Monte Carlo Methods for X-ray Dispersive Spectrometers

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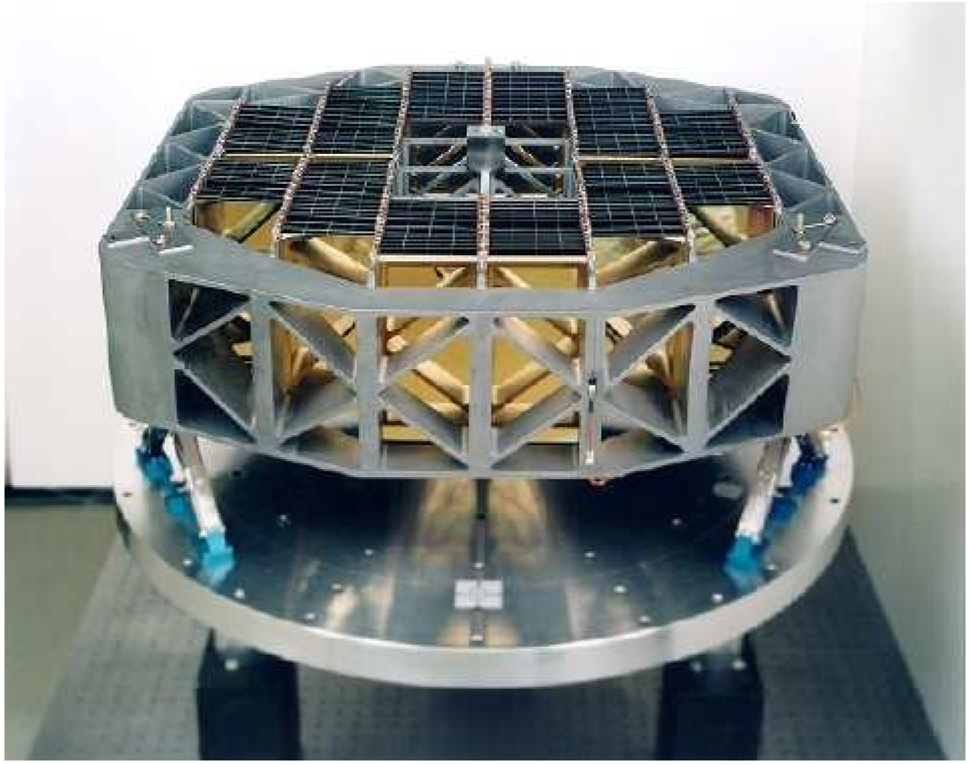
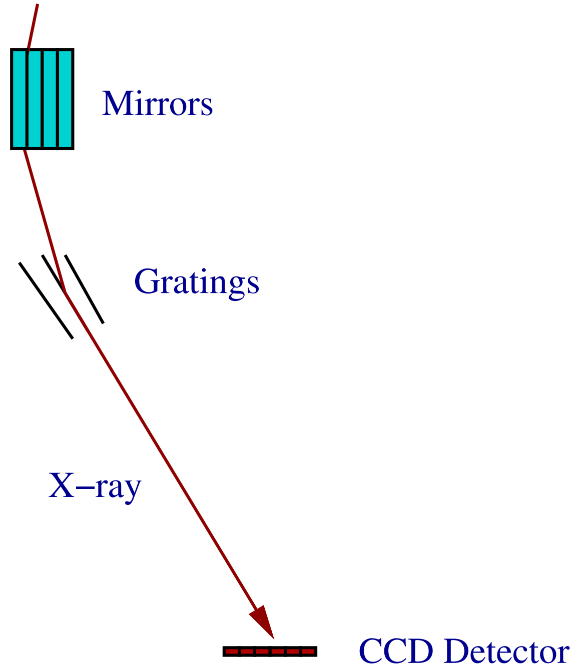
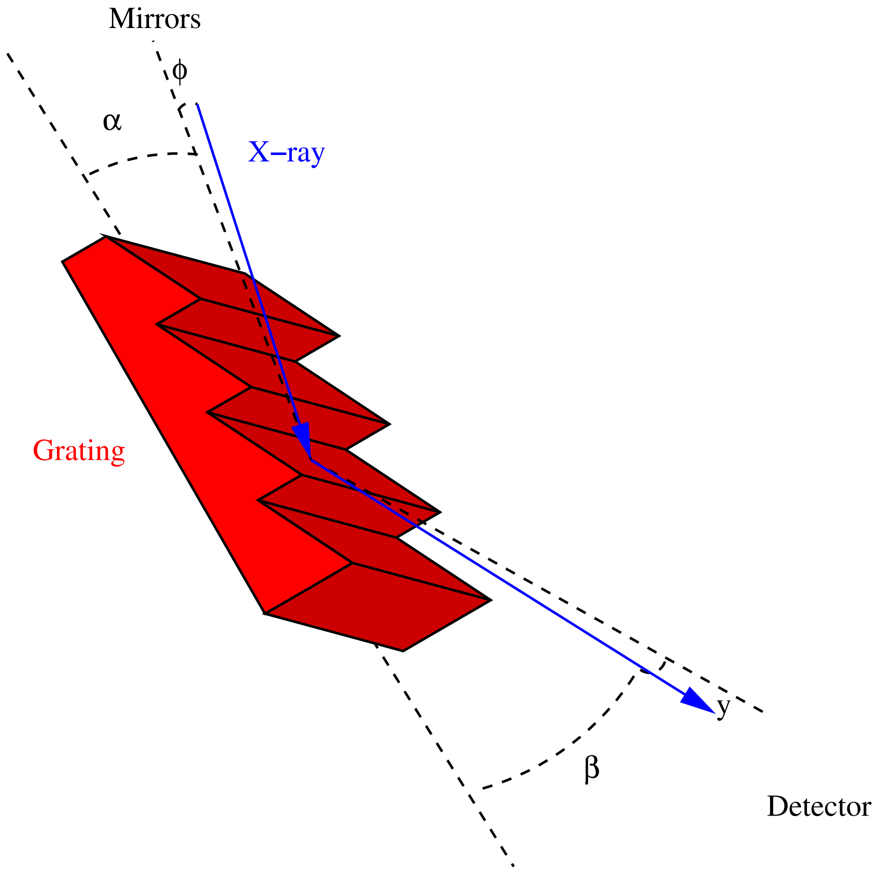
- Dispersive Spectrometers: Gratings on XMM-Newton and Chandra
- Multi-dimensional Data Sets Obtained with Dispersive Spectrometers
- Instrument Response Functions through Monte Carlo Methods
- Event Comparison with Multivariate Data





