Advanced SPEX models

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1. NEI models

- NEI = Non-Equilibrium Ionisation
- In most cases, plasmas assumed to be in *Collisional Ionisation Equilibrium (CIE):*

$$n_{z+1}R_{z+1}(T) - n_z R_z(T) + n_{z-1} I_{z-1}(T) - n_z I_z(T) = 0$$

- R(T) & I(T) (recombination & ionisation rates) only depend on T ➔ $n_z(T)$ only function of T

NEI: basic principle

- When T changes, ion concentrations need to adjust
- Change occurs through collisions: T higher, more collisional ionisation; T lower: more radiative & dielectronic recombination
- How fast plasma adjusts, depends on average collision time (read: density)

NEI: basic equation

$$\frac{1}{n_{\mathrm{e}}(t)}\frac{\mathrm{d}}{\mathrm{d}t}\vec{n}(Z,t) = \mathbf{A}(Z,T(t))\vec{n}(Z,t)$$

$$\mathbf{A} = \begin{pmatrix} -I_0 & R_1 & 0 & 0 & \dots & \\ I_0 & -(I_1+R_1) & R_2 & 0 & & \\ 0 & I_1 & \dots & \dots & & \\ & \vdots & \ddots & \vdots & \dots & \\ & & \dots & R_{Z-1} & 0 & \\ & \dots & 0 & I_{Z-2} & -(I_{Z-1}+R_{Z-1}) & R_Z \\ & & \dots & 0 & I_{Z-1} & -R_Z \end{pmatrix}.$$

See Kaastra & Jansen (1993) on how to solve this;

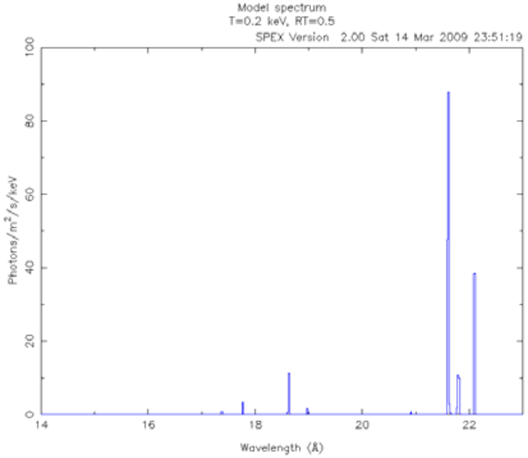
Solution depends on $\int n_e(t)\,dt$

Simple way:

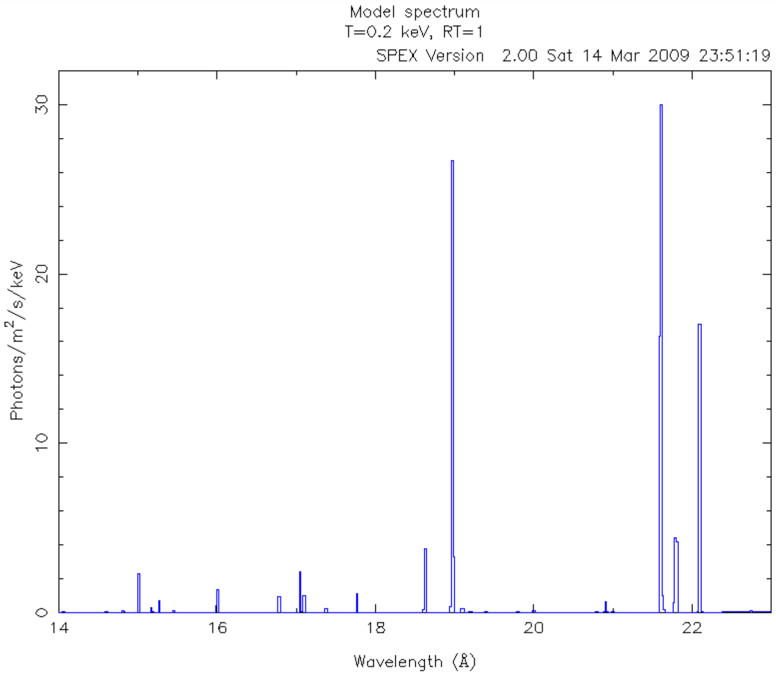
- Use CIE model in SPEX
- Parameter RT is ratio of T(balance) / T(spec)
- Ionisation balance (equilibrium) calculated using T(balance)
- Spectrum evaluated using T(spec)
- NON-physical model, but gives rough idea

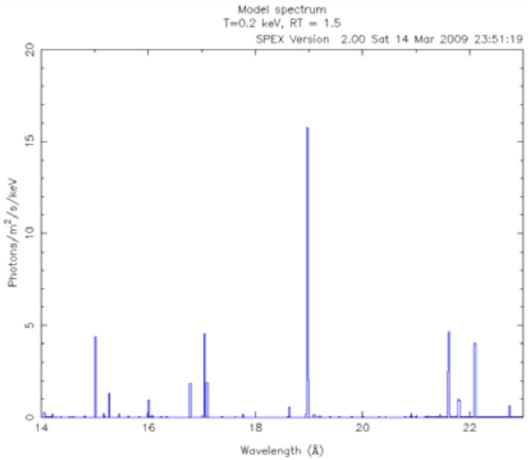
Examples

T=0.2 keV RT=0.5 (underionised)



T=0.2 keV RT=1 (equilibrium)





T=0.2 keV RT=2.0 (overionised)

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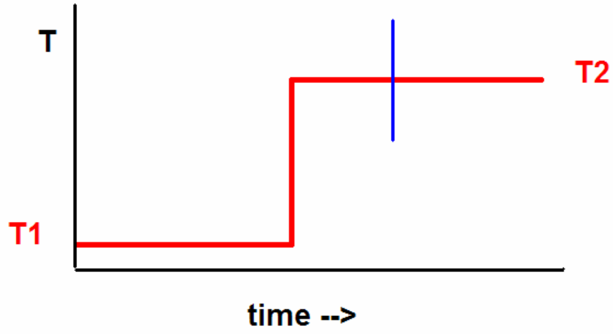
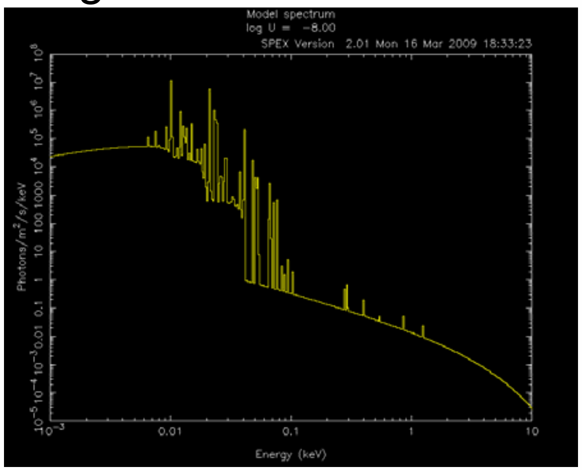
The NEI model of SPEX



