Free-Form Surface Description in Multiple Scales: Extension to Incomplete Surfaces

Nasser Khalili, Farzin Mokhtarian and Peter Yuen

Centre for Vision, Speech, and Signal Processing
Department of Electronic and Electrical Engineering
University of Surrey, Guildford, England GU2 5XH, UK

Tel: +44 1483 876035,
Fax: +44-1483-259554,
Email: N.Khalili@ee.surrey.ac.uk
Web: *http://www.ee.surrey.ac.uk/Research/VSSP/demos/css3d/index.html*

Abstract

A novel technique for multi-scale smoothing of a free-form 3-D surface is presented. Diffusion of the surface is achieved through convolutions of local parametrisations of the surface with a 2-D Gaussian filter. Our method for local parametrisation makes use of semigeodesic coordinates as a natural and efficient way of sampling the local surface shape. The smoothing eliminates the surface noise together with high curvature regions such as sharp edges, therefore, sharp corners become rounded as the object is smoothed iteratively. During smoothing some surfaces can become very thin locally. Application of decimation followed by refinement removes very small/ thin triangles and segments those surfaces into parts which are then smoothed separately. Furthermore, surfaces with holes and surfaces that are not simply connected do not pose any problems. Our method is also more efficient than those techniques since 2-D rather than 3-D convolutions are employed. It is also argued that the proposed technique is preferable to *volumetric smoothing* or *level set methods* since it is applicable to incomplete surface data which occurs during occlusion. Our technique was applied to closed as well as open 3-D surfaces and the results are presented here.

1 Introduction

This paper introduces a new technique for multi-scale shape description of free-form 3-D surfaces represented by polygonal or triangular meshes. Although there are several methods available to model a surface, triangular meshes are the simplest and most effective form of polygons to cover a free-form surface. The common types of polygonal meshes include the triangular

F. Solina and A. Leonardis (Eds.): CAIP'99, LNCS 1689, pp. 293-300, 1999.

mesh and the four sided spline patches [5]. Triangular meshes have been utilised in our work. The multi-scale technique proposed here can be considered a generalisation of earlier multi-scale representation theories proposed for 2-D contours [12] and space curves [9].

In our approach, diffusion of the surface is achieved through convolutions of local parametrisations of the surface with a 2-D Gaussian filter. *Semigeodesic coordinates* [4] are utilised as a natural and efficient way of locally parametrising surface shape. The most important advantage of our method is that unlike other diffusion techniques such as volumetric diffusion [7] or level set methods [16], it has *local support* and is therefore applicable to partial data corresponding to surface-segments. This property makes it suitable for object recognition applications in presence of occlusions. The organisation of this paper is as follows. Section 2 gives a brief overview of previous work on 3-D object representations including the disadvantage(s) of each method. Section 3 covers implementation issues encountered when adapting semigeodesic coordinates coordinates to 3-D triangular meshes. Section 4 presents diffusion results and discussion. Section 5 contains the concluding remarks.

2 Literature Survey

This section presents a survey of previous work in representation of 3-D surfaces. Comprehensive surveys of 3-D object recognition systems are presented by Besl and Jain [1], *Generalised cones* or *cylinders* [19] approximate a 3-D object using globally parametrised mathematical models, but they are not applicable to detailed free-form objects. A form of 3-D surface smoothing has been carried out in [23] but this method has drawbacks since it is based on weighted averaging using neighbouring vertices and is therefore dependent on the underlying triangulation. In *volumetric diffusion* [7] or *level set methods* [16], an object is treated as a filled area or volume. The major shortcoming of these approaches is lack of local support. In other words, the entire object data must be available. This problem makes them unsuitable for object recognition in presence of occlusion.

3 Semigeodesic Parametrisation

Free-form 3-D surfaces are complex hence, no global coordinate system exists on these surfaces which could yield a natural parametrisation of that surface. Studies of local properties of 3-D surfaces are carried out in differential geometry using local coordinate systems called *curvilinear coordinates* or *Gaussian coordinates* [4]. Each system of curvilinear coordinates is introduced on a patch of a regular surface referred to as a *simple sheet.* A simple sheet of a surface is obtained from a rectangle by stretching, squeezing, and bending but without tearing or gluing together. Given a parametric representation

$\mathbf{r} = \mathbf{r}(u,v)$ on a local patch, the values of the parameters u and v determine the position of each point on that patch. Construction and implementation of semigeodesic coordinates in our technique is described in [10].