- Dispersive Spectrometers: XMM-Newton Reflection Grating Spectrometers (RGS) and Chandra High and Low Energy Transmission Grating (HETG/LETG)
- Currently have resolving powers ($\frac{\lambda}{\Delta\lambda}$) of 300 to 1000 and improve over non-dispersive CCD configurations with ($\frac{\lambda}{\Delta\lambda} \sim 20$)



Dispersion Equation:

$$\frac{m\lambda}{d} = \cos\beta - \cos\alpha \qquad (1)$$

Cross Dispersion Equation:

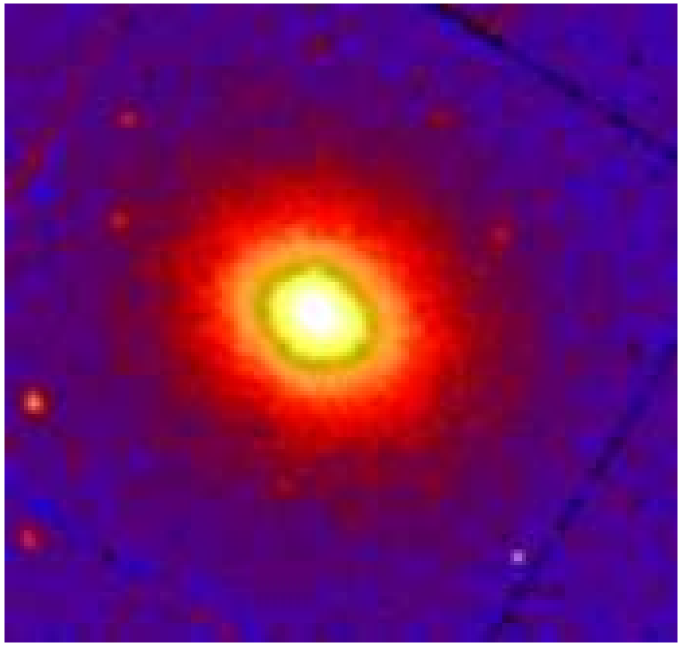
$$y = -\phi \qquad (2)$$

d is the grating spacing

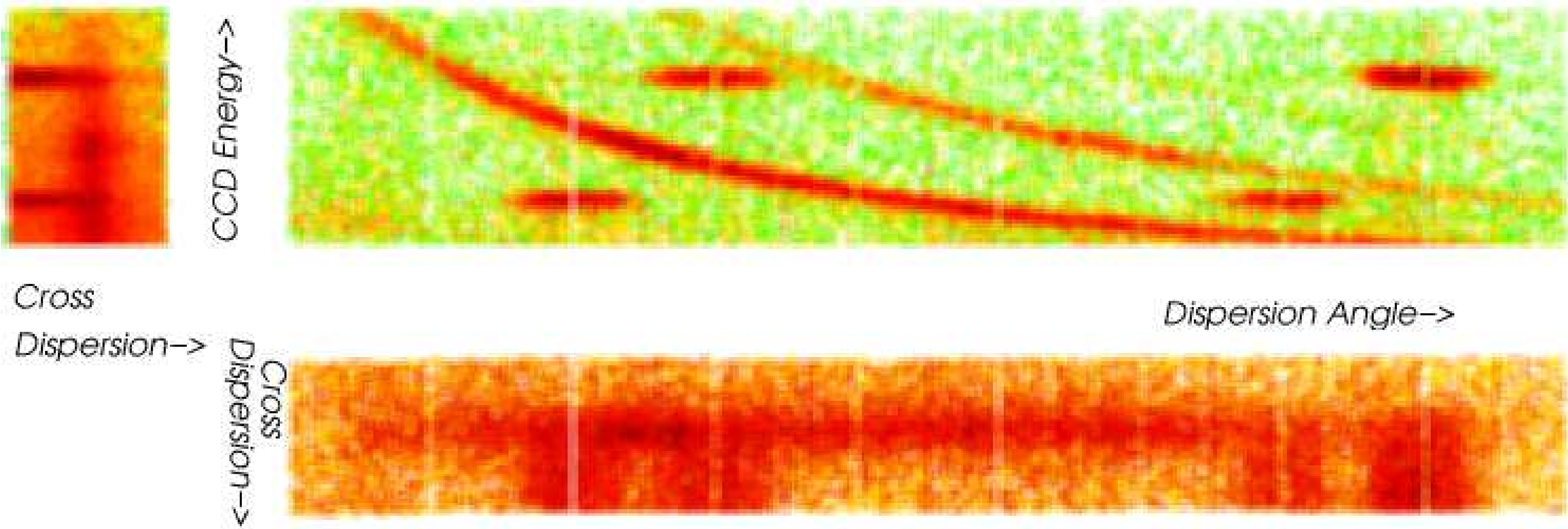
m is an integer spectral order

- Measure the dispersion angle β and the cross-dispersion angle y as well as a rough detector energy e
- Depends on the incident angles α, ϕ and wavelength, λ
- Relatively straight-forward for a point source, not so for extended