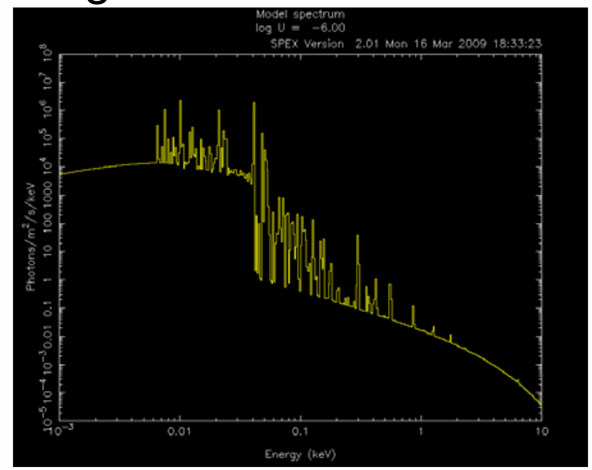
- Describes response of plasma jumping from T1 to T2
- Usually T1 low (e.g. 10^4 K, almost neutral gas)
- Time dependence through $U=\int n_e \, dt$
- For $U \rightarrow \infty$, plasma reaches equilibrium
- In practice often for $U \approx 10^{17}$ $m^{-3}s$

Example of NEI spectra

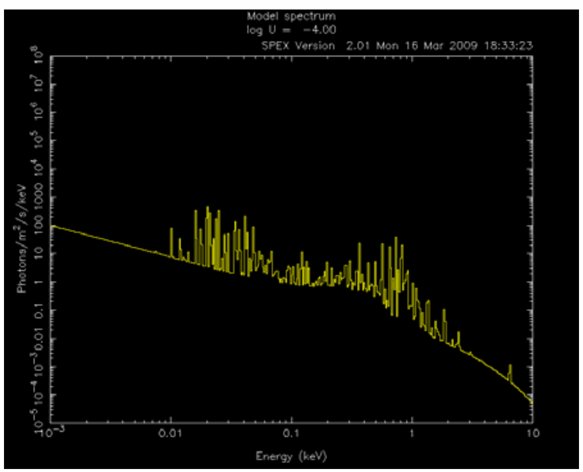
Log U=12



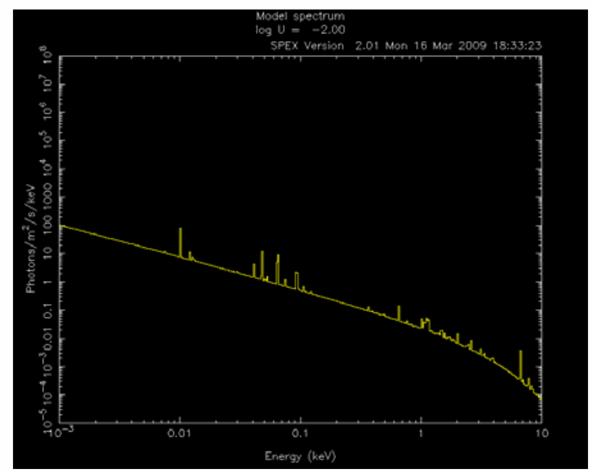
Log U=14



M



L



K

Log U=16

Log U=18

Examples of NEI spectra

- Supernova remnants ($n=1\ cm^{-3}$, $t=1000$ yr)
- Stellar flares ($n=10^{17}\ m^{-3}$, $t=10$ s)
- Cluster outskirts? ($n=100\ m^{-3}$, $t=10^{8}$ yr)

Did you know that …

- You can make T1>T2 to mimic a recombining plasma

2. DEM Modelling

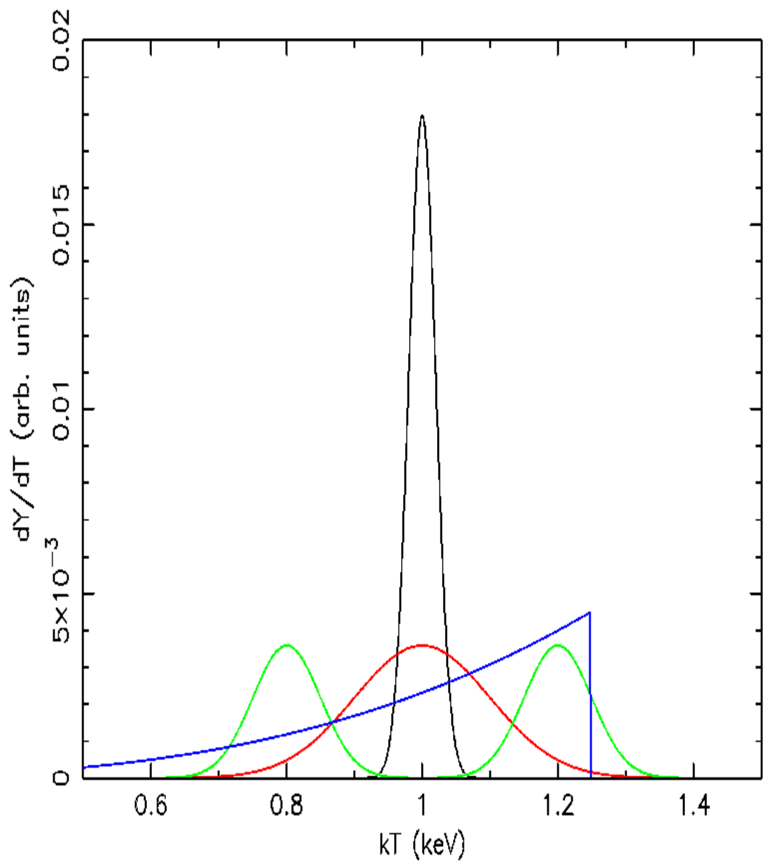
- Usual approach:
- Try 1T
- If fit not good, try 2T
- If still not good, try 3T
- *Ad infinitum* & often unstable (strong correlations between components)
- How to do a better job?

