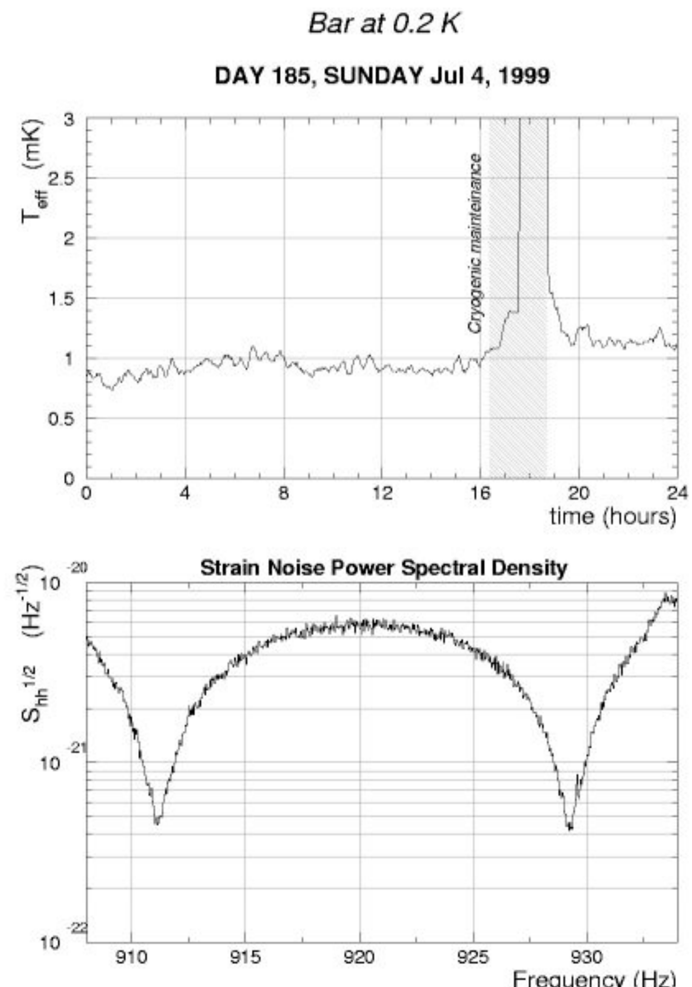


7

Detector noise is characterized by its power spectral density, which is noise power per unit Hz

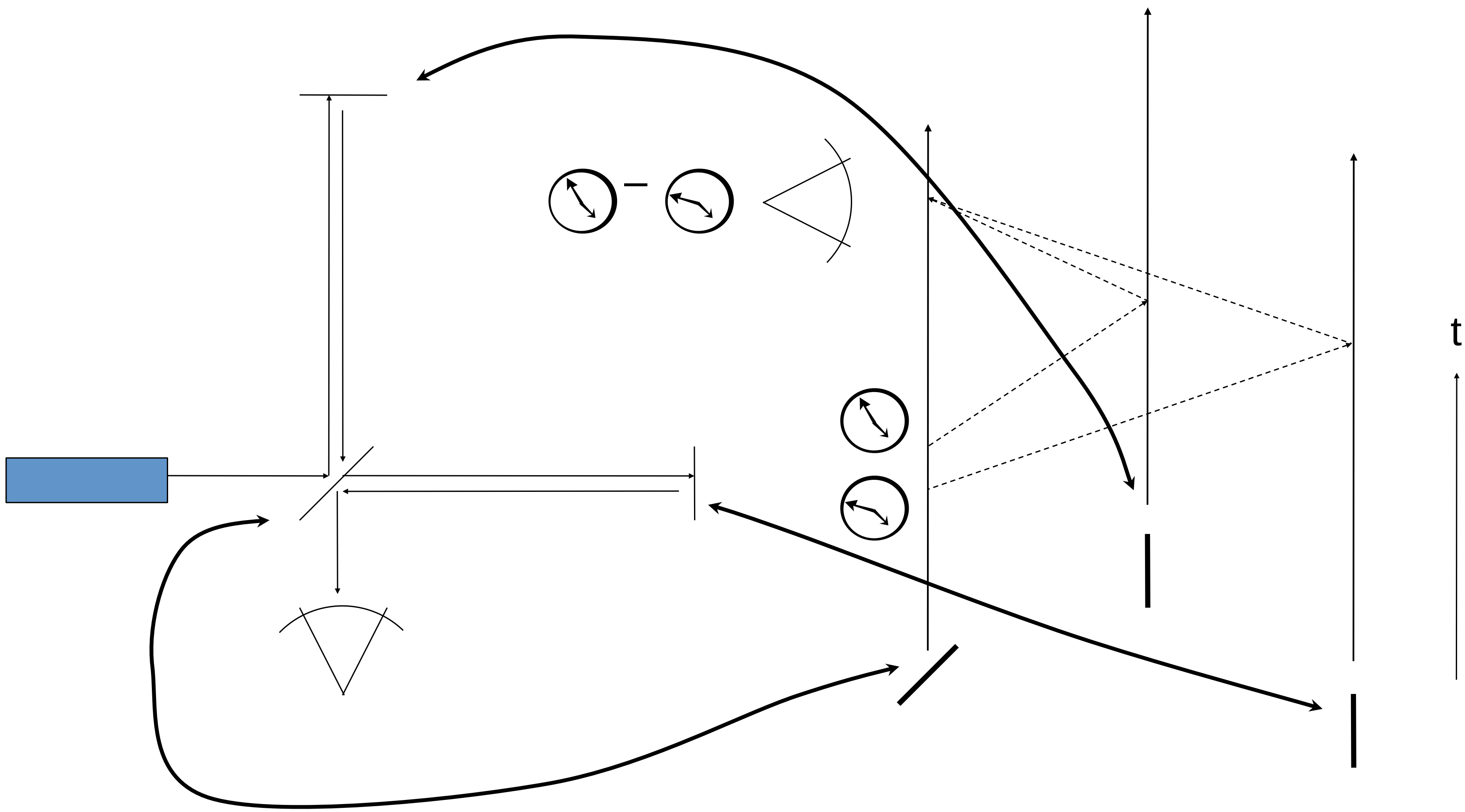
$$
\begin{aligned}
< n^2 > &= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} n(t)^2 dt \\
&= \lim_{T\to\infty} \frac{1}{2T} \int_{-\infty}^{\infty} |\tilde{n}_T(f)|^2 \, df \\
S_n(f) &= \lim_{T\to\infty} \frac{1}{2T} |\tilde{n}_T(f)|^2
\end{aligned}
$$