Data for the Galaxy Cluster, Sérsic 159-03, in two instruments



Left is a dispersive spectrometer, EPIC-MOS
Below is the grating data for the RGS



Response Functions

- Want to calculate the probabilities of photons landing where they did
- In one dimension:

$$P(\beta) = \int d\lambda \; R(\beta|\lambda) \; S(\lambda) \tag{3}$$

- P is the probability of a photon landing at a given β, S is the spectral model, and R is the response function

- General case:

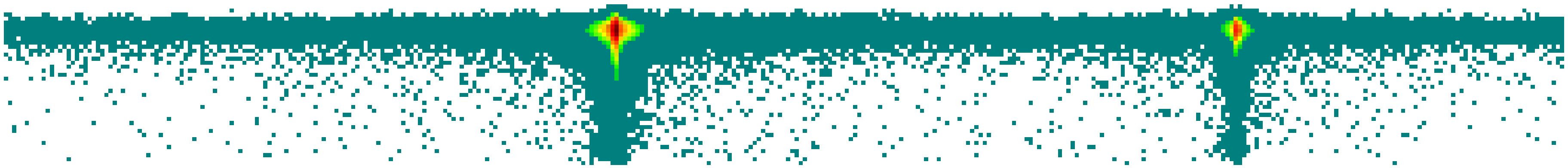
$$P(\beta, y, e) = \int d\alpha \; d\phi \; d\lambda \; R(\beta, y, e|\lambda, \alpha, \phi) \; S(\lambda, \alpha, \phi) \tag{4}$$

- Since three-dimensional– use a Monte Carlo

Not simple response functions:

- Simulation for a monochromatic on-axis point source

- Cross-Dispersion coordinate (y) vs. Dispersion coordinate (β)

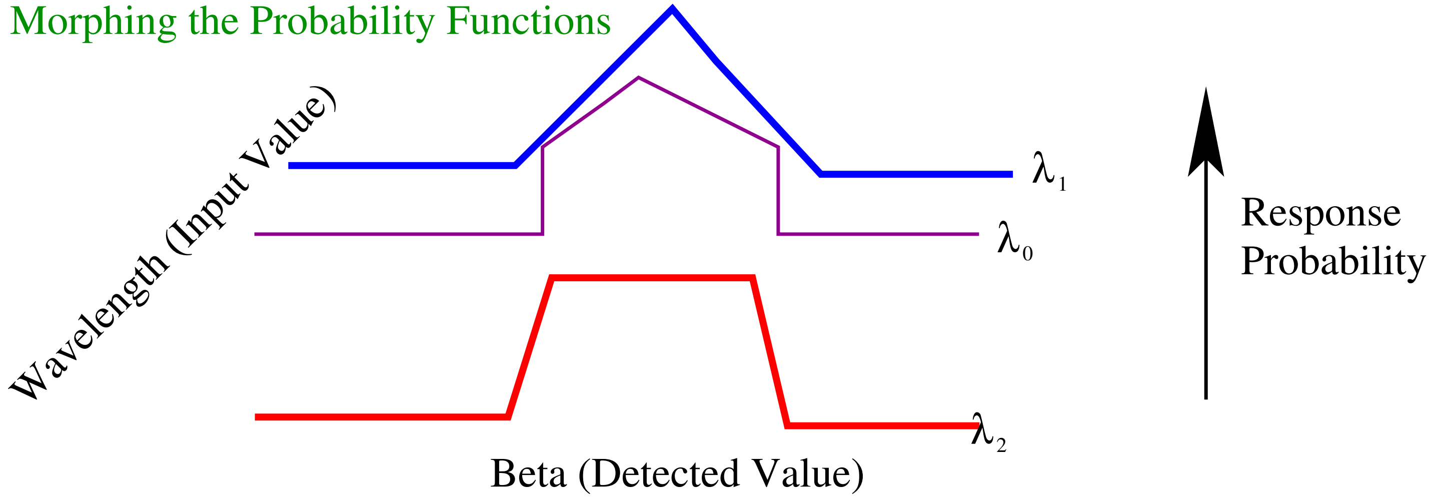


- CCD energy (e) vs. Dispersion coordinate (β)
- Real data is a convolution of these functions at a range of angles and wavelengths