Basic concept: DEM

- In real sources, emission measure ($Y=\int n_e n_H dV$) is integral over region with different physical properties
- T needs not to be constant over region (clusters, coronal loops, etc.)
- Introduce DEM = Differential Emission Measure, as function of T
- $\int DEM\,(T)\,dT \equiv Y$

Challenge with multi-T plasmas

- Line spectra insensitive to details DEM within T-range of factor 2
- All DEMs in example have same <T> and almost indistinguishable spectrum
- ➔ bin T-range with steps of factor 2



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DEM techniques

- Make a library of basis spectra $F_i(E)$ for a grid of temperatures
- Solve the equation:

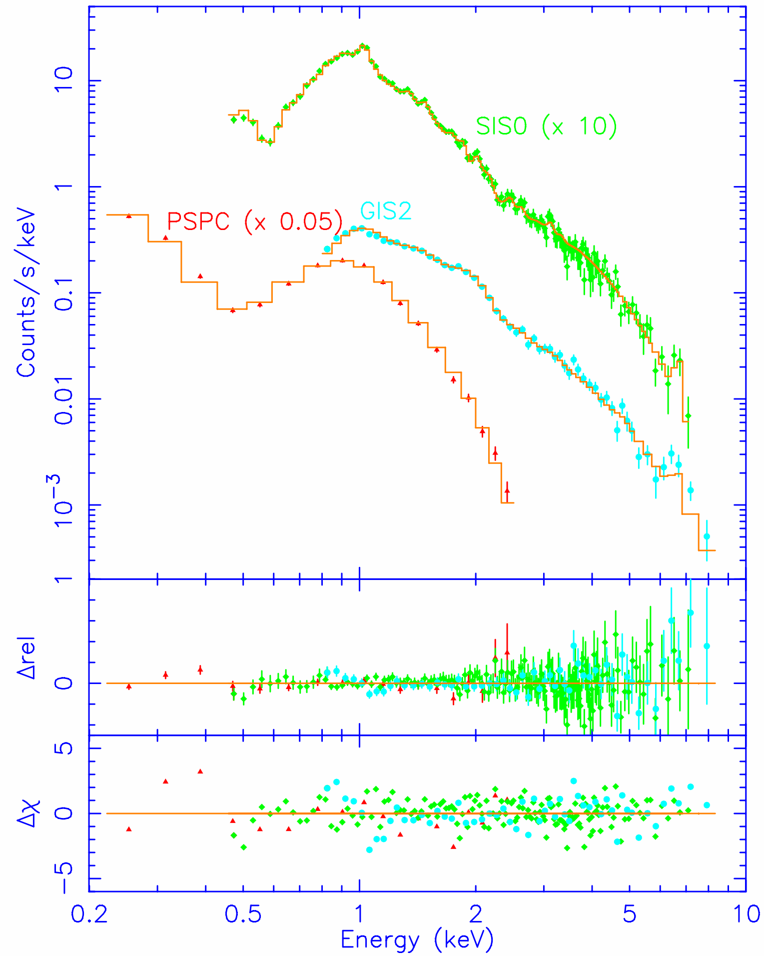
$$S(E)=\Sigma\, Y_i\, F_i(E)$$

- $S(E)$ is observed spectrum
- Y_i are the (differential) emission measures

Example: AR Lac

(Kaastra et al. 1996)

- Fit to Rosat PSPC & ASCA GIS & SIS spectra



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Regularisation method: principles

- Solve $S(E) = \Sigma\ Y_i\ F_i(E)$
- using constraint that 2^{nd} order derivative Y_i is as smooth as possible
- Degree of smoothing controlled by regularisation parameter R (essentially smoothness / χ^2)
- R adjustable (R=0: no smoothing)

Pro and contra regularisation

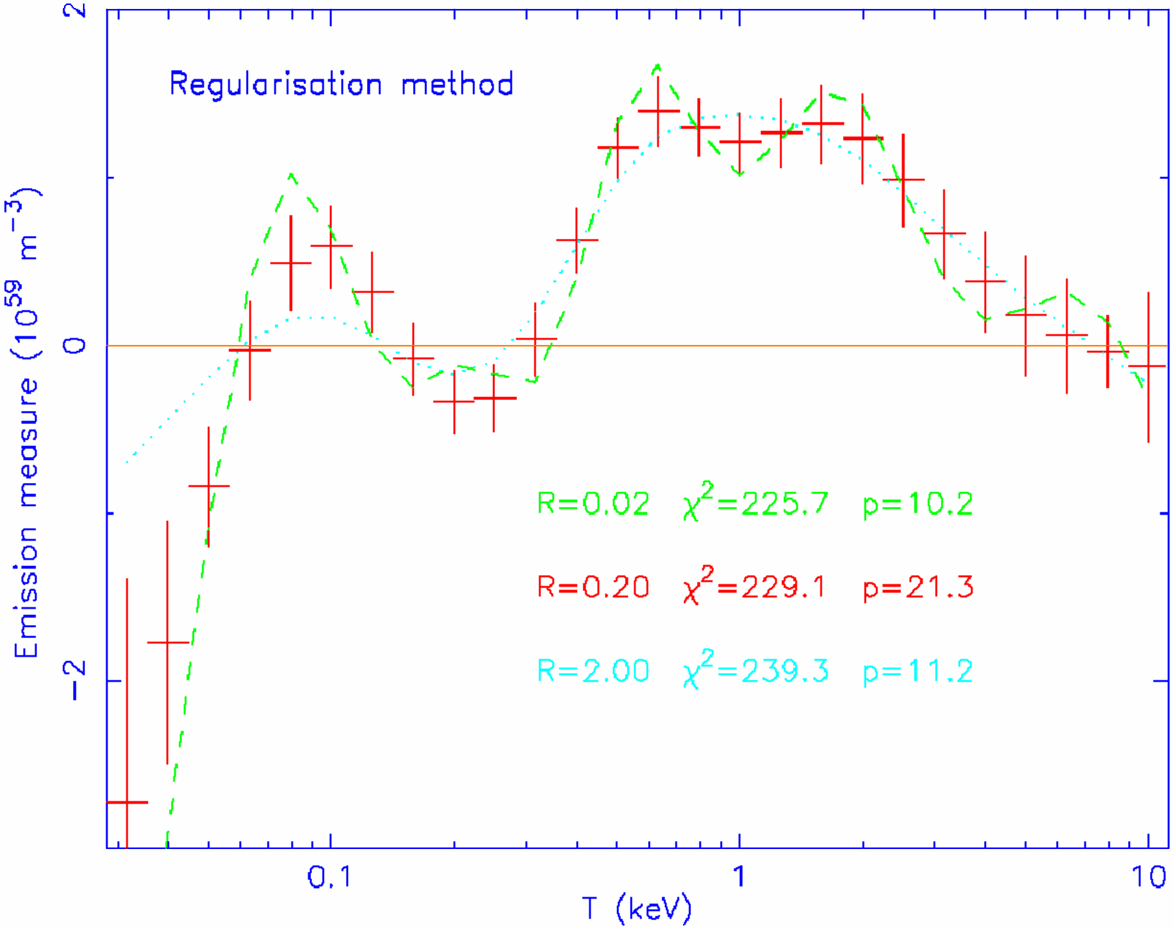
- Advantage: damp unwanted oscillations
- Disadvantage: solution can be negative
- SPEX solves this by introducing the "DEM penalty":