$$
\begin{aligned}
s &= R * h + n \\
\tilde{s} &= \tilde{R}\tilde{h} + \tilde{n} \\
S_n(f) &= \frac{S_n(f)}{\left|\tilde{R}(f)\right|^2}
\end{aligned}
$$



Auriga

Gravitational waves can be detected by monitoring phase shifts in a free-mass interferometer



Laser Interferometer Gravitational-wave Observatory

Two sites, 3 detectors

Hanford, Washington: 4, 2 km IFO

Livingston, Louisiana: 4 km IFO

Nov 2005 - Oct 2007: "S5" – First design sensitivity science run

"mLIGO" upgrade (initial LIGO x 2 sensitivity) over next 18 months

"Astrowatch": 2 km IFO operates through mLIGO upgrade

"S6": 2 years beginning ~ June 2009

Adv LIGO upgrade (mLIGO x 10)

Upgrade: June 2011 – Jun 2015

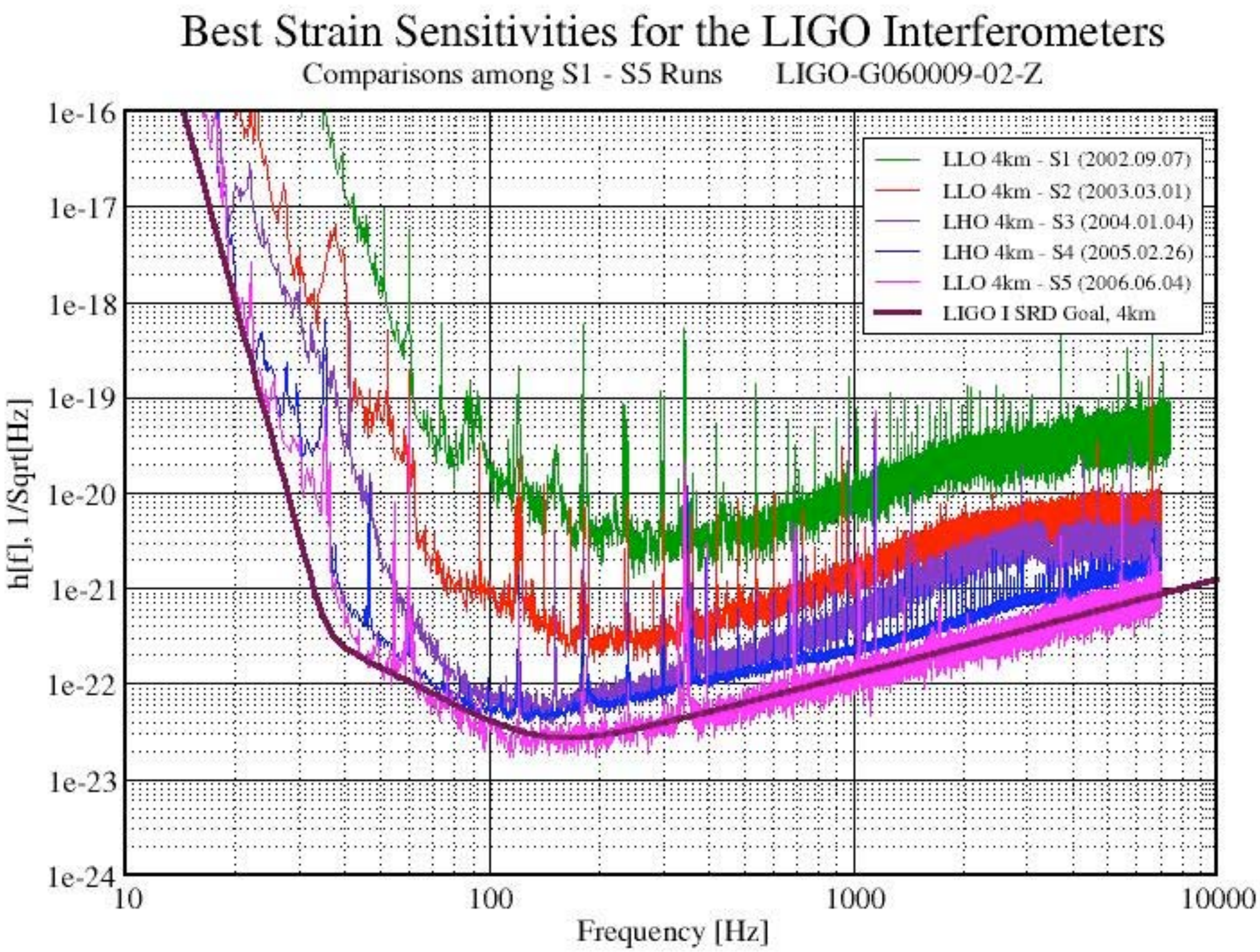
Adv LIGO Science Run: Jan 2015–

Laser Interferometer Detectors Worldwide

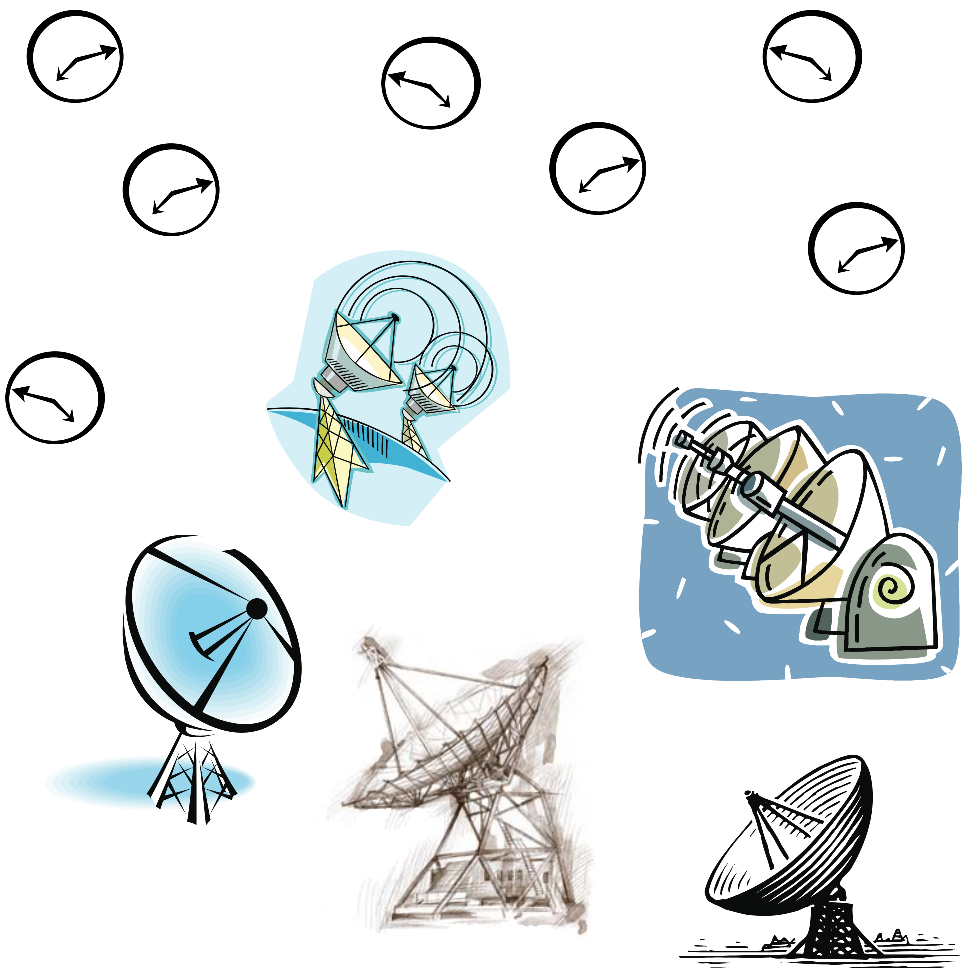
Virgo: Italy & France (3 Km)