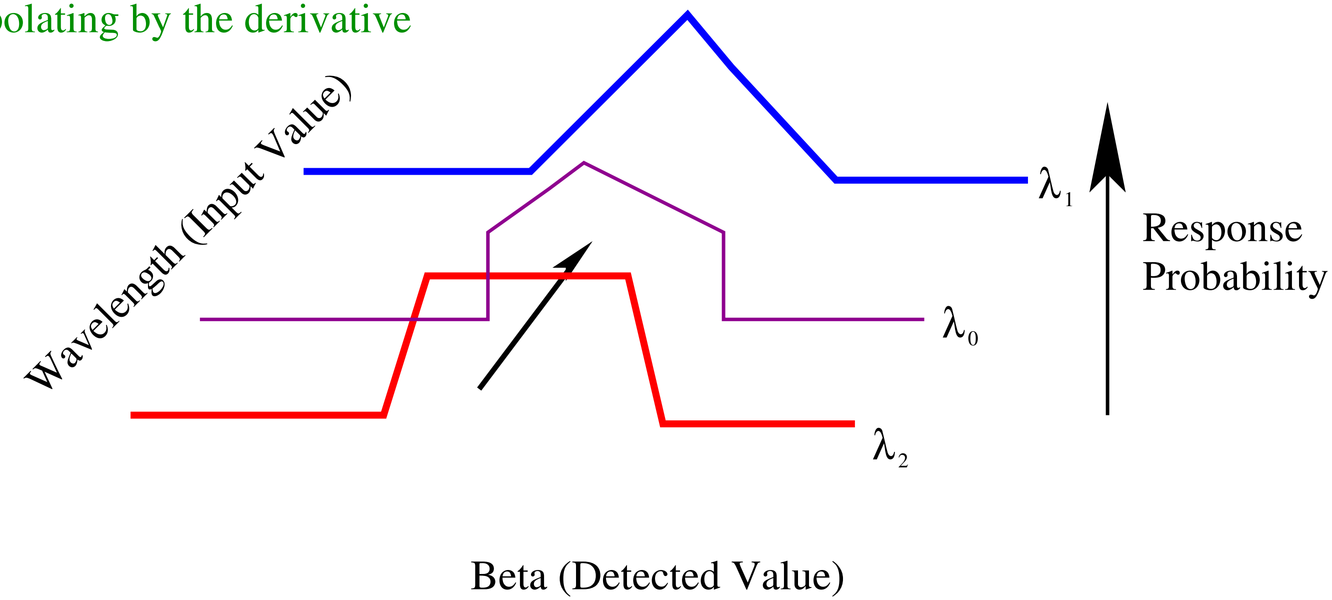
Want to save these functions on large grids, but take up too much memory