$$p \equiv \sum_{Y_i < 0} (Y_i / \Delta Y_i)^2$$

Advice: add p to χ^2 for tests of goodness

Regularisation method: practice

- All three solutions shown here acceptable



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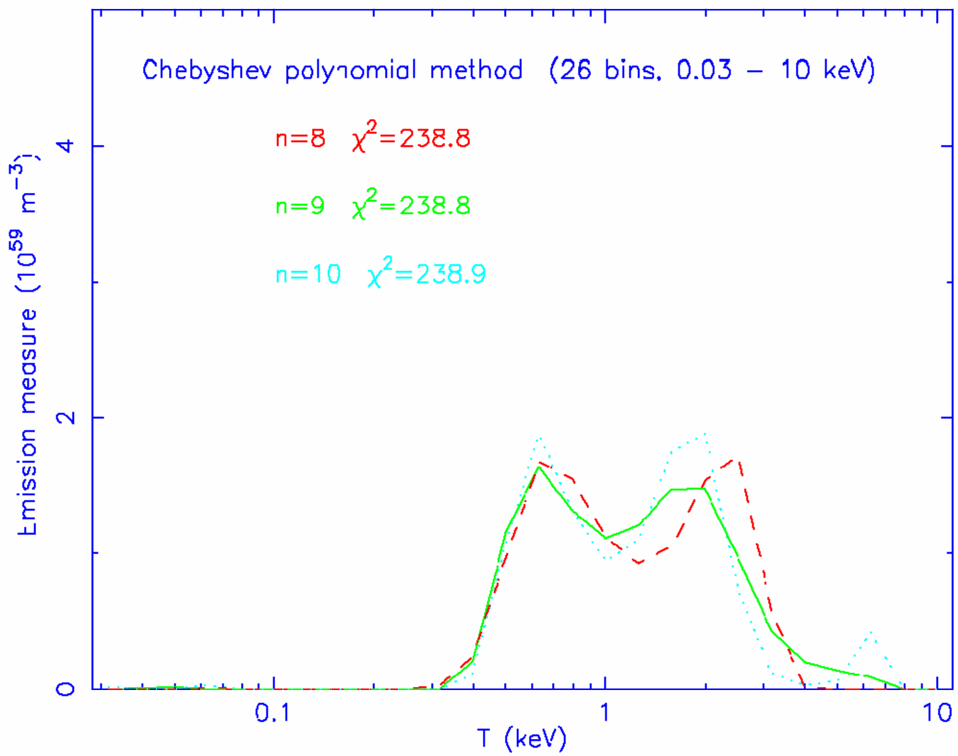
Polynomial method

- Model here log DEM(T) as an n-th degree polynomial
- n can be chosen, to get lowest χ^2
- Example here: n=8, 9 or 10 all give acceptable fits; look to the difference in solutions
- Works good for smooth DEMS



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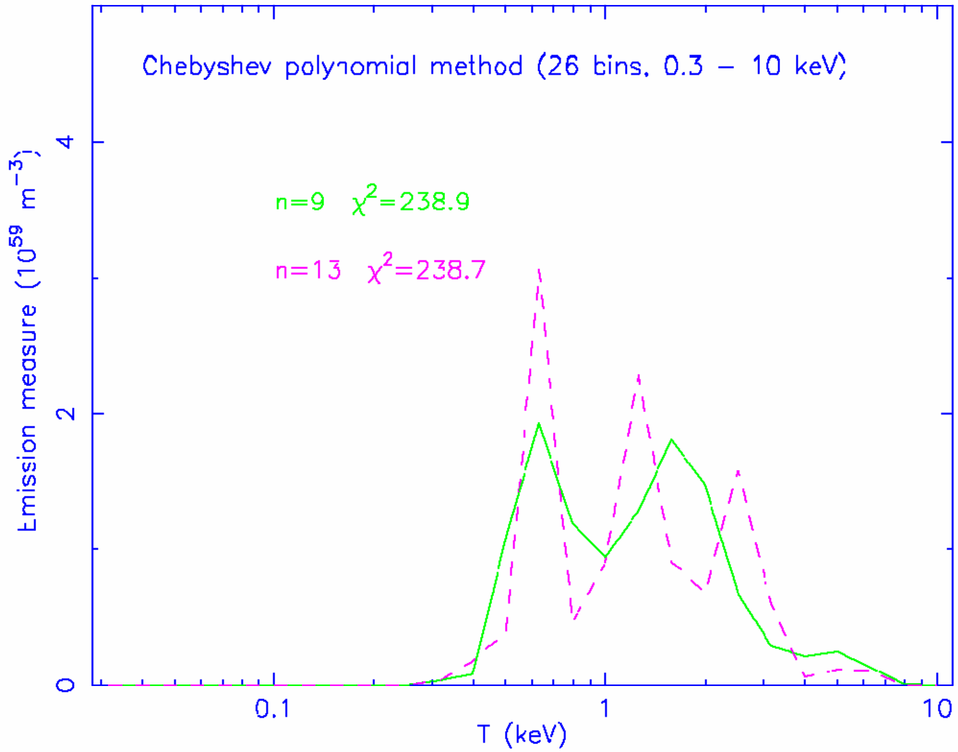
Polynomial method II

- Higher degrees of polynomial do not provide better fit
- They also are more spiky
- Advise: choose minimum n with acceptable χ^2 (e.g., $\chi^2 < \chi^2_{min} + 1$)