3.1 Construction of Semigeodesic Coordinates

A geodesic line is defined as a line, which represents the shortest distance between two given points on a 3-D surface. Semigeodesic coordinates are constructed at each vertex of the mesh which becomes the local origin. The following procedure is employed:

1. Construct a geodesic from the origin in an arbitrary direction such as the direction of one of the incident edges.
2. Construct the other half of that geodesic by extending it through the origin in the reverse direction.
3. Parametrise that geodesic by the arclength parameter at regular intervals to obtain a sequence of sample points.
4. At each sample point on the first geodesic, construct a perpendicular geodesic and extend it in both directions.
5. Parametrise each of the geodesics constructed in the previous step by the arclength parameter at regular intervals.

Due to the displacement of vertices which occurs as a result of smoothing, very small and/or very thin triangles can be generated during smoothing. These odd triangles can cause computational problems and are therefore removed or merged with neighbouring triangles using known existing algorithms for mesh decimation and refinement [6]. Detection of these triangles is based on the length of the shortest side or the smallest angle. When the smallest side or the smallest angle of a triangle is less than a small threshold, that triangle is removed by merging it with neighbouring triangles. Decimation and refinement are applied after each iteration to simplify the mesh. As a result, the number of triangles gradually decreases during smoothing. It is also possible for a surface to become very thin locally as a result of smoothing. When this happens, smoothing can not continue without segmentation of the surface into parts. Such a segmentation also occurs as a result of mesh decimation and refinement since the thinned area of the surface always consists of very small and thin triangles. Smoothing can then continue after segmentation with each part of the object smoothed independently.

3.2 Semigeodesic coordinates on open surfaces

The algorithm described above should be modified to make it also applicable to open surfaces. The algorithm for smoothing an open surface is defined in the following way:

- Grid construction and smoothing at internal vertices is carried out as on closed surfaces. Any geodesic line that reaches the boundary will stop. The last sample point at or near the boundary will be duplicated until the grid is filled. Likewise, if some geodesic lines can not be constructed, the last geodesic line near the boundary will be duplicated until the grid is filled.
- If the vertex V of triangle T resides on the boundary, measure the angle α between the two edges of T that are incident on V. Choose the first geodesic line as the bisector of α. Only half of the first geodesic line is constructed because the other half falls outside the surface boundary.
- At the same vertex, construct another geodesic line perpendicular to the first one.
- One of those geodesic lines might soon intersect the boundary, so compare the lengths of those lines and choose the longer one. This allows the maximum size grid to be constructed.
- Construct the second family of geodesic lines as perpendicular to the longer geodesic line determined above.
- As before, any geodesic line that reaches the boundary will stop, and the last sample point at or near the boundary will be duplicated until the grid is filled.

3.3 Gaussian Smoothing of a 3-D surface

The procedures outlined above can be followed to construct semigeodesic coordinates at every point of a 3-D surface $\mathcal{S}$. In case of semigeodesic coordinates, local parametrisation yields at each point P:

$$\mathbf{r}(u,v) = (x(u,v),\ y(u,v),\ z(u,v)).$$

The new location of point P is given by:

$$\mathbf{R}(u,v,\sigma) = (\mathcal{X}(u,v,\sigma),\ \mathcal{Y}(u,v,\sigma),\ \mathcal{Z}(u,v,\sigma))$$

where

$$\mathcal{X}(u,v,\sigma) = x(u,v) \otimes G(u,v,\sigma)$$
$$\mathcal{Y}(u,v,\sigma) = y(u,v) \otimes G(u,v,\sigma)$$
$$\mathcal{Z}(u,v,\sigma) = z(u,v) \otimes G(u,v,\sigma)$$

and

$$G(u,v,\sigma) = \frac{1}{2\pi\sigma^2}\ e^{-\frac{(u^2+v^2)}{2\sigma^2}}$$

$\otimes$ denotes convolution. This process is repeated at each point of $\mathcal{S}$ and the new point positions after filtering define the smoothed surface. Since the coordinates constructed are valid locally, the Gaussian filters have $\sigma = 1$. In order to achieve multi-scale descriptions of a 3-D surface $\mathcal{S}$, the smoothed surface is then considered as the input to the next stage of smoothing. This procedure is then iterated many times to obtain multi-scale descriptions of $\mathcal{S}$.