Save on coarse grids and do the following technique

Morphing the Probability Functions

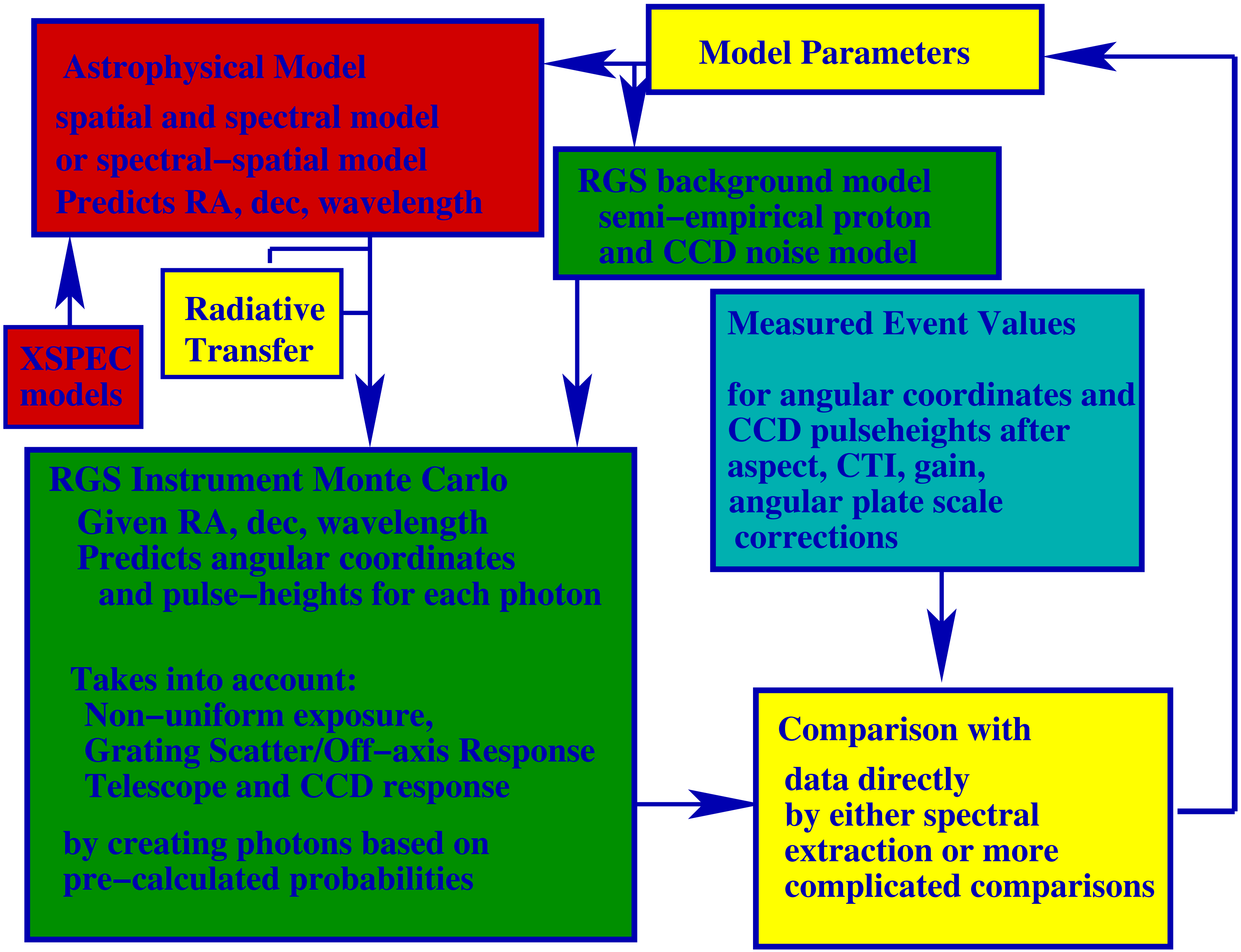


Interpolating by the derivative

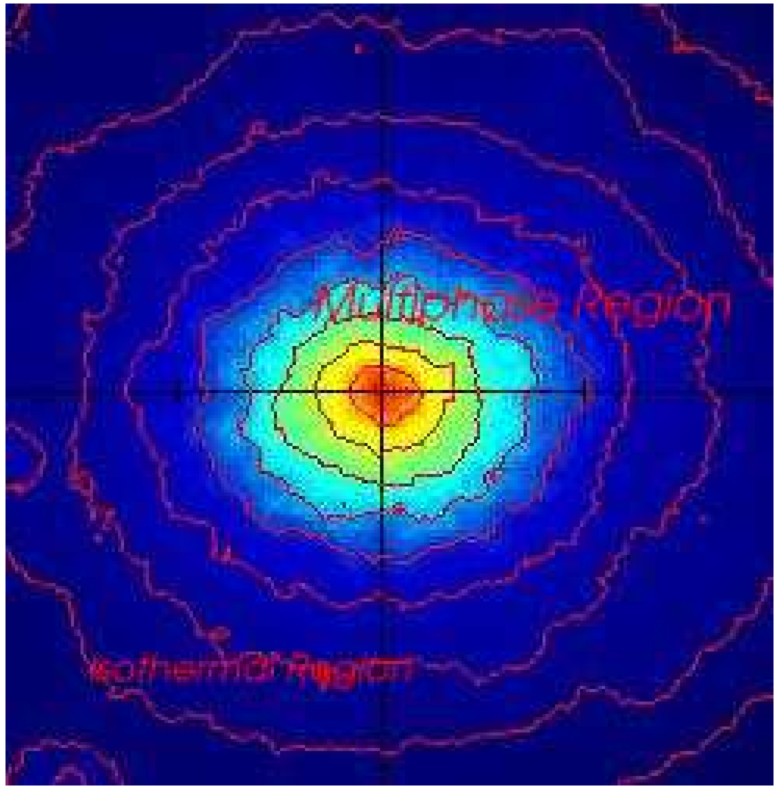


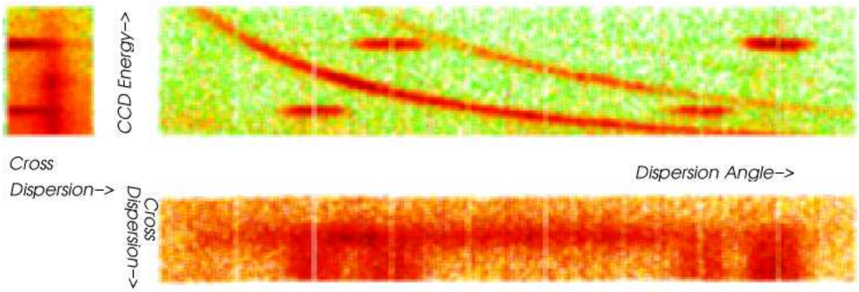
General Method using the Monte Carlo

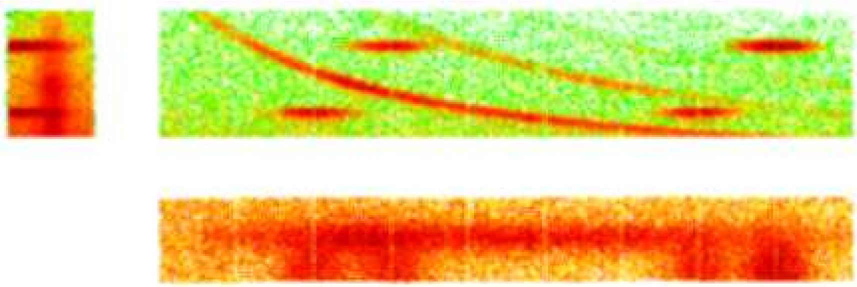
- First, models are formulated in terms of a set of parameters to predict probability distributions for the spectral and spatial models.
- Photons are then drawn from these probability distributions and assigned a wavelength and two sky coordinates. (λ, α, ϕ).
- The dispersion, cross-dispersion, and pulseheight values (β, y, e) are predicted from the instrument Monte Carlo.
- Finally, the simulated events are compared with the measured photon events and parameters are modified to improve the consistency of the data with the model.