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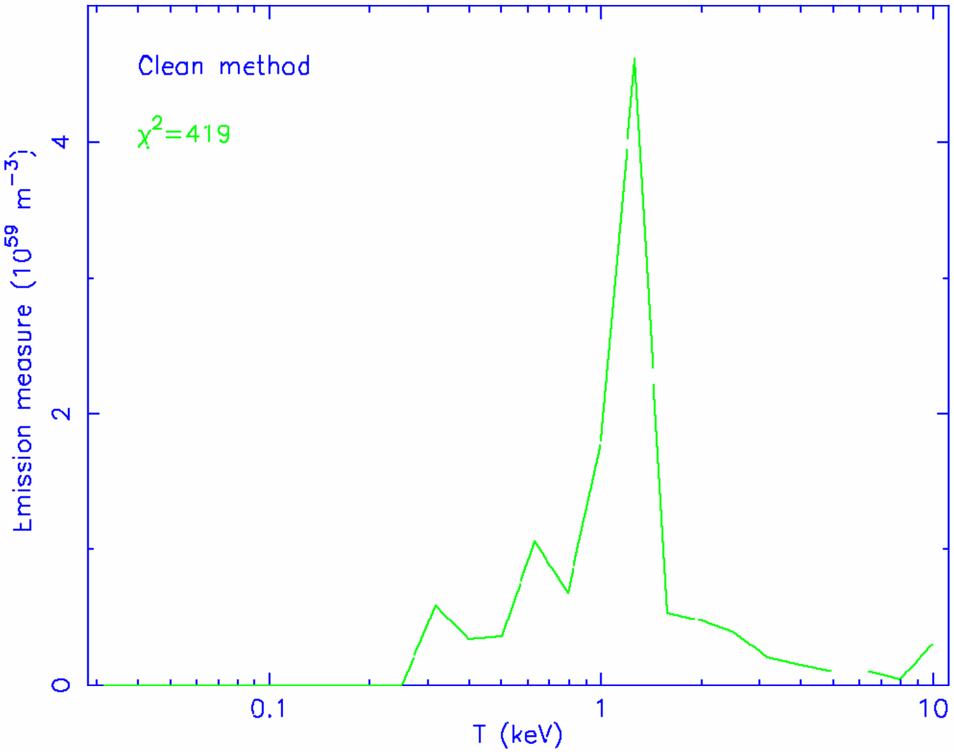
Clean method

- Clean method: derived from radio beam synthesis methods (Högbom 1974)
- Adds small (low Y) components that give best improvement χ^2 until convergence is reached
- Works good for "spiky" DEMS



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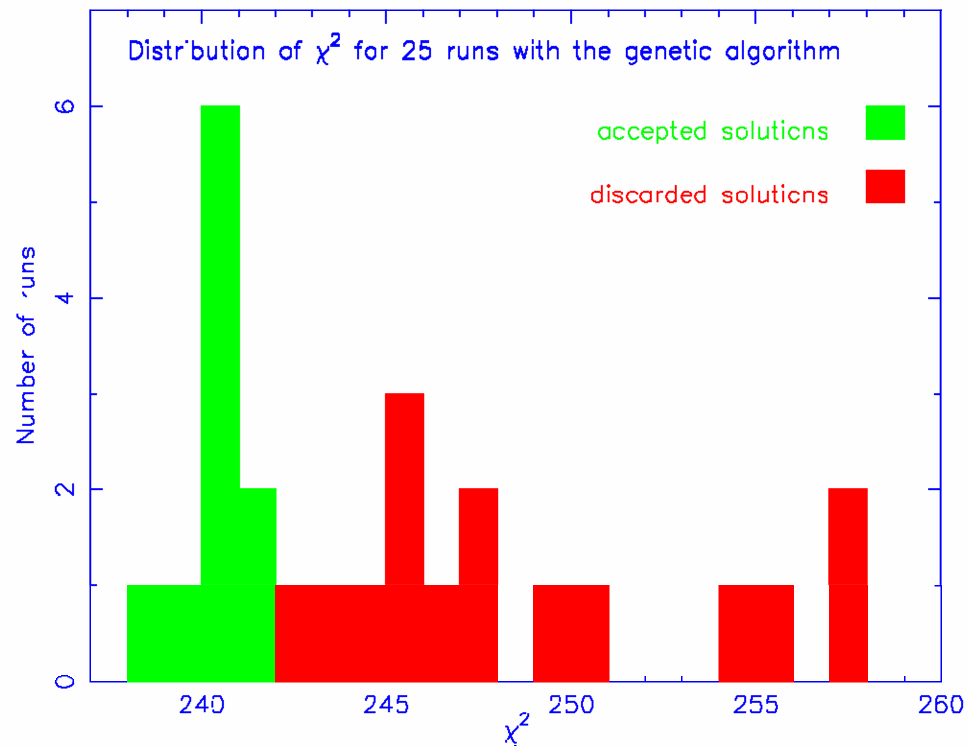
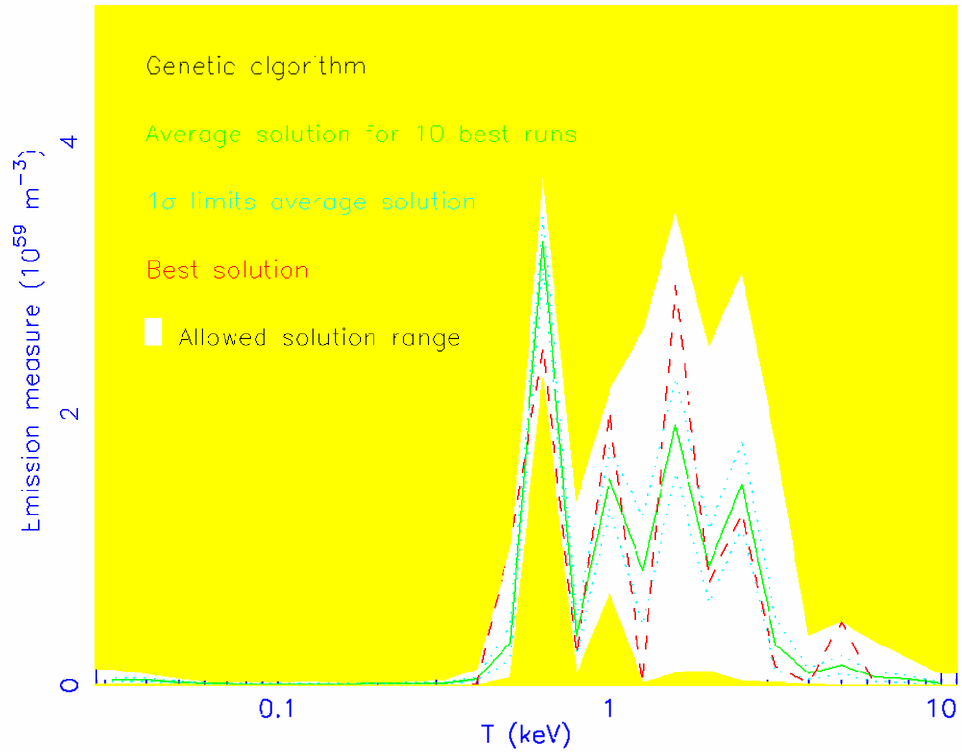
Genetic algorithm