GEO: Germany & UK (600 m)

TAMA: Japan (300m)


Best Strain Sensitivities for the LIGO Interferometers
Comparisons among S1 - S5 Runs LIGO-G060009-02-Z
LLO 4km - S1 (2002.09.07)
LLO 4km - S2 (2003.03.01)
LHO 4km - S3 (2004.01.04)
LHO 4km - S4 (2005.02.26)
LLO 4km - S5 (2006.06.04)
LIGO I SRD Goal, 4km
h[f], 1/Sqrt[Hz]
1e-16
1e-17
1e-18
1e-19
1e-20
1e-21
1e-22
1e-23
1e-24
10
100
1000
10000
Frequency [Hz]

Gravitational waves can be detected by searching for correlations in pulsar timing residuals



Pulsars: extremely stable metronomes

Passing gravitational wave disturbs beat reception

- Different for pulsars in different directions
- Correlation with wave propagation direction

Gravitational wave signal embedded in timing residual correlations

LISA: the laser interferometer space antenna

Joint ESA, NASA proposal

Under evaluation in US, Europe

Possible launch date ~ 2020

Advantages

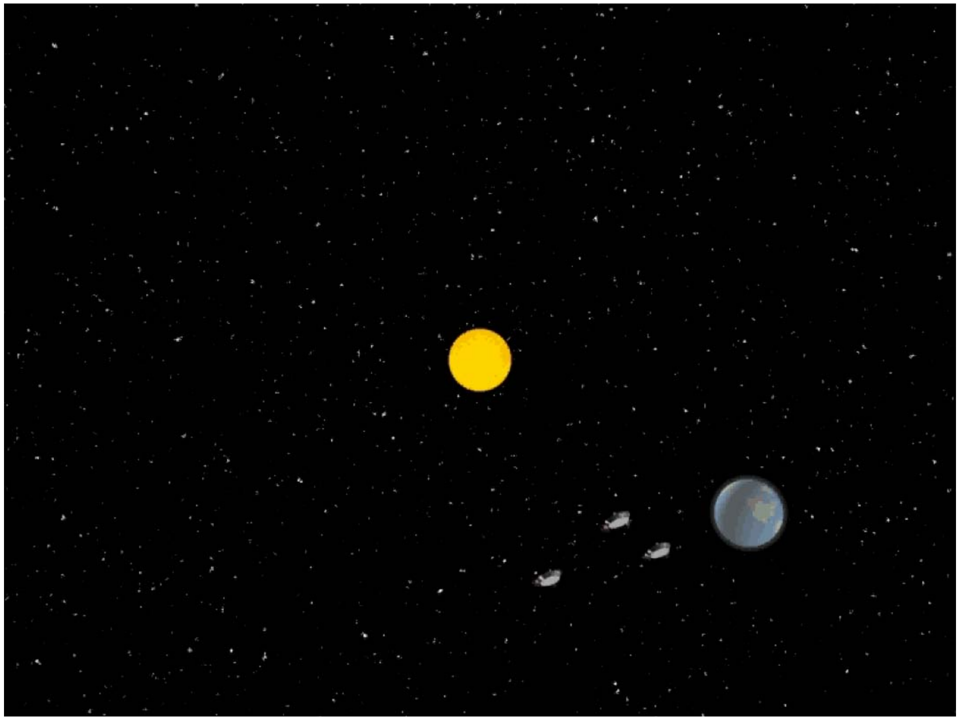
Longer arms ($5x10^6$ km)

No anthropomorphic, seismic or gravity-gradient noises

Critical technologies

“Drag-free” flight

Space interferometry



Courtesy Rutherford Appleton Laboratory

14

References and things to think about...

References...

L. S. Finn. 2009. *Response of interferometric gravitational wave detectors.* Phys. Rev. D **79**:022002

F. B. Estabrook and H. D. Wahlquist. 1975. *Response of doppler spacecraft tracking to gravitational waves*. Gen. Rel. Grav. **6**:439 – 447

S. Detweiler. 1979. Pulsar timing measurements and the search for gravitational waves. Ap. J. **234**:1100 – 1104.

L. Ju, D. G. Blair, C. Zhau. 2000. *Detection of gravitational waves*. Reports on Progress in Theoretical Physics **63**:1317 – 1427