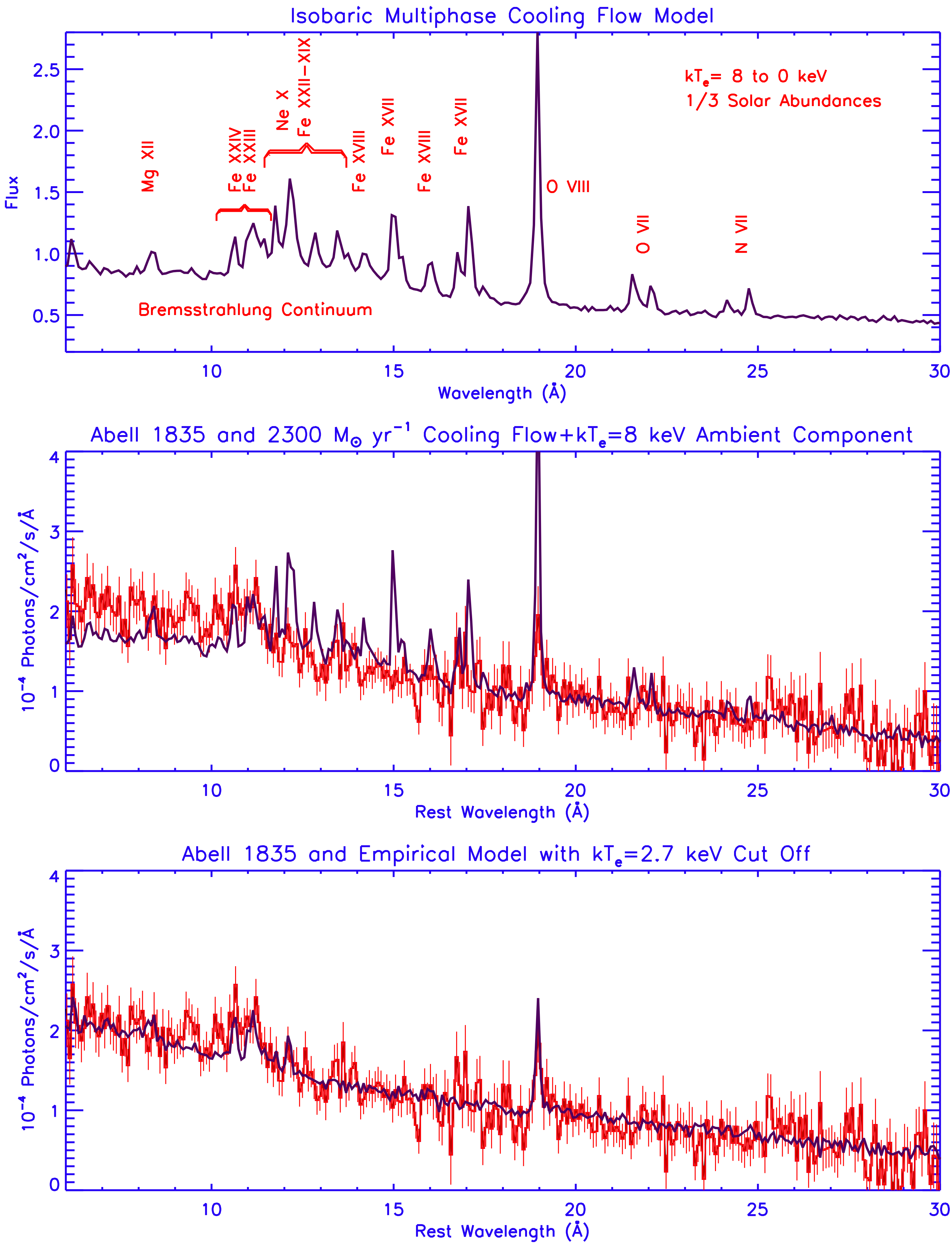

Astrophysical Model
spatial and spectral model
or spectral−spatial model
Predicts RA, dec, wavelength
Model Parameters
RGS background model
semi−empirical proton
and CCD noise model
Radiative
Transfer
XSPEC
models
Measured Event Values
for angular coordinates and
CCD pulseheights after
aspect, CTI, gain,
angular plate scale
corrections
RGS Instrument Monte Carlo
Given RA, dec, wavelength
Predicts angular coordinates
and pulse−heights for each photon
Takes into account:
Non−uniform exposure,
Grating Scatter/Off−axis Response
Telescope and CCD response
by creating photons based on
pre−calculated probabilities
Comparison with
data directly
by either spectral
extraction or more
complicated comparisons

- Multiphase Model according to standard cooling-flow model inside cooling radius
- Isothermal envelope surrounding this cooling region
- Temperature distribution predicts a spectrum at each radius
- Project the three-dimensional distribution with through a Monte Carlo