- Uses geneitic algorithm (PIKAIA, Charbonneau 1995) to find minimum solution
- Example: results of 25 runs
- Sort solutions according to χ^2
- Pick out best solutions



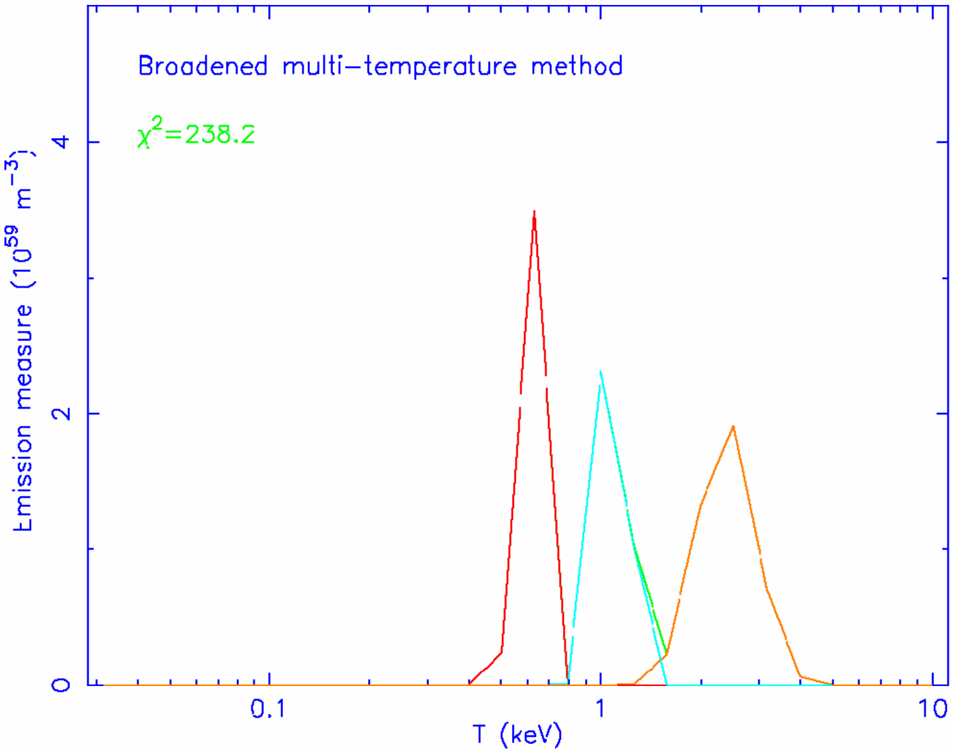
Genetic algoriothms II

- Doing multiple runs, you get an idea of allowed range of solutions



Multi-temperature method

- Alternative: use multiple Gaussian components (in log T)
- Works good for bimodal distributions
- Not always convergence, check the χ^2 (as for any DEM method)



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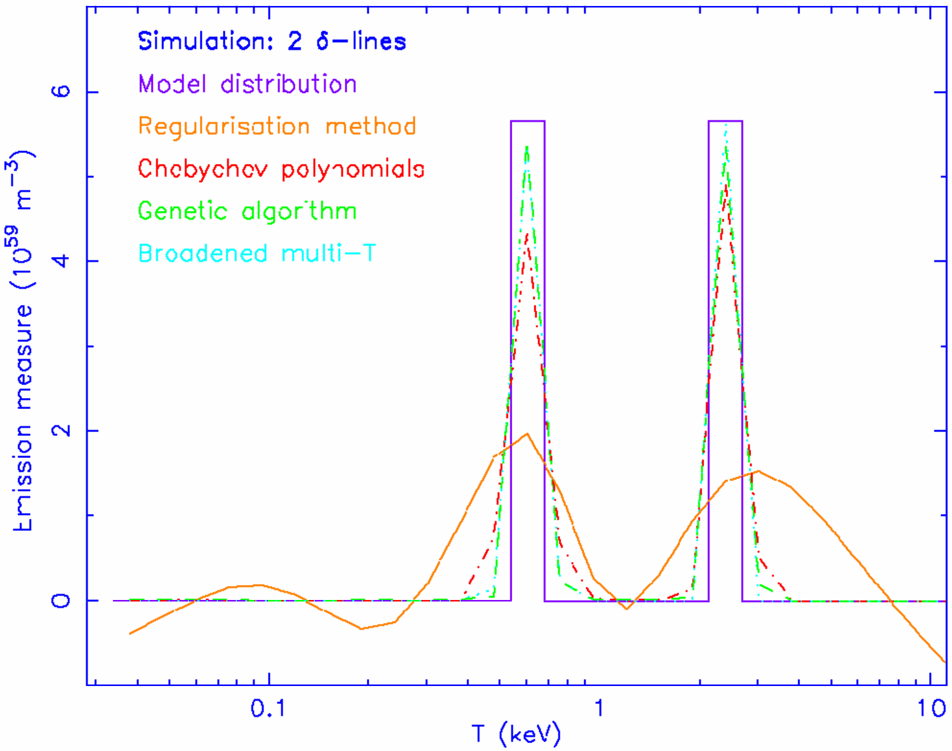
Comparison DEM methods I

- Input model: two delta-lines @ 0.6 & 2.4 keV
- See how different methods perform