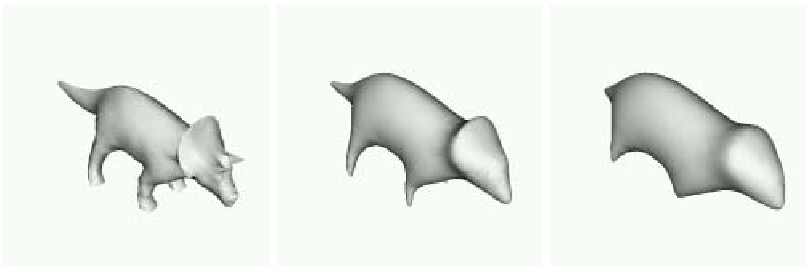
4 Results and Discussion

The smoothing routines were implemented entirely in C++ and complete triangulated models of 3-D objects used for our experiments are constructed at our canter [5]. In order to experiment with our techniques, 3-D objects with different numbers of triangles were used. Triangular meshes have been utilised in our work. The first test object was a dinosaur with 2996 triangles and 1500 vertices as shown in Figure 1. The object becomes smoother gradually and the legs, tail and ears are removed after 10 iterations. The second test object was a cow with 3348 triangles and 1676 vertices as shown in Figure 2. The surface noise is eliminated iteratively with the object becoming smoother gradually where after 12 iterations the legs, ears and tail are removed. Figure 3 shows the third test object which was a telephone handset with 11124 triangles and 5564 vertices. Notice that the surface noise is eliminated iteratively with the object becoming smoother gradually and after 15 iterations the object becomes very thin in the middle. Decimation and refinement then removes the thin handset and segments the object into two parts. Smoothing then continues for each part as shown in Figure 3.

Our smoothing technique was also applied to a number of open/incomplete surfaces. Figure 4 shows the results obtained on a part of the telephone handset. This object also has a triangle removed in order to generate an internal hole. Figure 5 shows smoothing results obtained on a partial rabbit. The object is smoothed iteratively and the ears disappear as well. These examples show that our technique is effective in eliminating surface noise as well as removing surface detail. The result is gradual simplification of object shape. Animation of surface diffusion can be observed at the web site: *http://www.ee.surrey.ac.uk/Research/VSSP/demos/css3d/index.html*

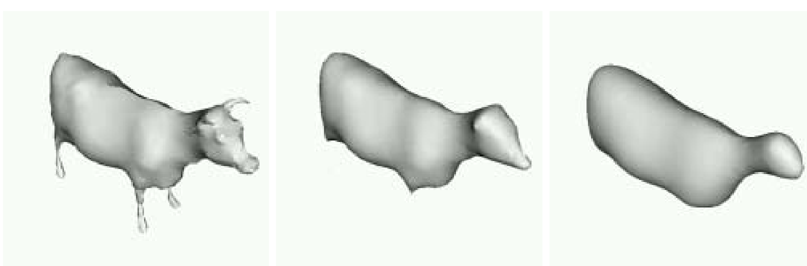
5 Conclusions

A novel technique for multi-scale smoothing of a free-form triangulated 3-D surface was presented. This was achieved by convolving local parametrisations of the surface with 2-D Gaussian filters iteratively. Our method for local parametrisation made use of semigeodesic coordinates as natural and efficient ways of sampling the local surface shape. The smoothing eliminated the surface noise and small surface detail gradually, and resulted in gradual