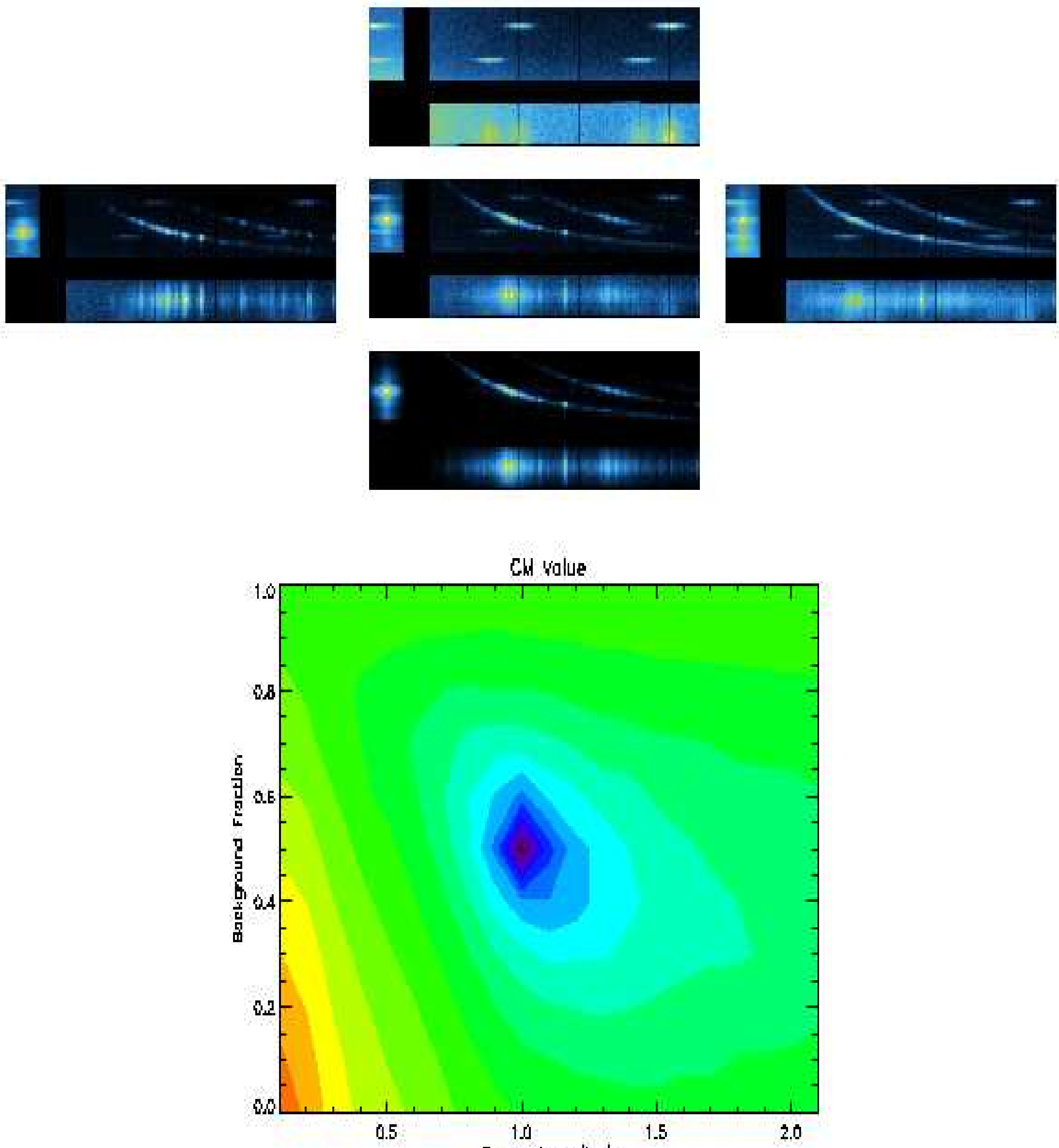

Isobaric Multiphase Cooling Flow Model
Flux
Wavelength (Å)
Mg XII
Fe XXIV
Fe XXIII
Ne X
Fe XXII–XIX
Fe XVIII
Fe XVII
Fe XVIII
Fe XVII
O VIII
O VII
N VII
kT_e= 8 to 0 keV
1/3 Solar Abundances
Bremsstrahlung Continuum
Abell 1835 and 2300 $M_\odot$ yr^{-1} Cooling Flow+kT_e=8 keV Ambient Component
10^{-4} Photons/cm^2/s/Å
Rest Wavelength (Å)
Abell 1835 and Empirical Model with kT_e=2.7 keV Cut Off
10^{-4} Photons/cm^2/s/Å
Rest Wavelength (Å)

Multi-dimensional Cramér-von Mises Statistic

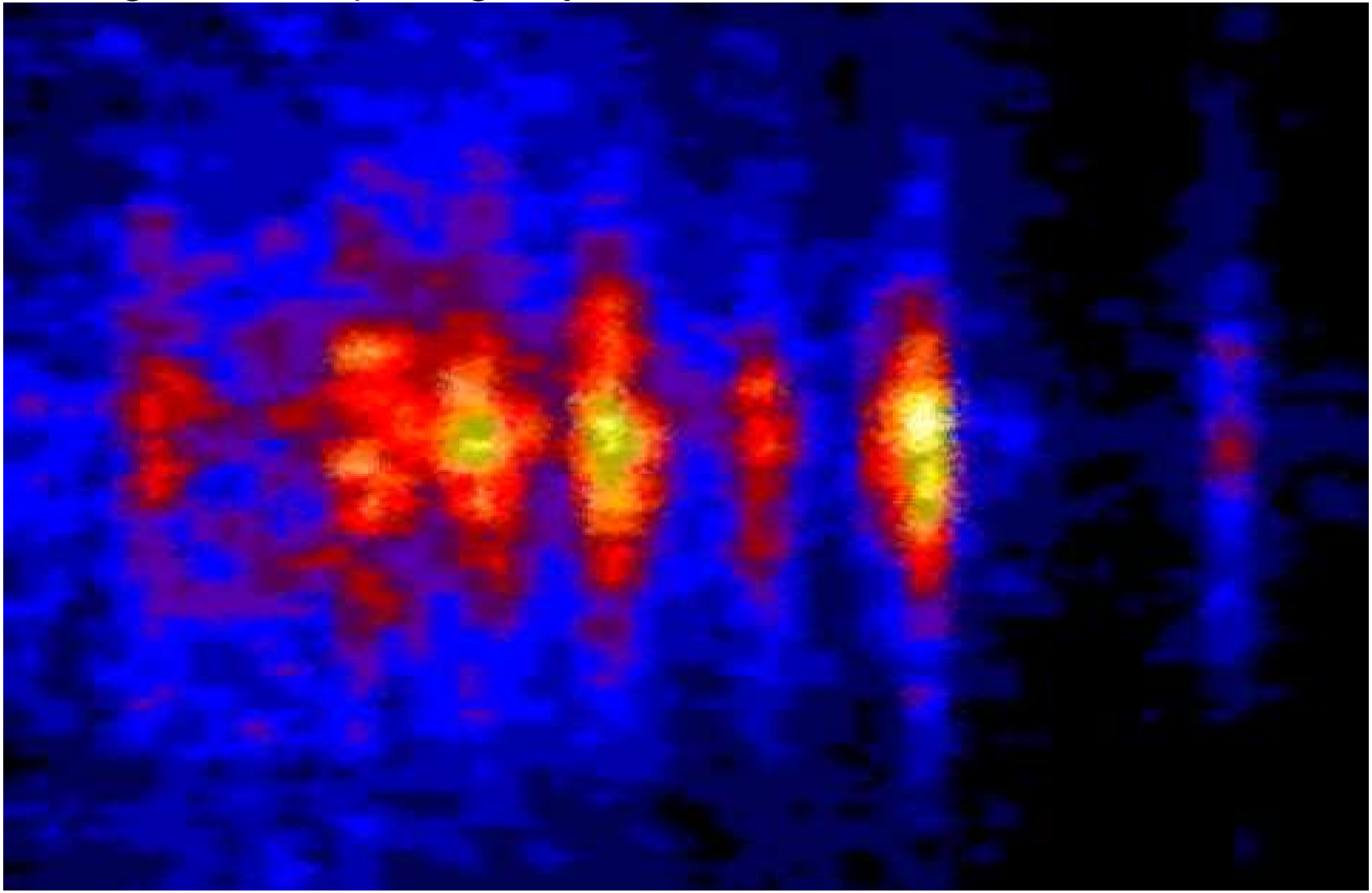
- m photons in the data
- n photons in the data
- x is the measured value of some quantity
- C(x) is the cumulative value of x