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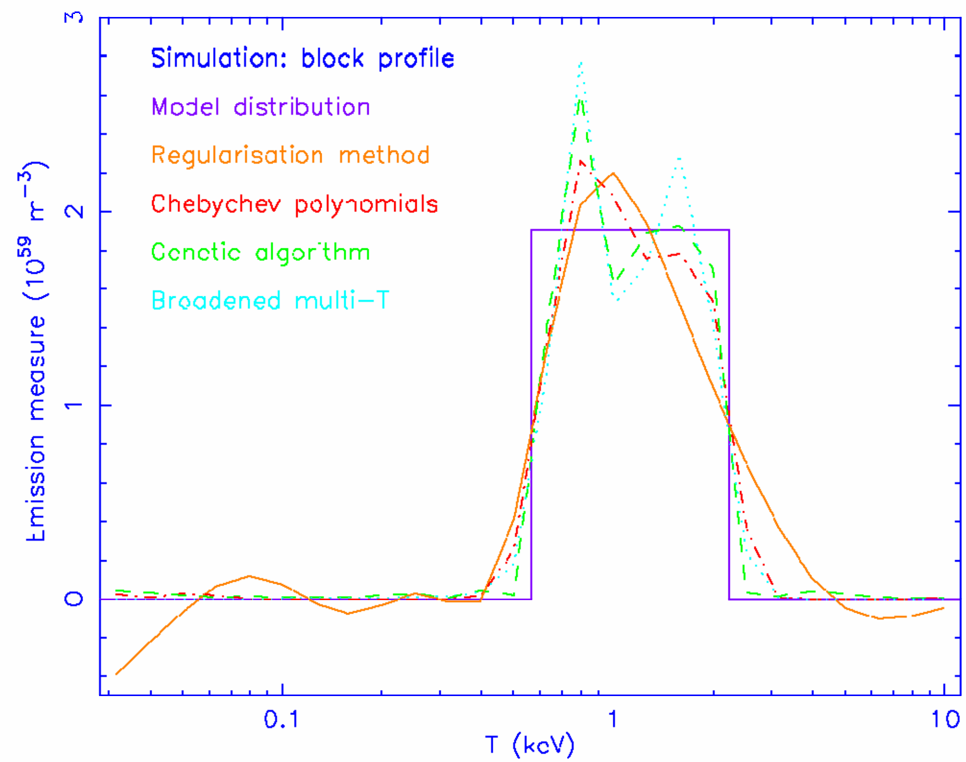
Comparison DEM methods II

- Input model continuous distribution between 0.6 @ 2.4 keV
- See how different methods perform



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3. Extended sources

- Two main challenges:
- Multi-temperature structure
- For grating spectra, additional spectral broadening due to way gratings work, usually $\Delta\lambda = c\,\Delta\theta$ ➔ degradation of resolution, depending upon spatial distribution photons along dispersion axis

WDEM model

We also try to model the emission of the hot plasma with a differential emission measure (DEM) model with a cut-off power-law distribution of emission measures versus temperature (*wdem*). The *wdem* model appears to be a good empirical approximation for the spectrum in cooling cores of clusters of galaxies (e.g. Kaastra et al. 2004; Werner et al. 2006; de Plaa et al. 2006). The emission measure $Y = \int n_e n_H dV$ (where n_e and n_H are the electron and proton densities, V is the volume of the source) in the *wdem* model is specified in Eq. (1) adapted from Kaastra et al. (2004):

$$\frac{dY}{dT} = \begin{cases} AT^{1/\alpha} & T_{\min} < T < T_{\max}, \\ 0 & \text{elsewhere.} \end{cases} \quad (1)$$

The emission measure distribution has a cut-off at $T_{\min} = cT_{\max}$. For $\alpha \to \infty$ we obtain a flat emission measure distribution. The emission measure weighted mean temperature T_{mean} is given by:

$$T_{\text{mean}} = \frac{\int_{T_{\min}}^{T_{\max}} \frac{dY}{dT} T \, dT}{\int_{T_{\min}}^{T_{\max}} \frac{dY}{dT} dT}. \quad (2)$$

By integrating this equation between $T_{\min}$ and $T_{\max}$ we obtain a direct relation between T_{mean} and $T_{\max}$ as a function of α and c:

$$T_{\text{mean}} = \frac{(1 + 1/\alpha)(1 - c^{1/\alpha+2})}{(2 + 1/\alpha)(1 - c^{1/\alpha+1})} T_{\max}. \quad (3)$$

A comparison of the *wdem* model with the classical cooling-flow model can be found in de Plaa et al. (2005). We note

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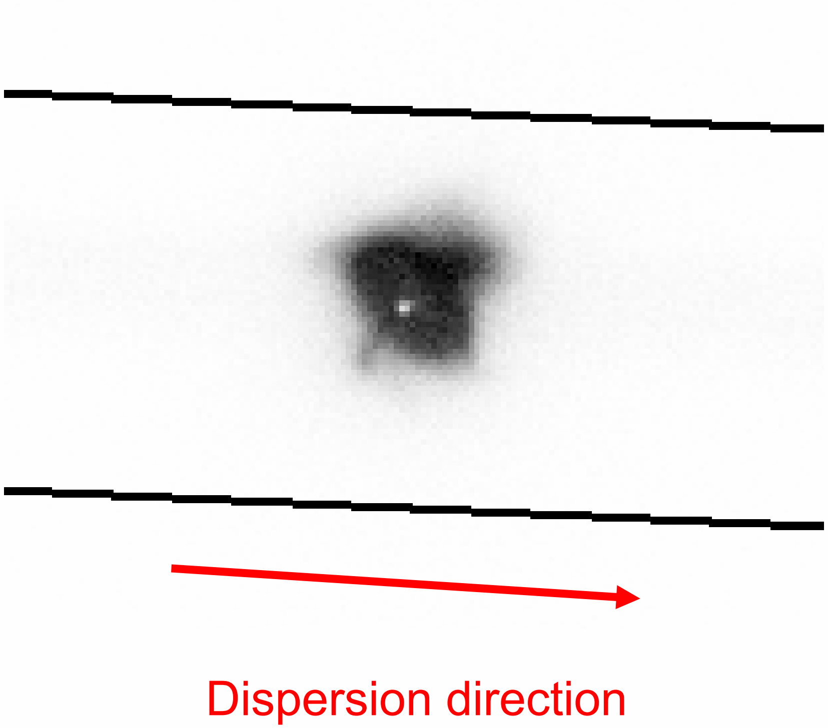
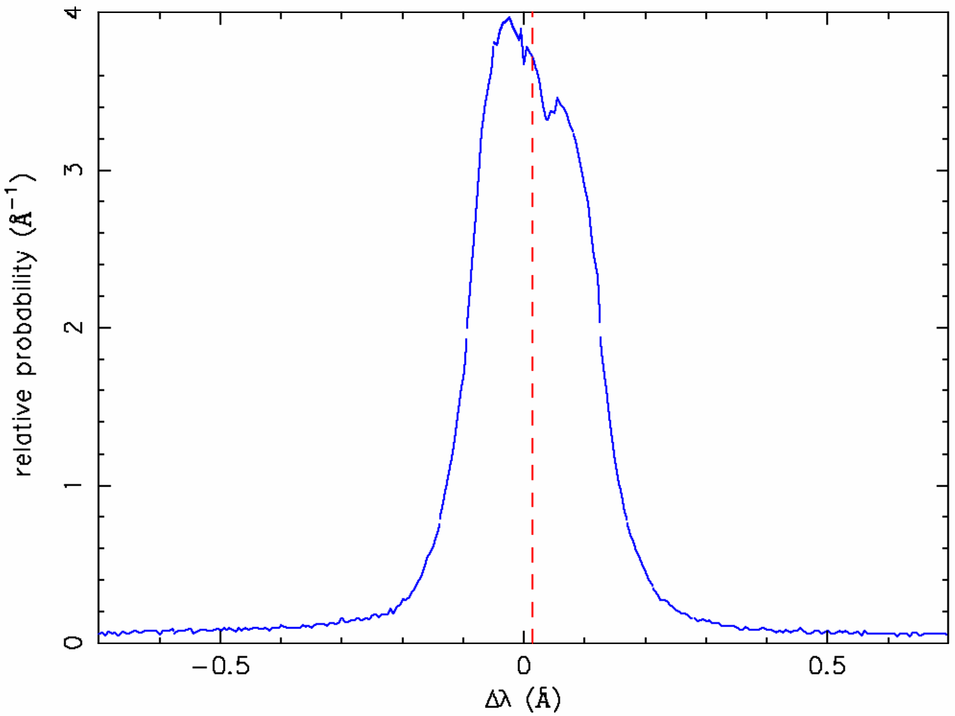
Extended source example: Crab