(a) Original **(b) 5 iterations** **(c) 10 iterations**

Figure 1: Smoothing of the dinosaur



(a) Original **(b) 3 iterations** **(c) 12 iterations**

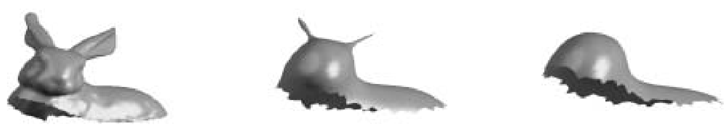
Figure 2: Smoothing of the cow

(a) Original **(b) 15 iterations** **(c) 25 iterations**

Figure 3: Diffusion of the telephone handset

(a) Original **(b) 3 iterations** **(c) 5 iterations**

Figure 4: Diffusion of the partial telephone handset



(a) Original **(b) 9 iterations** **(c) 15 iterations**

Figure 5: Smoothing of the rabbit

simplification of object shape. The method was independent of the underlying triangulation. During smoothing some surfaces can become very thin locally. Application of decimation followed by refinement removes very small/thin triangles and segments those surfaces into parts which are then smoothed separately. Our approach is preferable to *volumetric smoothing* or *level set methods* since it is applicable to incomplete surface data which occurs during occlusion. Finally, surfaces with holes and surfaces that are not simply connected do not pose any problems. Our approach is preferable to *volumetric smoothing* or *level set methods* since it is applicable to incomplete surface data which occurs during occlusion.

References

[1] P J Besl and R C Jain. Three dimentional object recognition. *ACM Computing Surveys*, 17:75–145, 1985.

[2] T W Chen and W C Lin. A neural network approach to csg-based 3-d object recognition. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 16(7):719–726, 1994.

[3] R T Chin and C R Dyer. Model-based recognition in robot vision. In *ACM Computing Surveys*, volume 18, pages 67–108, 1986.

[4] A Goetz. *Introduction to differential geometry*. Addison-Wesley, Reading, MA, 1970.

[5] A Hilton, A J Stoddart, J Illingworth, and T Windeatt. Marching triangles: Range image fusion for complex object modelling. In *Proc IEEE International Conference on Image Processing*, pages 381–384, Lausanne, Switzerland, 1996.

[6] H Hoppe. Progressive meshes. In *Proc SIGGRAPH*, pages 99–106, 1996.

[7] J J Koenderink. *Solid shape.* MIT Press, Cambridge, MA, 1990.

[8] A K Mackworth and F Mokhtarian. Scale-based description of planar curves. In *Proc Canadian Society for Computational Studies of Intelligence*, pages 114–119, London, Ontario, 1984.

[9] F Mokhtarian. A theory of multi-scale, torsion-based shape representation for space curves. *Computer Vision and Image Understanding*, 68(1):1–17, 1997.

[10] F Mokhtarian, N Khalili, and P Yuen. Multi-scale 3-d free-form surface smoothing. In *Proc British Machine Vision Conference*, pages 730–739, 1998.

[11] F Mokhtarian and A K Mackworth. Scale-based description and recognition of planar curves and two-dimensional shapes. *IEEE Trans Pattern Analysis and Machine Intelligence*, 8(1):34–43, 1986.

[12] F Mokhtarian and A K Mackworth. A theory of multi-scale, curvature-based shape representation for planar curves. *IEEE Trans Pattern Analysis and Machine Intelligence*, 14(8):789–805, 1992.

[13] M Pilu and R Fisher. Recognition of geons by parametric deformable contour models. In *Proc European Conference on Computer Vision*, pages 71–82, Cambridge, UK, 1996.

[14] H Samet. The design and analysis of spatial data structures. *Addison-Wesley*, 1990.

[15] M Seibert and A M Waxman. Adaptive 3-d object recognition from multiple views. In *IEEE Trans Pattern Analysis and Machine Intelligence*, volume 14, pages 107–124, 1992.

[16] J A Sethian. *Level set methods.* Cambridge University Press, 1996.

[17] S S Sinha and R Jain. Range image analysis. In *Handbook of Pattern Recognition and Image Processing: Computer Vision (T Y Young, ed.)*, volume 2, pages 185–237, 1994.

[18] F Solina and R Bajcsy. recovery of parametric models from range images: Thee case for superquadrics with global deformations. *IEEE Trans. on Pattern Analysis and Machine intelligence*, 12:131–147, 1990.

[19] B I Soroka and R K Bajcsy. Generalized cylinders from serial sections. In *Proc IJCPR*, 1976.

[20] A J Stoddart and M Baker. Reconstruction of smooth surfaces with arbitrary topology adaptive splines. In *Proc ECCV*, 1998.

[21] P Suetens, P Fua, and A J Hanson. Computational strategies for object recognition. *ACM Computing Surveys*, 24(2):5–61, 1992.

[22] G Taubin. Curve and surface smoothing without shrinkage. In *Proc ICCV*, pages 852–857, 1995.

[23] G Taubin. Optimal surface smoothing as filter design. In *Proc ECCV*, 1996.