$$CM = \frac{mn}{(m+n)^2} \sum_{all\ x} \left[C_{data}(x) - C_{model}(x)\right]^2$$

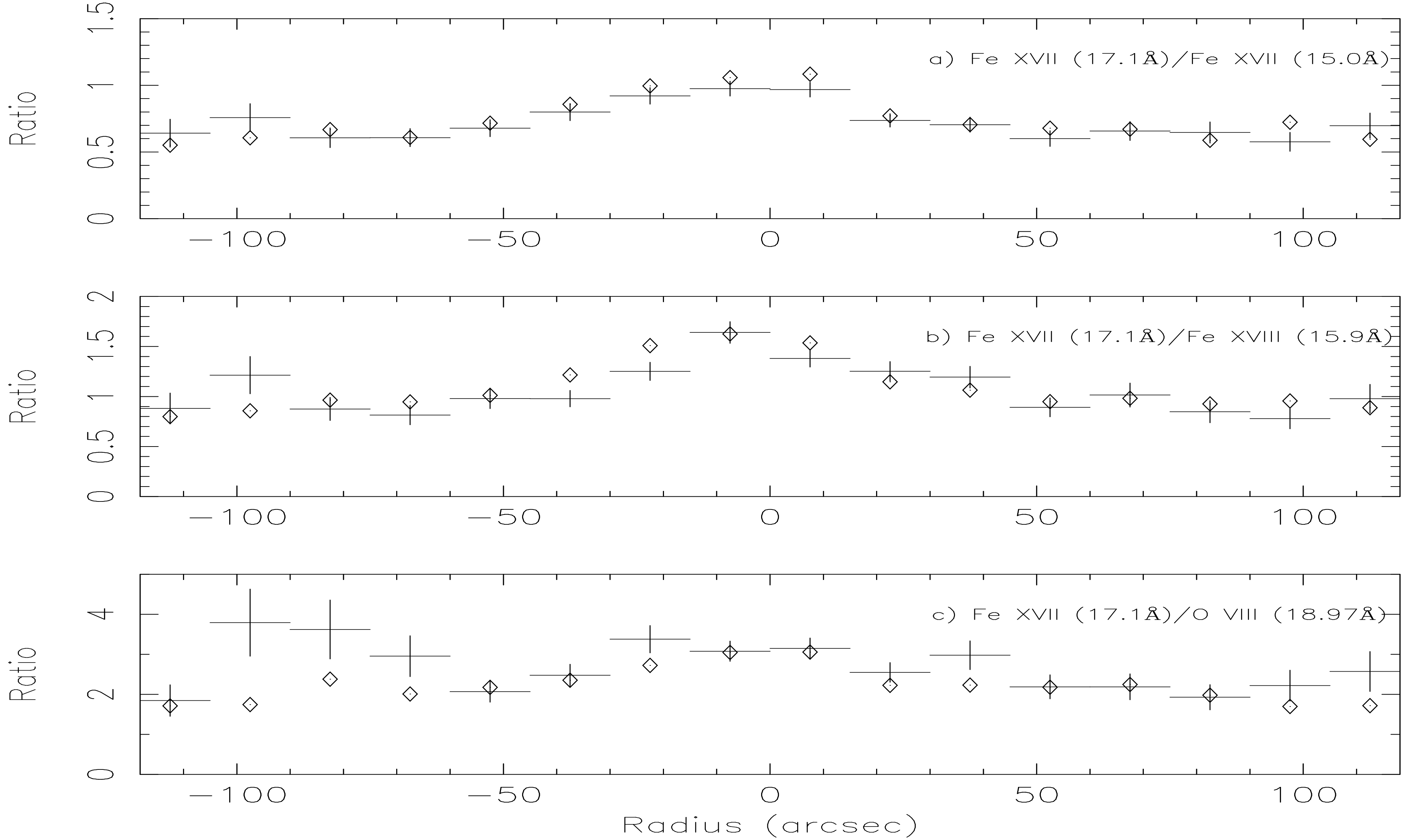
- Multivariate generalization achieved by making x a linear combination of the value in each dimension (β, y, e)


CM Value
Background Fraction
Temperature (keV)
0.0
0.2
0.4
0.6
0.8
1.0
0.5
1.0
1.5
2.0

Images of each emission line in a Cross-Dispersion (y) vs. Dispersion (β) image of the elliptical galaxy, NGC 4636



Sensitive to temperature, abundance, and radiative transfer effects on different spatial scales



Xu et al. (2002)

Conclusions and Future Work

– Still difficulties in convergence and uniqueness of the model

– May be useful to extend to non-dispersive

– May be useful in other applications with sparse, mildly multi-dimensional data

– Promises to be an extremely useful method in Diffuse X-ray Spectroscopy