- Shown is MOS1 image
- Image collapsed in cross-dispersion direction and projected onto dispersion axis RGS



Effective line-spread function

- Plot shows spatial profile projected onto dispersion axis



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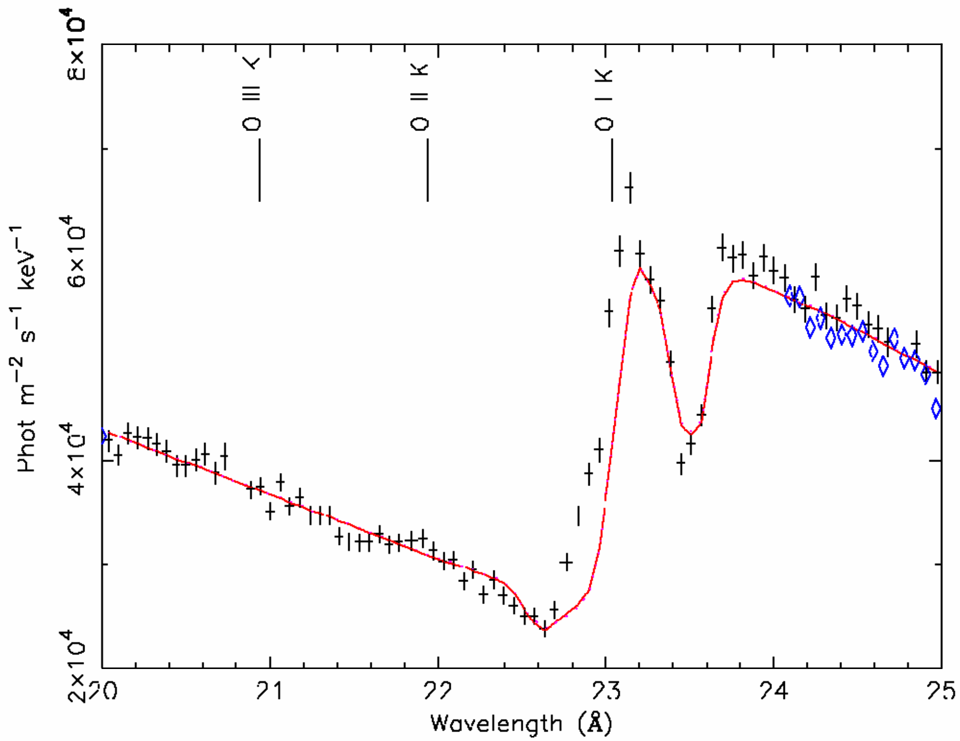
How is this implemented in SPEX?

- Provide an ascii file with two columns: cumulative probability distribution (between 0 and 1) versus λ
- Define the "lpro" component, and point there to the proper filename
- Apply the lpro component to any additive component in model that is broadened
- You can have multiple lpro components, with different files

Example: Fit to RGS Crab spectrum

- The O I 1s-2p line @ 23.5 Å (& full spectrum) clearly broadened by spatial extent Crab
- Why is fit not perfect? (answer: dust)



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4. Some other models in SPEX

- Emission models
- Convolution models

Other emission models in SPEX

- Cf: classical cooling flow model
- Spln: spline (continuum)
- File: table model
- Refl: reflection model Zycki
- SNR models (Sedov, Band, Chevalier, Solinger)
- Rrc: radiative recombination continua
- etc

Convolution models

- Laor profiles (relativistic lines)
- Gaussian broadening (vgau)
- Rectangular broadening (vblo)
- Arbitrary velocity broadening (vpro)

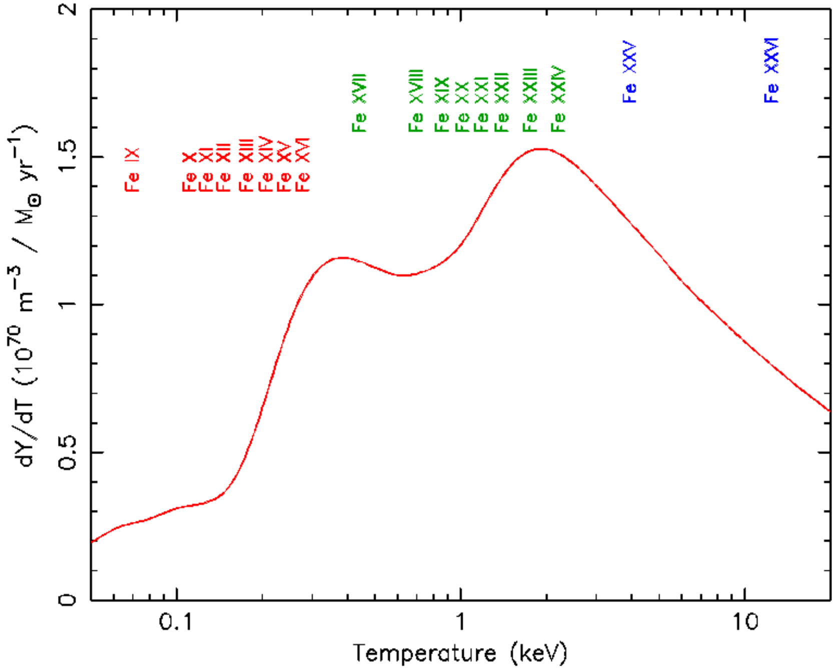
Other goodies

- Sectors & regions: see SPEX manual

Cooling flows

- Isobaric cooling gas:

$$\frac{dY}{dT} = \frac{5k\dot{M}}{2\mu m_H \Lambda(T)}$$



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